RADIO SOURCES IN CLUSTERS OF GALAXIES
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I. Historical Perspective

Mills (1960) and van den Bergh (1961) noticed nearly two decades ago that clusters of galaxies tend to contain radio sources. Both studies were based upon the positional coincidences between Abell clusters and sources from radio surveys. From the 85 MHz Sydney survey, Mills found that 5% of the Sydney radio sources lie in the directions of Abell clusters whereas only 1.4% are expected by chance. Similarly, van den Bergh found, using the Cambridge 3C catalog, that 10% of the radio sources were coincident with Abell clusters whereas only 3% are expected random foreground/background objects.

Pilkington (1964) and Wills (1966) extended the coincidence study of the Sydney and Cambridge catalogs including the then new data from the 4C survey. Both investigations revealed that cluster radio sources tend to congregate near the centers of Abell clusters (i.e. within 0.3 cluster radii); in addition the probability of radio emission from a cluster appeared to be proportional to its richness. Matthews, Morgan, and Schmidt (1964) further concluded that the probability of radio emission from a cluster which contains a cD galaxy is especially high.

Much of the early work on cluster radio sources was limited to coincidence studies from surveys with low angular resolution. Very little information was available on source structure. As a result, the nature of the cluster sources was essentially unknown until the development of radio interferometry in the mid-sixties. Radio telescopes with better pointing and spatial resolution meant more secure optical identifications of cluster sources. Fomalont and Pogstad (1966) examined 111 Abell clusters with distance class 3 or less using the Caltech Owens Valley interferometer at 1445 MHz. This survey detected 48 of the clusters down to a limiting flux density of 200 mJy. The limited
structure data seemed to indicate that these cluster radio sources were complex. A weak correlation between the number of coincidences and richness was noted.

The correlation of cluster radio emission with the number of member galaxies was further investigated by Rogstad and Ekers (1969) using a control sample. They observed 200 E and SO galaxies with the Owens Valley interferometer. They found that these types of galaxies in the "field" were as likely to be radio emitters as those in clusters. Their data suggest that the large number of detected radio sources in clusters may be due to the preponderance of E and SO galaxies in rich clusters.

The constantly improving sensitivity of aperture synthesis instruments produced detections of many weak sources and extended low-level emission features connected with previously known survey sources. Possibly the most significant discovery concerning cluster radio sources was made by Ryle and Windram (1968) using the Cambridge one-mile telescope at 408 and 1407 MHz. Up to this time crude structure information indicated that the vast majority of extended radio sources were composed of two lobes of emission which lie at the extremities of the source and are placed on either side of the optical identification (i.e. classical doubles). Ryle and Windram's high resolution radio maps of two galaxies in the Perseus cluster, NGC 1265 and IC310, revealed spatial brightness distributions which peaked on the optical galaxies and trailed away approximately exponentially ending about 10 arcminutes from the galaxies. Such configurations were labeled "head-tail" radio galaxies. Interestingly, the two tails in the Perseus cluster pointed away from an active Seyfert galaxy in the cluster, NGC 1275. Ryle and Windram speculated that a wind of relativistic particles may be generated by NGC 1275; this wind may blow past the two radio galaxies interacting with their gaseous component and igniting
non-thermal radio emission.

Several major problems exist with this relativistic particle stream model (e.g., Miley et al., 1972). First, the energy production rate required for NGC 1275 is an order of magnitude greater than what is presently observed. Second, the detailed process involved with the interaction between the particle stream and the galaxy remained unexplained. Third, it is curious that galaxies closer to NGC 1265 are not head-tail galaxies. Explanations which attribute the lack of radio emission to "less significant gaseous components" seem a little ad hoc and contrived. But the most damaging piece of evidence for this model came with the discovery of a third head-tail galaxy in the Perseus cluster. Miley, Perola, van der Kruit, and van der Laan (1972) found a head-tail galaxy whose tail pointed toward NGC 1275 on a map made with the Westerbork array at 1415 MHz.

The Leiden group (Miley et al., 1972) proposed an alternative model for the head-tail sources. They suggested that these galaxies undergo periodic explosions in the nucleus which result in the ejection of two oppositely directed radio plasmas (i.e., magnetic fields and relativistic particles in a thermal gas) reminiscent of double sources. The direction of ejection of these radio components lies at a large angle (∼90°) with respect to the motion of the galaxies through a dense (∼5x10^4 cm^-³) intracluster medium. The dynamic pressure slows the radio components with respect to the galaxy, resulting in the formation of two tails emanating from the optical galaxy. The high velocity difference of NGC 1265 from the cluster mean, 2000 to 3000 km/sec, seems to provide the necessary dynamic pressure. These observations present the first well documented evidence that a dense environment in clusters of galaxies may play an important role in shaping the extended structure of radio sources.
A rapid succession of discoveries of cluster sources with distorted features followed the publication of Ryle and Windram's paper. Hill and Longair (1971) mapped a pair of head-tail radio galaxies, 3C 129 and 3C 129.1, with the Cambridge interferometer. Although these sources lie in a region of heavy galactic obscuration, Hill and Longair speculated that both galaxies lie within a cluster and may, therefore, be subject to dynamic pressure from a dense IGM similar to NGC 1265. Riley (1973) further observed these sources at 2.7 and 5 GHz with the Cambridge one-mile telescope and noted that there was a marked increase in the spectral index, $\alpha$, along the tails. This is what one might expect from a distribution of relativistic electrons ejected from the radio galaxy nucleus and "aging" through synchrotron losses.

Further observations of NGC 1265 and 3C 129 by Miley (1973) and Miley, Wellington, and van der Laan (1975) revealed the increasing complexity of head-tail galaxies at higher resolutions. The dual frequency total intensity observations with the Westerbork array confirmed the steepening of the synchrotron spectrum down the tail. Polarization data indicate, furthermore, that the fractional polarization, $P^*$, also increases rapidly down the tail reaching close to 60% in 3C 129. This result combined with the high degree of alignment of the polarization vectors seem to indicate that the magnetic field in the tails are very uniform and lie parallel to the tail.

Until 1973 the analysis of cluster radio sources was confined primarily to a few strong cataloged radio galaxies which coincided with nearby clusters. No systematic investigations of the types and structures of cluster radio sources had been undertaken. Owen (1974, 1975) began a series of observations of over 500 Abell clusters of galaxies using the NRAO 300-ft telescope at 1.4 GHz. Clusters were searched out to 0.4 A for radio emission with $S \geq 100$ mJy; 127 clusters were detected.

$$P^* = \frac{\sqrt{Q^2 + U^2}}{I}$$  where $I, Q,$ and $U$ are Stokes parameters.
From the 300-ft statistical investigation, Owen found that the probability of radio emission in an Abell cluster correlates with the degree of dominance of the brightest galaxies (as suspected earlier by Matthews, Morgan, and Schmidt (1964)). In particular, those clusters of Rood-Sastry type cD, B, C or L or BM class I to II-III are prone to have radio emission. Furthermore, Owen noted that over the range of richness classes 0 to 3, there is very little difference in the probability of radio emission. Although it is clear that the number density of cluster radio sources is considerably greater than the background, any correlation between the richness of clusters within the Abell catalog and radio emission is small for the most radio-luminous clusters.

Owen (1975) speculated on the basis of the appearance of his 300-ft cluster luminosity function that cluster radio emission could arise from several (averaging about five) individual radio galaxies of differing brightness rather than one cluster-wide source. An NRAO interferometer survey by Owen and Rudnick (1976a, 1976b), Rudnick and Owen (1976, 1977), and Owen, Rudnick, and Peterson (1977) of Abell clusters with $S > 200$ mJy, $\theta > 20'\text{ }$, and within 0.3 A of the cluster center seemed to indicate that this is the case. A relatively large number (57) of distorted double sources and head-tail radio galaxies were mapped with the Green Bank interferometer at 2.7 and 8 GHz. Examples of the various types of radio sources found in the Owen-Rudnick survey is shown in Figure 1. It became increasingly obvious that such distorted structure may be the rule rather than the exception for sources in rich clusters. The nature of these sources seems to confirm earlier speculations that the cluster IGM plays a significant role in determining the morphology of these extended features.

Among the results of their survey of cluster source structures, Owen and Rudnick found that only 5% (corresponding to two sources) of
MORPHOLOGICAL TYPES OF RADIO SOURCES IN RICH CLUSTERS

CLASSICAL DOUBLES

WIDE-ANGLE HEAD-TAILS

NARROW-ANGLE HEAD-TAILS
sample show the classical double structure which is prevalent among the strong sources of the 3C catalog. The other sources show some degree of distortion; (2) radio sources are associated with dominant galaxies but also are associated with galaxies of average brightness in clusters which do not contain prominent galaxies. The structure of radio sources associated with Owen-Rudnick Type 1 galaxies most often appear to be wide-angle tails or small doubles with size of about 50 kpc. This structure may result from the expectedly slower motion of these central galaxies with respect to the IGM. Narrow-angle head-tail sources all appear to be associated with Types 2 or 3 galaxies in clusters. The galaxy position within the cluster and the shape of the tail indicate a rapid motion through the IGM; (3) the tails of the narrow-angle head-tail galaxies do not have preferred directions with respect to the cluster centers. This seems to imply that there is no tendency for the gas in the IGM to flow outward (via a wind as suggested by Yahil and Ostriker, 1973) or inward (by collapse as suggested by Gunn and Gott, 1972) in clusters which contain these sources; (4) There is an overall increase in luminosity from head-tail radio galaxies to classical doubles with the classical doubles being the most luminous. (Fanaroff and Riley, 1974) previously had found that 3C sources which are classical doubles generally appear more luminous than "complex" sources.) The reason for this hierarchy in source brightness may arise from the different IGM densities surrounding the various types of sources.

Similar investigations of the properties of cluster radio sources were performed using data collected by the Cambridge and Westerbork arrays. Guthrie (1974), McHardy (1974), Tovmasyan and Shirkanyan (1974) and Lari and Perola (1977) noted once again that the strongest radio sources are most often associated with clusters that contain dominant galaxies, in particular RM class I clusters. Lari and Perola concluded from their
sample of Bologna sources which coincide with rich clusters that although the fraction of radio emitting first ranked galaxies is not a function of richness, the number of other sources not associated with the brightest cluster galaxies seems to increase proportional to the cluster richness.

A number of studies (e.g. see reviews by van der Laan (1977) and Harris (1977)) have also revealed that the distribution of integrated spectral indices in cluster radio sources is broader than for sources not associated with clusters. Baldwin and Scott (1973), Slingo (1974), and Riley (1975) have found that 3C and 4C sources which lie in Abell clusters tend to have steeper spectral indices than the average of sources in the catalogs. A similar preponderance of steep spectrum sources from the 408 MHz Bologna survey which coincide with clusters was noted by Rcland, Veron, Pauliny-Toth, Preuss, and Witzel (1977). One explanation for this association is that the dense IGM in clusters strongly confines the source; synchrotron losses dominate rather than losses from adiabatic expansion as in the case of a source surrounded by little or no medium. More recently Lea and Holman (1978) have suggested that steep-spectrum radio sources in clusters heat the intergalactic medium to temperatures necessary to produce the observed x-ray emission. This, rather than the confinement of the radio sources, may explain the correlation between extragalactic x-ray sources (see below), radio sources with large and their association with clusters.

Studies of the morphological types of cluster sources mapped with the Westerbork interferometer (e.g. Lari and Perola, 1977; Vallee and Wilson, 1976; Colla et al., 1975; Wilson and Vallee, 1977; Valentijn and Percla, 1977) produced conclusions which agreed quite well with those of Owen and Rudnick. Less than 20 of the cluster sources observed at high resolution could be classified as double sources. The majority of the
remaining sources show more complex or head-tail morphology. The relative absence of head-tail and distorted structure in sources associated with field galaxies seems to demonstrate that a dense IGM cluster environment is necessary for the formation and maintenence of these extended features.

Recently, several head-tail galaxies have been discovered in environments which contain closer associations of galaxies. Schilizzi and Ekers (1975) have mapped a head-tail galaxy, NGC 7385, which lies in the Zwicky cluster 2247.3+1107. Fomalont and Pridle (1978) recently discovered a wide-angle head-tail galaxy (about 4' in extent) which is in a very poor group of galaxies not cataloged by either Abell or Zwicky. Ekers, Fanti, Lari, and Ulrich (1978) have mapped a head-tail, 1615+35, which lies within a poor Zwicky cluster. Interestingly, 1615+35 is also near two Abell clusters, 2199 and 2162, which possess similar redshifts and velocity dispersions possibly forming a supercluster of galaxies. It is conceivable, then, that the overall supercluster medium density and velocity dispersion are the important quantities which have produced the observed radio structure rather than the more local environment. As radio observations of poor clusters and groups continue to grow, considerations of the effects on the radio structure due to the overall supercluster may become increasingly important.

II. THE ROLE OF THE IGM IN CLUSTERS

If the local neighborhood around radio galaxies were completely devoid of gas and dust, one would expect the associated extended radio sources to be simple single or double spheres. Such sources would have linear synchrotron spectra and would be short lived because of the rapid adiabatic expansions into the surrounding vacuum. However, observations of sources in clusters of galaxies (see e.g. Figure 1.9) reveal
structures which significantly deviate from a spherical geometry. Many cluster double sources have leading edges which are flattened and compressed as well as trails of steep-spectrum emission leading back to the optical identification; both are signs of confinement by a dense medium. The very existence of head-tail radio galaxies in clusters argues strongly in favor of the presence of an intracluster gas. There appears, then, to be a definite coupling between cluster radio source structure and motion through a dense IGM. The study of either one separately requires some knowledge of the other. With the expanding data from the direct x-ray observations of cluster IGMs and the radio structure maps from a variety of different richness class clusters, a coherent picture of the production and evolution of radio sources and the IGM is beginning to emerge.

III. Direct observations of the Intracluster Medium

The deviation of extended source structure from the classical double morphology is fairly convincing, although circumstantial, evidence for the existence of an intergalactic medium in clusters. Recently, direct detections of cluster-wide IGMs have been reported at both radio and x-ray frequencies. Since such independent observations bear heavily upon the construction of a complete picture of extended radio sources, it is appropriate to briefly review these experiments.

A. Diminution of the 3 K Microwave Background

Sunyaev and Zel'dovich (1972) postulated that a dense ICM might be detected through the scattering or "cooling" of the 3 K microwave background as the radiation travels through the clusters. The microwave
photons are "cooled" through the inverse compton scattering by a hot intergalactic gas in the clusters. The effect, then, is a depletion in the microwave background in the directions of rich clusters. Gull and Northover (1976) and Lake and Partridge (1977) have attempted to detect such microwave diminutions at the Rayleigh-Taylor radio portion of the electromagnetic spectrum (3 cm and 9 mm, respectively). Lake and Partridge have particularly high level detections of three richness class 4 Abell clusters. Using an adiabatic model with $T=10^6$ K and a cluster "core" radius of 250 kpc, Lake and Partridge find an intracluster medium density of about $10^3$ to $10^2$ cm$^{-3}$ in the three clusters. Rudnick (1978) in a similar experiment at 2 cm has placed upper limits on the microwave diminution for several nearby clusters.

B. Cluster x-ray emission

The Uhuru "all-sky" survey at x-ray wavelengths suggested that a large number of strong extragalactic sources are associated with clusters of galaxies (e.g. Gursky et al., 1972; Kellogg et al., 1973), as suspected earlier from rocket flights. These x-ray clusters range in richness from nearby loose groups (e.g. Virgo) to dense clusters (e.g. Abell 2256). The emission may emanate from individual active galaxies (e.g. M87 and NGC 1275, Wolff et al., 1974) or extend spatially to follow the galaxy density contours of the cluster (e.g. Perseus). The extended emission (e.g. Kellogg and Murray, 1974) is of particular interest since current theories contend that it arises from an intracluster medium.

The models for the x-ray emission can be divided into two categories, inverse Compton scattering or thermal bremsstrahlung, both of which in some cases fit the low energy (i.e. 1 to 10 keV) spectrums fairly well. In the case of the inverse Compton model, the x-ray
radiation is produced by the scattering of 3 K background photons to higher energies by relativistic electrons in the IGM (e.g. Brecher and Burbidge, 1972). This model, at first, seemed attractive since it would provide a natural explanation for the observed correlation between x-ray and radio emission in clusters (Cwen, 1974; Bahcall, 1974). The thermal bremsstrahlung model assumes that the x-ray radiation originates from a hot (10^5 K) intergalactic gas that permeates the cluster. The heating of the IGM may be produced, for example, by random motions of cluster galaxies (e.g. Schipper, 1974; Lea and De Young, 1976) or by relativistic electrons which diffuse out from cluster radio galaxies (Lea and Holman, 1978).

More recent observations with rockets (e.g. Davidson et al., 1975) and the OSO-7 (e.g. Ulmer, Paity, and Peterson, 1973), Ariel V (Cooke et al., 1978) and OSO-8 satellites strongly point to a thermal bremsstrahlung interpretation of cluster x-ray emission. The key to fitting the appropriate model lies in the energy spectrum. In the hard x-ray portion of the spectrum (~10 keV), the theoretical thermal and Compton scattering curves substantially deviate from their close agreement at lower energies. The observed hard x-ray cluster spectra seem to agree with the predictions of the thermal model in almost all cases. Present data, however, does not yet allow a distinction between isothermal IGM and non-static (e.g. hydrostatic models of Lea, 1975) and Rephaeli (1977) models. But probably the most important spectral evidence substantiating a thermal interpretation was the discovery of an x-ray line feature apparently due to highly ionized iron in the Coma and Perseus clusters (Mitchell et al., 1976; Serlemitsos et al., 1976). The densities and temperatures necessary to produce this feature (i.e. T=10^8 K and n=10^3 cm^-3) clearly indicate that a hot, dense gas containing processed material exists between the galaxies in clusters (e.g. Bahcall
and Sarazin, 1977). The origin of the abundant heavy elements in the IGM pose an interesting question concerning stellar mass loss and galaxy evolution (e.g., De Young, 1977).

A number of interesting correlations have been noted between x-ray emission and cluster radio and optical properties. McHardy (1978), Bahcall (1974, 1977a, 1977c) and Owen (1974) have each pointed out that the x-ray luminosity of clusters increases with the degree of dominance of one or several bright galaxies (cE, B, or L clusters). Numerous authors (e.g., Rudnick and Owen, 1977; Lari and Perola, 1977), as was mentioned in the previous section, have found that these same types of clusters with optically dominate galaxies are also strong radio emitters. These correlations may indicate that a dense gaseous environment near the center of clusters results in the formation of giant E and D galaxies, and possibly trigger explosions which produce the strong radio sources as well.

In addition to the dominance correlation, McHardy (1978) finds from Ariel V data that the probability of cluster emission depends upon the cluster richness. He concludes that the x-ray luminosity, $L_X$, of rich clusters increases monotonically with richness. Similarly, Bahcall (1977a) discovered a correlation between x-ray luminosity and the central density of galaxies in clusters. She attributes this to thermal bremsstrahlung from a hot IGM whose density is proportional to the galaxy density. This may also explain the weak proportionality between $L_X$ and the cluster velocity dispersion (Solinger and Tucker, 1972; Silk, 1977; McHardy, 1978) since the virial theorem velocity dispersion is proportional to the square-root of the galaxy number density.

Both Bahcall (1977b) and McHardy (1978) have noted that the presence of a dense gas in clusters may have a direct effect upon the optical galaxy morphology. As the x-ray luminosity increases, the fraction of
spiral galaxies in the cluster is observed to decrease. This may be attributable to the stripping of the intergalactic gas from spiral galaxies by the ram pressure of the surrounding intracluster medium, a process that was found viable in the calculations of Lea and De Young (1976).

Radio observations of x-ray clusters of galaxies at decametric frequencies (Baldwin and Scott, 1973; Erickson, Matthews, and Viner, 1978) have revealed an interesting relationship between $L_x$ and the the low-frequency integrated cluster emission. $L_x$ is found to increase with the spectral index and to a lesser extent with the radio power, $P_R$. It has been suggested that the confinement of the radio source by a surrounding hot IGM will halt the rapid expansion so that energy losses steepen the spectrum. Lea and Holman (1978) have noted, however, that such an explanation may be invalid for many observed cluster radio sources which are larger than the apparent gas distribution. They suggest instead that an interaction occurs between the non-thermal electrons which escape from cluster radio galaxies and the IGM. Lea and Holman conclude that the IGM can be heated by these relativistic electrons and this process results in the observed correlation between the radio and x-ray emission.

Several authors have suggested that x-ray sources may also be associated with superclusters of galaxies (e.g. Maccagni et al., 1978; Murray et al., 1978; Kellcqq, 1978; Forman et al., 1978). It is conceivable that the x-ray emission may emanate from an intracluster gas which is distributed throughout the supercluster. Although recent data cannot rule out this possibility, McHardy (1978) and Ricketts (1978) feel that, statistically, the x-ray sources are most likely identified with individual clusters within the supercluster. Higher resolution observations with the HEAO-P satellite may clarify this situation.
With the independent knowledge of the existence of a hot dense intracluster medium from x-ray data, one can draw some interesting conclusions concerning radio source and general cluster structure. For example, information on IGM temperatures and the spatial variation in cluster IGM density are of great use in determining the degree of confinement of the radio source and therefore the source energetics. This data will then provide more complete tests of theoretical models and may stimulate the production of new models as well. On the other hand, radio source structure maps and models of distant poorer clusters may provide predictions of x-ray emission which could be confirmed by future more sensitive x-ray satellites such as the HEAO series (e.g. Gursky and Schwartz, 1977). Through this type of iterative process with improving radio and x-ray instrumentation, convergence upon a more complete picture of cluster evolution is rapidly approaching.
IV. SYNCHROTRON RADIATION

An indepth expose on the theory of extragalactic synchrotron emission, from which much of the following is taken, can be found in Pacholczyk (1970, 1977).

If we consider an optically thin plasma containing relativistic electrons with a power-law energy distribution

\[ N(E) \, dE = N_0 E^{-\gamma} \, dE, \]  

spiraling in a magnetic field, \( B \), the spectral flux density of synchrotron emission observed is given by

\[ S_\nu = N_0 R^3 B_\perp (\gamma+1)/2 \nu^{(1-\gamma)/2}, \]  

where \( R \) = radius of the emission region, \( B_\perp \) = magnetic field perpendicular to the line of sight, and \( \nu \) = frequency. Thus the observed spectrum is a power law with

\[ S_\nu \propto \nu^{-\alpha}, \]  

where \( \alpha \equiv (\gamma-1)/2 \) = spectral index.

From the initial distribution defined by (1) at some starting time \( t_0 \), further changes in the distribution function can occur from both electron sinks \( p(E, t) \) and sources \( q(E, t) \). These changes can, for most extended extragalactic sources, be determined by the equation of continuity. For a uniform and isotropic distribution, \( N(E, t) \), the continuity equation is
\[
\frac{\partial N(E,t)}{\partial t} + \nabla E \cdot \left[ N(E,t) \frac{\partial E}{\partial t} \right] = q(E,t) - p(E,t). \tag{5}
\]

If we assume that \( p(E,t) = 0 \) and that \( \frac{\partial E}{\partial t} = \phi(E) \), the total loss rate can be written as

\[
\phi(E) = \zeta - nE - \xi E^2, \tag{6}
\]

where \( \zeta = \) ionization losses, \( nE = \) free-free losses, and \( \xi E^2 = \) synchrotron + inverse Compton losses. For the radio source conditions which we will consider, only the last term in equation (6) is important or

\[
\frac{\partial E}{\partial t} = -\xi_E = - (\xi_s + \xi_c)E^2. \tag{7}
\]

Synchrotron losses are given by

\[
\xi_s = 2.37 \times 10^{-3} B \text{ G}^2 \tag{8}
\]

and inverse Compton losses by

\[
\xi_c = 3.97 \times 10^{-2} u_{\text{rad}}, \tag{9}
\]

where \( u_{\text{rad}} \) is the local radiation density.

Now, in the simplest case of a single injection of energetic particles (e.g., one particle accelerating explosion in a radio galaxy which produces two lobes of a classical double source), the initial electron energy distribution would be given by

\[
N(E,0)dE = \begin{cases} 
N_0 E^{-\gamma} dE & \text{for } E_1 \leq E \leq E_2 \\
0 & \text{for } E < E_1 \text{ and } E > E_2 
\end{cases} \tag{10}
\]

Using (5) and (7), this distribution at some time \( t \) later becomes
\begin{equation}
N(E,t)\,dE = \begin{cases} 
\frac{N \cdot E^{-\gamma}}{(1-\xi Et)^{2-\gamma}} & E_1' \leq E \leq E_2' \\
0 & E < E_1' \text{ and } E > E_2'
\end{cases}, \quad (11)
\end{equation}

where $E' = E/(1+\xi Et)$. The distribution, shown schematically below, is characterized by the presence of a cutoff energy, $E_T = \frac{1}{\xi t}$, above which there are no electrons; even if the energy ranges extends to infinity, an electron will have its energy reduced to the value $1/\xi t$ within some finite time.

![Schematic diagram of electron energy distribution](image)

The effect of this energy distribution as a function of time upon the observed synchrotron flux density is shown below. As the source "ages", the spectral index between two frequencies, $\nu_1$ and $\nu_2$, will increase; the observed spectral index in this case is defined as

\begin{equation}
\alpha = \frac{\log (S_{\nu_1}/S_{\nu_2})}{\log (\nu_2/\nu_1)} . \quad (12)
\end{equation}

At some future time, synchrotron losses alone will cause the source flux density to drop below the detection limit of the radio telescope. The spectral index, then, can be used as a measure of the current state of energy of relativistic particles and $B$-fields in a source.
Implicit in the above illustration is the relationship between the energy of an electron, $E$, and the characteristic frequency at which it radiates. From synchrotron theory, this relationship is given by

$$v_s = 6.27 \times 10^{18} B \sqrt{E^2}.$$  \hspace{1cm} (13)

As a lower limit to the typical lifetime of an ensemble of relativistic electrons at an observed frequency $v$, equations (7) and (13) can be combined to produce

$$t_s \sim \frac{E}{dE/dt} = 3 \times 10^4 B^{-3/2} v^{-1/2} \text{ years.} \hspace{1cm} (14)$$

If one has an estimate of the $B$-field in a source, then equation (14) can be used to obtain an approximate value of the source lifetime (and velocity, as well). This information will provide further physical insight into such processes as source confinement and electron reacceleration. One standard method for estimating $B$ and the total source energy density is described below.

V. EQUIPARTITION CALCULATION OF THE MINIMUM ENERGIES AND MAGNETIC FIELDS IN RADIO SOURCES

From the results of synchrotron radiation theory, it is possible to obtain estimates of the two physical parameters which dominate the emission
process: (1) the minimum magnetic field plus relativistic particle energy, 
\[ E_T = E_B + (E_e + E_p) \], and (2) the strength of the magnetic field. With the assumptions outlined below, we can compute these quantities in terms of observables (the source luminosity and total linear extent).

First of all, the total energy of the relativistic electrons, assuming a power-law energy distribution \( N(E) = N_0 E^{-\gamma} \), between \( E_1 \) and \( E_2 \), is given by

\[ E_e = \int_{E_1}^{E_2} E N(E) dE = N_0 \int_{E_1}^{E_2} E^{-\gamma+1} dE. \]  

The quantity \( N_0 \) can be determined from the total energy or luminosity of the radio source (see, e.g., Pacholczyk 1970) since

\[ L = - \int_{E_1}^{E_2} \frac{dE}{dt} N(E) dE = N_0 C_2 B_{\perp}^2 \int_{E_1}^{E_2} E^{-\gamma+2} dE, \]

where \( B_{\perp} \) = the magnetic field perpendicular to the line of sight and \( C_2 = \frac{\lambda e^4}{3 m_e c^6} = 2.37 \times 10^{-3} \). Placing equation (A2) into equation (A1), the expression for the electron energy becomes

\[ E_e = C_2^{-1} L B_{\perp}^{-2} \left( \frac{\gamma-3}{\gamma-2} \right) \frac{E_1^{-\gamma+2} - E_2^{-\gamma+2}}{E_1^{-\gamma+2} - E_2^{-\gamma+3}}. \]

Synchrotron theory provides a conversion between the energy of an electron and the characteristic frequency in the form

\[ \nu_s = C_b E_{\perp}^2, \]

where \( C_b = \frac{3}{4\pi m_e c^5} \). Using the above formula, equation (17) can be rewritten as
\[ E_e = C_{12}^{(\alpha_1, \nu_1, \nu_2)} B_{\perp}^{-3/2} L, \]  

where \( C_{12} = C_2^{-1} C_1^{1/2} \left(2 \alpha - 1\right) \left(1 - 2 \alpha\right) \left(1 - 2 \alpha\right) / \left(1 - \nu_1\right) \) and \( \alpha = (\gamma - 1)/2 \) is the spectral index.

Since the observable synchrotron emission is independent of the protons in the source (and therefore we know nothing about the proton energy), we must make an assumption about the proton energy in order to estimate the total particle energy \( E_p + E_e \). Assume that the ratio of proton to electron energies is \( k \) so that the particle energy becomes \( E_p (1+k) \). The quantity \( k \) can range from 1, if the acceleration of the electrons is by matter-antimatter annihilation, up to \( \sim 2000 \), for an induction-type acceleration mechanism. Typically, \( k \) is thought to be \( \sim 100 \) which will be the case for electrons and positrons formed by the collisions of a proton flux with the dust and gas of a radio source.

Next, the magnetic field contribution to the total source energy is

\[ E_B = \phi \frac{B^2}{8\pi} V, \]  

where \( \phi \) is the fraction of the source's geometrical volume, \( V \), which is occupied by magnetic field and relativistic particles. If we allow \( B_{\perp} \ll B \), then the total energy in the radio source is

\[ E_T = E_B + (E_p + E_e) \]

\[ = \phi \frac{B^2}{8\pi} V + (1+k) C_{12} B^{-3/2} L. \]
The minimum value of the total energy \( \frac{3E}{\partial B} = 0 \) occurs when the particle and field energies are approximately equivalent or

\[
E_B = \frac{3}{4} (1+k) E_e. \tag{21}
\]

This is the assumption of equipartition of energy. The minimum value of the total source energy is

\[
(E_T)_{\text{min}} = \frac{7}{4} (1+k) E_e = \frac{7}{4} (1+k) C_{12} B^{-3/2} L. \tag{22}
\]

The corresponding value of the magnetic field is

\[
B_{\text{min}} = (6\pi)^{2/7} (1+k)^{2/7} C_{12}^{2/7} \phi^{-2/7} V^{-2/7} L^{2/7}. \tag{23}
\]

Substituting (21) into (22), the expression for the total minimum energy becomes

\[
(E_T)_{\text{min}} = \frac{7}{4} (6\pi)^{-3/7} C_{12}^{4/7} (1+k)^{4/7} \phi^{3/7} V^{3/7} L^{4/7}. \tag{24}
\]

Assuming that the distance to the radio source is known, estimates of the field strength and particle energies can be obtained under the assumption of equipartition of energy for a given source volume, luminosity, and spectral index (between two frequencies of observation).
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