Summer Student Lecture Series.

Galen Gisler

Physical Cosmology and The Early Universe.

Modern Cosmology starts with the Copernican Principle: the earth is not at the center of the universe, nor does it occupy any other special location. Our locale is typical of the universe as a whole.

The usual working hypothesis is the stronger Cosmological Principle: the universe as a whole is homogeneous and isotropic everywhere. There is no center, and all viewpoints are essentially equivalent. $\exists$ Cosmological standard coordinates, origin taken conveniently at the observer's position, which are called "comoving" coordinates, and are the same at all times for any given object which is at rest with respect to all other objects in its vicinity.

Under the Cosmological Principle, the line element used for measuring intervals in space and/or time is given by the Robertson-Walker metric:

$$d\tau^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right\}$$

τ is proper time or distance (for "time-like" or "space-like" intervals respectively), $\Delta\tau = 0$ is propagation time of light signals

r, θ, ϕ — comoving coordinates

t — cosmic standard time

$R(t)$ — an arbitrary function of t, sometimes called "the radius of the universe" or "the cosmic scale factor"

k — constant, + if r is chosen suitably, $k = \left(\begin{smallmatrix}+1\\0\\-1\end{smallmatrix}\right)$ curvature

The Cosmological Red Shift

We are at $r = 0$, a distant galaxy at r_1 (arbitrary θ and ϕ)

Light wave equation of motion

$$d\tau^2 = 0 = dt^2 - R^2(t)\,\frac{dr^2}{1-kr^2}$$

Light which leaves r_1 at time t_1 arrives at $r=0$ at time t_0

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \sin^{-1} r_1 \\ r_1 \\ \sinh^{-1} r_1 \end{cases} \equiv f(r_1)$$

Next wave crest leaves at $t_1 + \delta t_1$, arrives $t_0 + \delta t_0$

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{R(t)} = f(r_1) \quad \text{same as above}$$

$R(t)$ is constant over light period, so

$$\frac{\delta t_0}{R(t_0)} = \frac{\delta t_1}{R(t_1)}$$

Since $\lambda_0 = \frac{1}{\nu_0} = \delta t_0$, $\quad \frac{\lambda_0}{\lambda_1} = \frac{R(t_0)}{R(t_1)}$

Define the "redshift" $\quad z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{R(t_0)}{R(t)} - 1$

If the universe is expanding, so $R(t_0) > R(t_1)$, then z is a redshift

contracting $\rightarrow$ blue shift.

But these shifts result from the change in scale of the universe, and are not, properly speaking, velocity (Doppler) shifts. At small distances, however, they are identical to Doppler shifts.

The proper distance measured between our galaxy and one at r_1 at the same cosmic time (this requires an acausal conspiracy!) is, from the metric $\quad d = R(t_0) \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} \simeq R(t_0)\, r_1 \quad$ for r_1 small

$$v_{rad} = \dot{d} \simeq \dot{R}(t_0)\, r_1$$

For $t_0 - t_1$ small $\int_{t_1}^{t_0} \frac{dt}{R(t)} \simeq \frac{t_0 - t_1}{R(t_0)} \simeq r_1$

$$z = \frac{R(t_0)}{R(t_1)} - 1$$

expand $R(t_1) \simeq R(t_0) - \dot{R}(t_0)(t_0 - t_1)$

then $z \simeq \frac{\dot{R}(t_0)(t_0 - t_1)}{R(t_0)}$

$\simeq \dot{R}(t_0)\, r_1 \simeq v_{rad}$

But cosmological z's $[(t_0 - t_1)$ large$]$ do not represent velocities!

Distance measures:

1. Absolute luminosity compared to apparent luminosity
2. True diameter compared to angular diameter
3. Parallax
4. Proper motion

For distances greater than ~10^9 light years, these all differ from each other

1. Luminosity distance:

Fraction of all isotropically emitted photons that strike a telescope mirror of diameter A is

$$\frac{A}{4\pi R^2(t_0) r_1^2}$$

- each photon has energy $h\nu_1$, redshifted to $h\nu_1\left(\frac{R(t_1)}{R(t_0)}\right)$
- Time interval δt_1 between emitted photons dilated to $\delta t_1 \left(\frac{R(t_0)}{R(t_1)}\right)$

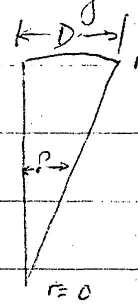
So Power received is

$$P = L\left(\frac{R^2(t_1)}{R^2(t_0)}\right)\left(\frac{A}{4\pi R^2(t_0) r_1^2}\right)$$

$$l = \frac{P}{A} = \frac{L R^2(t_1)}{4\pi R^4(t_0) r_1^2}$$

as in Euclidean geometry, define $d_L \equiv \left(\frac{L}{4\pi l}\right)^{1/2} = R^2(t_0)\frac{r_1}{R(t_1)}$

2. Angular diameter distance



Proper distance across source is $D = R(t_1) r_1 \delta$

In Euclidean geometry, $\delta = \frac{D}{d}$, so define

$$d_A \equiv \frac{D}{\delta} = R(t_1) r_1$$

Since $R(t_1)$ decreases as r_1 increases, some models can have a maximum d_A

3. Proper motion distance

Source with transverse velocity $V_\perp$, moves a distance ΔD in a time (local to us) Δt_0:

$$\Delta D = V_1 \Delta t_1 = V_1 \Delta t_0 \frac{R(t_1)}{R(t_0)}$$

source will appear to move an angular distance $\Delta \delta = \frac{\Delta D}{R(t_1) r_1}$
$= \frac{V_1 \Delta t_0}{R(t_0) r_1}$

in Euclidean space $\Delta \delta = \frac{V_\perp \Delta t_0}{d}$.

so define proper motion distance $d_M \equiv \frac{V_\perp \Delta t_0}{\Delta \delta} = R(t_0) r_1$

(need apriori knowledge of $V_\perp$!)

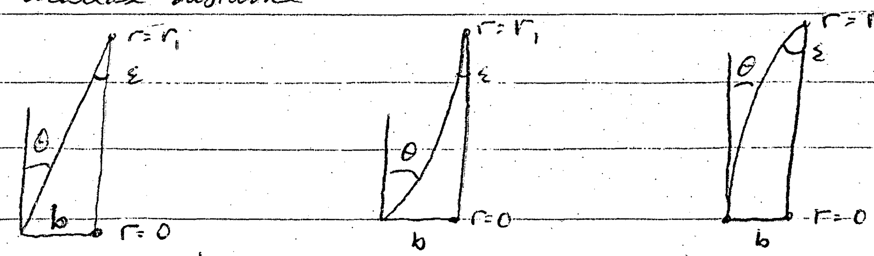
$$d_L = R^2(t_0) \frac{r_1}{R(t_1)} \qquad \frac{d_A}{d_L} = (1+z)^{-2}$$
$$d_A = R(t_1) r_1$$
$$d_M = R(t_0) r_1 \qquad \frac{d_M}{d_L} = (1+z)^{-1}$$

no new info

II

can check R.W metric, or cosmological origin of z

4. Parallax distance



$b = R(t_0) r_1 |\varepsilon|$
proper distance locally

$$\theta \simeq (1-kr^2)^{1/2} \varepsilon$$

In Euclidean geometry, $\theta \simeq \frac{b}{d}$

so define $d_P \equiv \frac{b}{\theta} = R(t_0) \frac{r_1}{(1-kr^2)^{1/2}}$

∴ measurement of d_P can give info on curvature!

Except require 10^{-9} sec resolution on 1AU baseline; earth-moon optical interferometer doesn't quite make it.
Also require point sources 10^{-9} sec in size at 10^9 l.y. away - unlikely.

Observational Cosmology

Red-shift versus distance relation

use $d_L = \left(\frac{L}{4\pi l}\right)^{1/2}$ (uncertainties in L !!)

$$d_L = r_1 \frac{R^2(t_0)}{R(t_1)} = r_1 R(t_0)(1+z)$$

$$1+z = \frac{R(t_0)}{R(t_1)} \qquad \text{also need } \int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_1} \frac{1}{\sqrt{1-kr^2}} dr = r_1 + O(r_1^3)$$

$d_L(z)$ is known only for small z, so we expand $R(t)$ in a Taylor series:

$$R(t) = R(t_0) + \dot{R}(t_0)(t-t_0) + \tfrac{1}{2}\ddot{R}(t_0)(t-t_0)^2 + \cdots$$

define $H_0 \equiv \frac{\dot{R}(t_0)}{R(t_0)}$

$$q_0 \equiv -\ddot{R}(t_0)\frac{R(t_0)}{\dot{R}^2(t_0)}$$

(just kinematics, no dynamics)

Then $R(t) = R(t_0)\left[1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0^2) + \cdots\right]$

{in fact, if Einstein's field equations are assumed, the whole $R(t)$ follows from just present values of H_0 and q_0.}

$$z = \frac{R(t_0)}{R(t_1)} - 1 = H_0(t_0 - t_1) + (1 + \tfrac{1}{2}q_0) H_0^2 (t_0 - t_1)^2 + \cdots$$

inverting, we have

$$(t_0 - t_1) = \frac{1}{H_0}\left[z - \left(1 + \frac{q_0}{2}\right) z^2 + \cdots\right]$$

To find r_1, we use

$$\int_{t_1}^{t_0} \frac{dt}{R(t_0)} = r_1 + O(r_1^3)$$

$$r_1 = \frac{1}{R(t_0)} \int_{t_1}^{t_0} \left[1 + H_0(t_0 - t) + (1 + \tfrac{1}{2}q_0) H_0^2 (t_0 - t)^2 + \cdots\right] dt$$

$$r_1 = \frac{1}{R(t_0)} \left[(t_0 - t_1) + \tfrac{1}{2} H_0 (t_0 - t_1)^2 + \cdots\right]$$

in terms of z,

$$r_1 = \frac{1}{R(t_0) H_0}\left[z - \tfrac{1}{2}(1 + q_0) z^2 + \cdots\right]$$

then $d_L = r_1 R(t_0)(1+z) = \frac{1}{H_0}\left[z + \frac{1}{2}(1 - q_0) z^2 + \cdots\right]$

$$m - M = 5 \log d_L - 5 = 5 \log_{10} z + \frac{5}{2 \ln 10}(1 - q_0) z - 5 \log_{10} H_0 - 5$$

Note that measurements of d_L and z cannot determine $R(t)$

m versus z should then yield H_0 and q_0, but lots of problems

(1) clusters of galaxies have large velocity dispersions
(2) aperture effects
(3) redshift effects ("k" term)
(4) absorption within our galaxy
(5) uncertainty in M due to poor distance calibration
(6) Scott Effect – Malmquist bias
(7) Local anisotropies
(8) galactic evolution $L(t)$

Einstein Field Equations

$$3\ddot{R} = -4\pi G(\rho + 3p)R \qquad \text{(not needed)}$$
$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2$$

energy conservation
$$\frac{d}{dR}(\rho R^3) = -3pR^2$$

equation of state
$$p = p(\rho)$$

if pressure is negligible, $\rho \propto R^{-3}$

if pressure is $p = \rho/3$, $\rho \propto R^{-4}$

E.F. equations plus energy conservation + equation of state ~~give~~ + Robertson-Walker metric give Friedmann models.

From acceleration equation:

$\frac{\ddot{R}}{R}$ is negative while $\rho + 3p$ is positive

Since $R > 0$ and $\dot{R}/R > 0$ (redshifts, not blueshifts),
$R(t)$ is concave down and must have reached 0 at some finite time in the past – singularity – call it $t=0$

~~R~~ Since $\ddot{R}$ is negative, $t_0 < H_0^{-1} = R(t_0)/\dot{R}(t_0)$

From $\dot{R}$ equation + conservation of energy: if p does not go negative, then ρ decreases with R at least as fast as R^{-3}, so that RHS of $\dot R$ equation vanishes as R^{-1} for $R \to \infty$. If $k = -1$, $\dot R^2$ is always positive definite, so $R(t)$ increases forever, with $\dot R \to 1$, so $R(t) \to t$ as $t \to \infty$

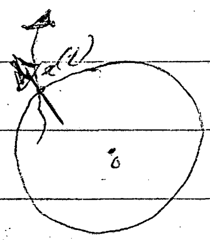
If $k = 0$, $\dot R^2$ is still positive definite, but $\dot R \to 0$ as $R \to \infty$

If $k = +1$, $\dot R^2$ is zero at $\rho R^2 = \frac{3}{8\pi G}$. Since $\ddot R$ is negative definite, $R(t)$ begins to decrease, + must reach $R = 0$ at finite time.

Newtonian derivation

Any gas particle has a trajectory

$$\vec x(t) = \vec x(t_0)\frac{R(t)}{R(t_0)}$$ Newtonian gas, uniform expansion



Gravitational potential energy is

$$V(t) = -\frac{4}{3}\pi |\vec x(t)|^3 \rho(t) \frac{mG}{|\vec x(t)|}$$

$$= -\frac{4}{3}\pi m G |x(t_0)|^2 \rho(t) \frac{R^2(t)}{R^2(t_0)}$$

Kinetic energy is

$$T(t) = \frac{1}{2} m |\dot{\vec x}(t)|^2 = \frac{1}{2} m |\vec x(t_0)|^2 \frac{\dot R^2(t)}{R^2(t_0)}$$

Total energy is $E \equiv T(t) + V(t) = \frac{1}{2} m \frac{|\vec x(t_0)|^2}{R^2(t_0)}\left[\dot R^2(t) - \frac{8\pi G}{3}\rho(t) R^2(t)\right]$

is the same as Einstein's $\dot R$ equation, provided we write $k = \frac{-2E}{m}\frac{R^2(t_0)}{|\vec x(t_0)|^2}$

then $k = 0$ means $E = 0$, so gas is just able to expand indefinitely.

Density + ~~pressure~~ of present universe

$$\rho_0 = \frac{3}{8\pi G}\left(\underbrace{\frac{k}{R_0^2}}_{\text{spatial curvature}} + H_0^2\right)$$

if $\rho = \rho_c \equiv \frac{3 H_0^2}{8\pi G}$, $k = 0$ ∴ for $H_0 = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\rho_c \simeq 1.1\times 10^{-29}\,\frac{\mathrm{gm}}{\mathrm{cm}^3} \sim 6\times10^{-6}\,\frac{m_H}{\mathrm{cm}^3}$

Pressure of present universe $p_0 = -\frac{1}{8\pi G}\left[\frac{k}{R_0^2} + H_0^2(1 - 2q_0)\right]$

If $p_0 \ll \rho_0$, then $\frac{k}{R_0^2} = (2q_0 - 1) H_0^2$
(matter-dominated universe, as our own is)

and $\rho_0 = \frac{3}{8\pi G} H_0^2 (2q_0)$

$$\frac{\rho_0}{\rho_c} = 2q_0$$

$\rho > \rho_c$, $q_0 > \frac{1}{2}$, $k = +1$, universe "closed", positively curved

$\rho < \rho_c$, $q_0 < \frac{1}{2}$, $k = -1$ universe "open", negatively curved

Calculation of $R(t)$ in the Matter-Dominated Era

Use $$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2 \qquad (1)$$

and $$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{R}{R_0}\right)^{-3} \qquad (2)$$

$$\frac{k}{R_0^2} = (2q_0 - 1) H_0^2 \qquad (3)$$

$$\frac{8\pi G \rho_0}{3} = 2 q_0 H_0^2 \qquad (4)$$

Substituting (2), (3), (4) into (1) gives

$$\left(\frac{\dot{R}}{R_0}\right)^2 = H_0^2\left[1 - 2q_0 + 2q_0 \frac{R_0}{R}\right]$$

invert and integrate
$$t = \frac{1}{H_0}\int_0^{R/R_0}\left[1 - 2q_0 + \frac{2q_0}{x}\right]^{-1/2} dx \qquad (5)$$

In particular, the age of the universe is given by
$$t_0 = \frac{1}{H_0}\int_0^1 \left[1 - 2q_0 + \frac{2q_0}{x}\right]^{-1/2} dx < \frac{1}{H_0} \text{ for } q_0 > 0$$

A. Easiest to integrate (5) when $q_0 = \frac{1}{2}$
$$t = \frac{1}{H_0}\int_0^{R/R_0}\sqrt{x}\, dx = \frac{1}{H_0}\frac{2}{3}\left(\frac{R(t)}{R_0}\right)^{3/2}$$

"Einstein-de Sitter universe" or $\frac{R(t)}{R_0} = \left(\frac{3}{2}H_0 t\right)^{2/3}$

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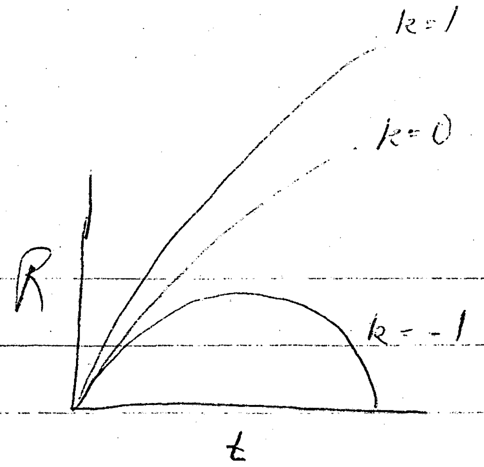

Early on, all models behave like $R(t) \propto t^{2/3}$

(Matter-Dominated, remember)

B. For other values of q_0, define θ by

$$1 - \cos\theta = \left(\frac{2q_0 - 1}{q_0}\right)\frac{R(t)}{R_0}$$

$$= \left(\frac{2q_0-1}{q_0}\right)x$$

$\boxed{\text{note: if } q_0 < \tfrac{1}{2},\ \theta \text{ is imaginary, } \theta = i\psi,\ \cos\theta = \cosh\psi}$

then: $t = \frac{1}{H_0}\int_0^{\theta_0} \frac{q_0}{(2q_0-1)^{3/2}}(1-\cos\theta)\,d\theta$ for $q_0 \geq \tfrac{1}{2}$

$$H_0 t = q_0(2q_0-1)^{-3/2}\left[\theta - \sin\theta\right]$$

∴ equation of a cycloid, maximum at $\theta = \pi$, $t_m = \frac{\pi q_0}{H_0(2q_0-1)^{3/2}}$

present instant $\cos\theta_0 = \frac{1}{q_0} - 1$

C. or $t = \frac{1}{H_0}\int_0^{\theta_0}\frac{q_0}{(1-2q_0)^{3/2}}(\cosh\psi - 1)\,d\psi$ for $q_0 < \tfrac{1}{2}$

$H_0 t = q_0(1-2q_0)^{-3/2}\left[\sinh\psi - \psi\right]$, $\cosh\psi - 1 = \frac{1-2q_0}{q_0}\frac{R(t)}{R_0}$

as $t \to \infty$, $\frac{R(t)}{R_0} \to \frac{1}{2}q_0(1-2q_0)^{-1}e^{\psi}$

$\to (1-2q_0)^{1/2}H_0 t$

present, $\cosh\psi_0 = \left(\frac{1}{q_0} - 1\right)$

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Summaries of Friedmann zero pressure models

Closed universe

$$\frac{R(t)}{R_0} = \frac{q_0}{(2q_0-1)}(1-\cos\theta)$$

$$H_0 t = \frac{q_0}{(2q_0-1)^{3/2}}(\theta - \sin\theta)$$

Open universe

$$\frac{R(t)}{R_0} = \frac{q_0}{(1-2q_0)}(\cosh\psi - 1)$$

$$H_0 t = \frac{q_0}{(1-2q_0)^{3/2}}(\sinh\psi - \psi)$$

for θ small, $\cos\theta \simeq 1 - \frac{\theta^2}{2}$

$\sin\theta \simeq \theta - \frac{\theta^3}{6}$

for ψ small $\cosh\psi \simeq 1 + \frac{\psi^2}{2}$

$\sinh\psi \simeq \psi + \frac{\psi^3}{6}$

Einstein - de Sitter universe

$$\frac{R(t)}{R_0} = \left(\frac{3H_0}{2}t\right)^{2/3}$$

give, for young matter dominated universe, all q_0,

$$\frac{R(t)}{R_0} \simeq \left[\frac{3}{2}\sqrt{2q_0}\,(H_0 t)\right]^{2/3}$$

$q(t)$ changes with time for $k \neq 0$

$k = +1$: $q = \frac{1}{1+\cos\theta}$ starts at 1/2, goes to ∞, then back to 1/2

$k = -1$: $q = \frac{1}{1+\cosh\psi}$ starts at 1/2, goes to 0

∴ So in early universe, makes no difference if space is open or closed.

The Pressure Dominated Universe

- in early universe, pressure is given by relativistic particles + photons, so $p = \frac{1}{3}\rho$ and $\left(\frac{\rho}{\rho_0}\right) = \left(\frac{R_0}{R}\right)^4$

neglecting curvature,
$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2$$
$$\dot{R}^2 = \frac{8\pi G\rho_0}{3}\left(\frac{R_0}{R}\right)^4 R^2$$
$$R\dot{R} = \left(\frac{8\pi G\rho_0}{3}\right)^{1/2} R_0^2$$

solution $R(t) = \left(\frac{32\pi G\rho_0}{3}\right)^{1/4} R_0\, t^{1/2}$

or $t = \left(\frac{3}{32\pi G\rho}\right)^{1/2}$

Microwave Background Radiation

At early times when $R(t)$ was very small, ρ large, matter + radiation were presumably in thermal equilibrium -

matter + radiation cooled as universe expanded, + finally when $T \sim 4000°K$, free electrons joined atoms, so opacity dropped sharply + matter and radiation were decoupled.

presently observed microwave background radiation, discovered 1965, is vastly redshifted image of the "surface of last scattering" at a temperature of 4000°K Present observed temperature is 2.7°K, so redshift can be deduced from the Wien displacement law

$\lambda_{max} T = \text{const}$ for blackbody rad

$$1+z = \frac{\lambda_0}{\lambda_1} = \frac{T_1}{T_0} \simeq \frac{4000}{2.7} \simeq 1500$$

The black body character of the radiation must be preserved by the expansion of the universe, if there is no interaction between matter and radiation, so

$$\lambda_{max} T = \text{constant}$$

requires

$$\frac{T}{T_0} = \frac{R_0}{R}$$

A more rigorous way of looking at this is the following: (ideal gas in equilibrium with black body radiation)

energy density of radiation $\rho_\gamma(t) = a T_\gamma^4(t)$

where $a = \frac{8\pi^5 k^4}{15 h^3 c^3}$

total pressure $p = nkT + \frac{1}{3}aT^4$

total energy density $\rho = nm + \frac{1}{(\gamma-1)} nkT + aT^4$ (m in energy units)

particle conservation $nR^3 = n_0 R_0^3$, $\therefore \frac{d}{dR}(nR^3) = 0$

energy conservation $\frac{d}{dR}(\rho R^3) = -3pR^2$

$$\frac{d}{dR}\left[\cancel{nm}R^3 + \frac{nkTR^3}{\gamma-1} + aT^4R^3\right] = -3nkTR^2 - aT^4R^2$$

get $$\frac{R}{T}\frac{dT}{dR} = \frac{-3nkR^3 - 4aT^3R^3}{\frac{nkR^3}{\gamma-1} + 4aT^3R^3}$$

define $\sigma = \frac{4aT^3}{3nk}$: the photon entropy per gas particle is σk

$$\frac{R}{T}\frac{dT}{dR} = -\left(\frac{1+\sigma}{\frac{1}{3(\gamma-1)} + \sigma}\right)$$ ~~note if $\sigma \ll 1$, get adiabatic~~ expansion of an ideal gas

in present universe, $\sigma \simeq 2.4 \times 10^8$ for $n = \frac{\rho_c}{m_H}$

so $\frac{R}{T}\frac{dT}{dR} \simeq -1$ and $T \propto R^{-1}$

As matter goes out of equilibrium with radiation, must talk of matter temps T_m and rad temp T_γ, in that case if σ is ever very large, it stays very large + tends to the present value: if $T \propto R^{-1}$ and $n \propto R^{-3}$, σ = constant

Scenario of the Early Universe

The existence of the microwave background, and the fact that $T \propto \frac{1}{R}$ holds through most of the history of the universe, enable us to study much of the physics of the early universe. The only complication is that one must keep track of the multitude of particles, both massive (nucleons, mesons, leptons) and massless (photons, neutrinos, gravitons), their interactions, and decays, when they drop out of equilibrium and begin to annihilate each other; because the equation of state of the universe depends strongly upon what particles are playing the game. Further, at temperatures greater than 10^{12} K, or $t < 10^{-4}$ sec, there were enormous numbers of strongly interacting particles with mean interparticle distances less than a typical Compton wavelength, and no equation of state (or no believable one) is available for that case.

Most discussions of the early universe begin, then, at $t > 10^{-4}$ sec. At this point most of the protons + antiprotons ~~have~~, neutrons and antineutrons have already annihilated each other, leaving a small ($\sim$ 1 part in 10^{10}) residue that will eventually become galaxies and stars and planets and people. Muons all annihilate before $T \sim 1.3 \times 10^{11}$ K, neutrinos, antineutrinos decouple, leaving only electrons, positrons, and photons in thermal equilibrium with the radiation field. Electrons and positrons annihilate each other at $\sim 5 \times 10^{9}$ °K ($t \sim 4$ sec), so that only photons and neutrinos are left in free expansion, at slightly different temperatures. ~~At~~ ~~[illegible] the remaining neutrons fused with protons~~ With no more electrons remaining, the interchange between protons and neutrons ceased, and the proton to neutron ratio was frozen at ~5:1. The neutrons remaining then fused with the protons to make helium and a few heavier nuclei, yielding about 27% He by weight. The gas remains ionized, its temperature locked to the radiation temperature until $T \sim 4000°K$. At about the same time (may be a little after, maybe a little before, depending upon ρ_0/ρ_c), the energy density of photons + neutrinos (declining as R^{-4}) drops below rest mass energy density of hydrogen and helium (declining as R^{-3}), and the universe becomes matter dominated.

As indicated before, the expansion of the pressure dominated universe is

$$R(t) \propto t^{1/2}$$
$$\text{or } t \propto \rho_\gamma^{-1/2}$$

$$\text{So } \underline{\rho_\gamma \propto t^{-2}} \text{ while } \rho_m \propto R^{-3} \propto t^{-3/2}$$

Once the universe is matter dominated,

$$R(t) \propto t^{2/3}$$
$$\rho_\gamma \propto R^{-4} \propto t^{-8/3} \text{ while } \underline{\rho_m \propto R^{-3} \propto t^{-2}}.$$

<u>Helium synthesis</u>

One of the easiest calculations which can be done in the early universe, and one which is very satisfying because a simple minded approach yields the right answer.

Two parts:

1. neutron/proton abundance ratio as a function of time
2. nuclear reactions leading to nucleosynthesis

For (1) need to watch reactions

$$\left.\begin{array}{l} n+\nu \leftrightarrow p+e^- \\ n+e^+ \leftrightarrow p+\bar{\nu} \\ n \leftrightarrow p+e^-+\bar{\nu} \end{array}\right]$$ from rate equations calculate net rates $\lambda(p \to n)$ and $\lambda(n \to p)$

define $Q \equiv m_n - m_p = 1.293\ MeV$
$\to 1.5 \times 10^{10} K$

Initially, $kT >> Q$, and neutron ratio X_n has its equilibrium value

$$X_n \simeq \frac{\lambda(p \to n)}{\lambda(p \to n) + \lambda(n \to p)}$$

These rates have the ratio (if $T_\nu \simeq T_e$)

$$\frac{\lambda(p \to n)}{\lambda(n \to p)} = \exp\left(\frac{-Q}{kT}\right)$$

So that $X_n \simeq \dfrac{1}{1+e^{Q/kT}}$ at $T \sim 10^{10}$, $X_n \sim 0.17$
use $N \sim 0.17$

$\simeq$ ½ at early times
drops slowly as temp falls.

After $T \sim 1.3 \times 10^9 K$, electrons and positrons all gone, so only reaction left is

$$\lambda^{-1}(n \to p + e^- + \bar{\nu}) = 1013\ sec$$

[neutron decay lifetime]

so after this time, neutron abundance is

$$X_n(t) = N \exp\left[\frac{-t}{1013\,sec}\right].$$

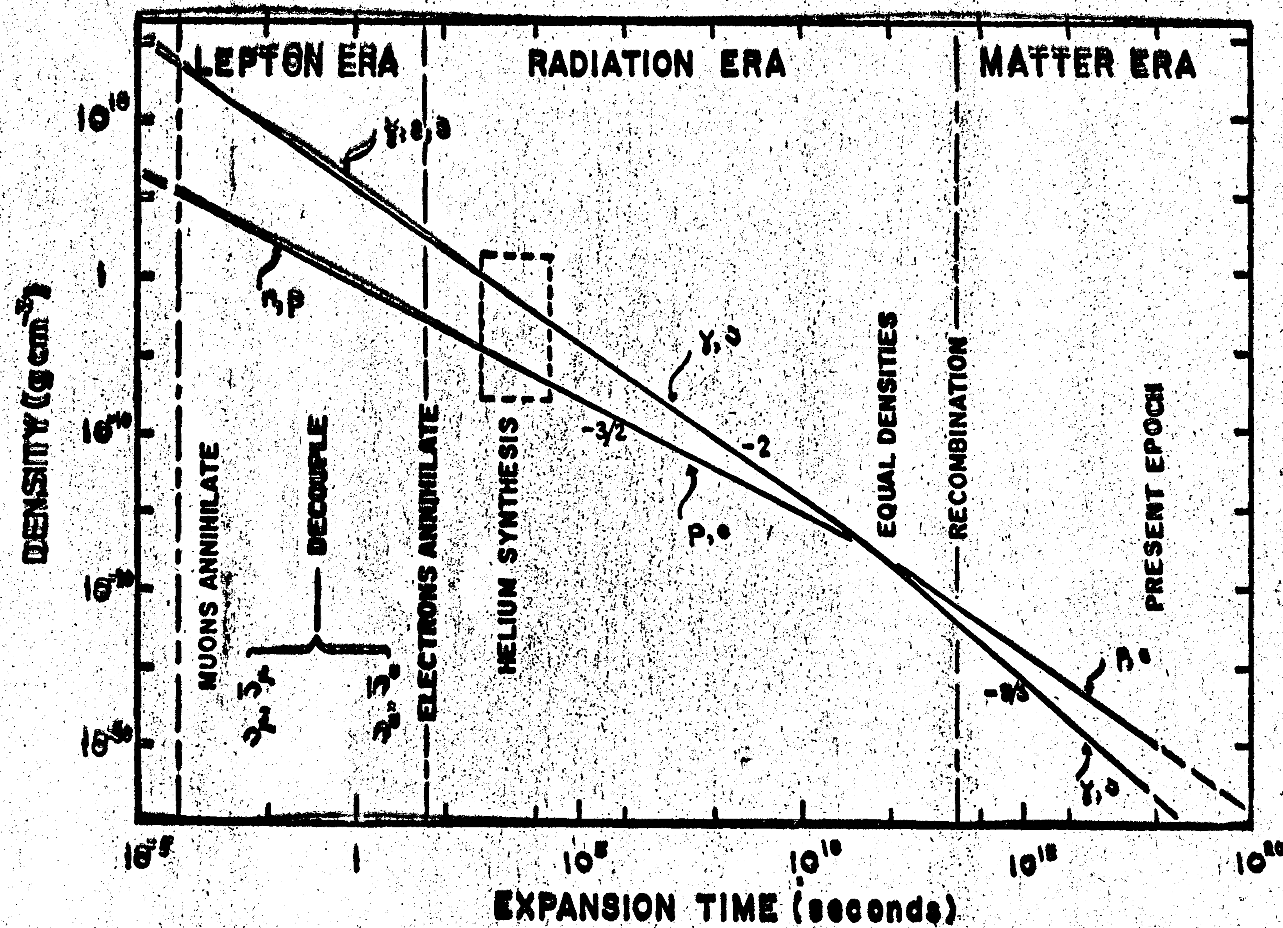
For pressure dominated universe $R(t) \propto t^{1/2}$, so
$t \propto T^{-2}$.

Photo dissociation of deuterium prevents nucleosynthesis until $T \lesssim 10^9 K$, or $t \sim 200 sec$. At that point, deuterium can last long enough to be made into helium, and the reaction proceeds until all available neutrons are used up, since the reaction times are much less than the expansion time.

Helium abundance by weight is approx. twice the neutron abundance by weight (since there are 4 nucleons and two neutrons in the helium atom), so the final helium abundance is

$$Y \sim 2 \times N \times \exp\left[\frac{t_{nuc}}{1013 sec}\right]$$
$$\sim 2 \times 0.17 \times 0.8$$
$$\sim 0.27$$

which is the same as the abundance calculated through more careful analysis, and is in agreement with observations.



LEPTON ERA
RADIATION ERA
MATTER ERA
DENSITY (g cm^{-3})
EXPANSION TIME (seconds)
MUONS ANNIHILATE
DECOUPLE
ELECTRONS ANNIHILATE
HELIUM SYNTHESIS
EQUAL DENSITIES
RECOMBINATION
PRESENT EPOCH
-3/2
-2
-8/3