Galen Gisler Summer Student Lecture July 8, 1981

Physical Cosmology.

Observations which suggest a cosmological model:

1. Redshifts of galaxies; $z \equiv \frac{\Delta\lambda}{\lambda} \propto D$
2. Isotropy of galaxies + radio sources.
3. Homogeneous distribution of "nearby" galaxies, scales > 50 Mpc.
4. "Distant" radio sources are more common and more powerful than "nearby" ones.
5. Cosmic objects are composed of 3 parts H to 1 part He, with traces of heavier elements.
6. Universe is filled with microwave radiation, highly isotropic, approximately black body, 2.7 °K.

Inferences

1 → we live in an expanding universe.

2,3 → the universe is approximately homogeneous + isotropic: its properties are (on average) the same at all points and in all directions [the universe has no center].

4 → the universe is evolving; its properties change with time.

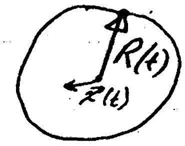
5,6 → the universe was once (briefly) hot enough and dense enough for fusion.

Field Equations for an Expanding Universe

Newtonian derivation

Uniform gas, density $\rho(t)$

Expansion scale factor $R(t)$



Consider the total energy of a particle of mass m

Any gas particle has a trajectory $\vec{x}(t) = \vec{x}(t_0)\,\dfrac{R(t)}{R(t_0)}$.

Gravitational potential energy $V(t) = -\frac{4}{3}\pi\,|\vec{x}(t)|^3 \rho(t)\,\dfrac{mG}{|\vec{x}(t)|}$

(ignore mass outside $|\vec{x}(t)|$)

$$= -\frac{4}{3}\pi m G\,|x(t_0)|^2 \rho(t)\,\frac{R^2(t)}{R^2(t_0)}$$

Kinetic energy of the particle is $T(t) = \frac{1}{2} m\,|\dot{\vec{x}}(t)|^2$

$$= \frac{1}{2} m\,|\vec{x}(t_0)|^2\,\frac{\dot{R}^2(t)}{R^2(t_0)}$$

The particle's total energy is $E(t) = T(t) + V(t)$

conserved $\quad E = \frac{1}{2} m\,\dfrac{|\vec{x}(t_0)|^2}{R^2(t_0)}\left[\dot{R}^2(t) - \dfrac{8\pi G}{3}\rho(t) R^2(t)\right]$

Define $k \equiv \dfrac{-2E}{m}\,\dfrac{R^2(t_0)}{|\vec{x}(t_0)|^2}$. Note $k=0 \iff E=0$.

Then $\dot{R}^2 + k = \dfrac{8\pi G}{3}\rho R^2$. Einstein Field Equation.

Newtonian treatment is illustrative, not rigorously correct. We can choose units for m such that $k = \pm 1$ or 0. k is called the curvature index.

Conservation of energy (we use units such that $c = 1$)

$$\frac{d}{dt}(\rho R^3) + p \frac{d}{dt}(R^3) = 0$$

(mass energy) (pressure-volume work)

a useful form $\rightarrow \dot{\rho} R = -3(\rho + p)\dot{R}$

another useful form $\frac{d}{dR}(\rho R^3) = -3p R^2$

To get acceleration equation, take time derivative of Field Equation:

$$2\dot{R}\ddot{R} = \frac{8\pi G}{3} R (\dot{\rho} R + 2\rho \dot{R})$$

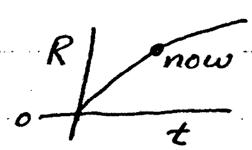
$$\ddot{R} = -\frac{4\pi G}{3}(\rho + 3p) R$$

These equations, plus an equation of state $[p = f(\rho)]$ are all you need to do cosmology, in the homogeneous and isotropic ("Robertson-Walker") universe.

<u>Elementary conclusions</u>

① Since $(\rho + 3p)$ is always positive, $\frac{\ddot{R}}{R}$ is always negative.
Since R is positive, $R(t)$ is concave down
From observations $\frac{\dot{R}}{R} > 0$ (we see redshifts, not blue shifts)

$\therefore$ $R = 0$ at some time in the past



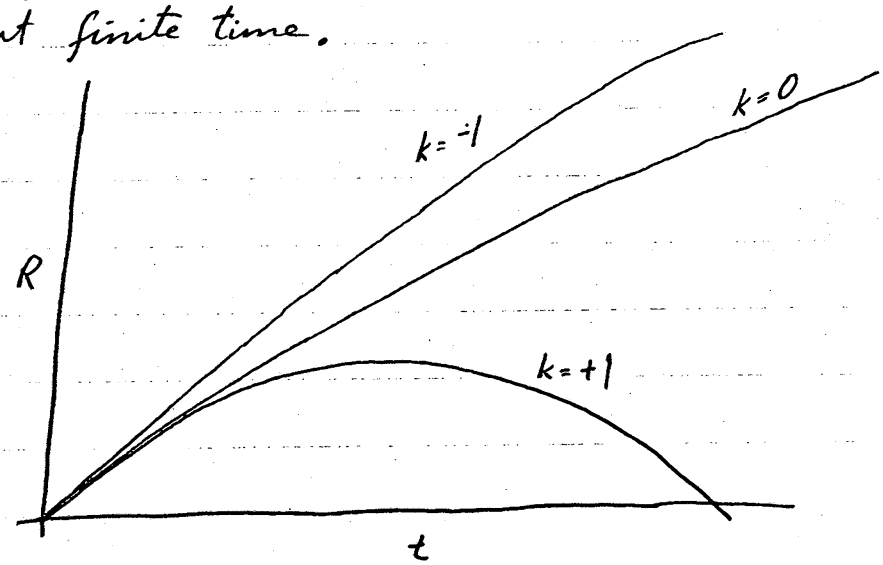
② From the energy conservation equation
(a) if $p = 0$ ("dust-filled universe") then $\rho \propto R^{-3}$
(b) if $p = \rho/3$ ("radiation-filled universe") then $\rho \propto R^{-4}$
in any case, $p \geq 0$ implies ρ decreases at least as fast as R^{-3}.

③ From the $\dot{R}$ equation: if ρR^2 vanishes as $R \to \infty$ at least as fast as R^{-1}, then

(a) if $k = -1$, $\dot{R}^2$ is always positive definite, in fact $\dot{R} \to 1$
$R(t)$ increases forever, $R(t) \to t$ as $t \to \infty$

(b) if $k = 0$, $\dot{R}^2$ is still positive definite but $\dot{R} \to 0$ as $t \to \infty$
$R(t)$ increases forever, to an asymptotic R_{max}

(c) if $k = +1$, $\dot{R}^2$ is zero at $\rho R^2 = \frac{3}{8\pi G}$. Since $\ddot{R}$ is negative definite, $R(t)$ must decrease again, reaching $R = 0$ at finite time.



Observational parameters

Invert the $\dot{R}, \ddot{R}$ equations to get expressions for density:

$$\rho = \frac{3}{8\pi G}\left(\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right)$$

$$\rho + 3p = \frac{-3}{4\pi G}\frac{\ddot{R}}{R}$$

<u>Define</u> $H_0 \equiv \frac{\dot{R}(t_0)}{R(t_0)}$ = present expansion <u>rate</u>, units t^{-1}
= Hubble "constant"

H_0^{-1} is expansion timescale
$= 1.3\times 10^{10}$ yr for $H_0 = 75 \frac{km}{sec\, Mpc}$

present density of universe is
$$\rho_0 = \frac{3}{8\pi G}\left(H_0^2 + \frac{k}{R_0^2}\right)$$
$R_0 \equiv R(t_0)$

if $k=0$, $\rho_0 = \rho_c \equiv \frac{3H_0^2}{8\pi G} \simeq 1.1\times 10^{-29}\ gm/cm^3$
$\simeq 6\times 10^{-6}\ m_H/cm^3$

observable density in galaxies $\rho_G \sim 0.1\rho_c$
$\rho < \rho_c \rightarrow k < 0$

<u>Deceleration parameter</u>.

If $p \ll \rho$, as in present (matter-dominated) universe, then we have another expression for the present density:
$$\rho_0 = \frac{-3}{4\pi G}\frac{\ddot{R}(t_0)}{R(t_0)} = \frac{3}{8\pi G}\left(\frac{k}{R_0^2} + H_0^2\right)$$
$$\frac{k}{R_0^2} = -H_0^2 - 2\frac{\ddot{R}(t_0)}{R(t_0)}$$
$$= H_0^2\left(-1 - 2\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)}\right)$$
or $\frac{k}{R_0^2} = H_0^2(2q_0 - 1)$ where $q_0 \equiv \frac{\ddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)}$

Substitute this expression into the first expression for ρ_0, to get

$$\rho_0 = \frac{3}{8\pi G} H_0^2 (2q_0)$$

$$\text{or} \quad \frac{\rho_0}{\rho_c} = 2q_0 \equiv \Omega$$

So we have 3 equivalent parameters by which to classify Robertson-Walker universes:

$\rho_0 > \rho_c$	$q_0 > \frac{1}{2}$	$k = +1$	universe "closed", will recontract
$\rho_0 = \rho_c$	$q_0 = \frac{1}{2}$	$k = 0$	universe "flat", } will expand
$\rho_0 < \rho_c$	$q_0 < \frac{1}{2}$	$k = -1$	universe "open" } forever

Integration of $R(t)$ in the Matter-Dominated Era.

Use

$$\dot{R}^2 + k = \frac{8\pi G}{3} \rho R^2$$

substitute $\rho = \rho_0 \left(\frac{R_0}{R}\right)^3$ and $\frac{8\pi G}{3}\rho_0 = 2q_0 H_0^2$

and $k = (2q_0 - 1) H_0^2 R_0^2$

$$\dot{R}^2 + (2q_0 - 1) H_0^2 R_0^2 = 2q_0 H_0^2 \left(\frac{R_0^3}{R}\right)$$

$$\text{or} \quad \frac{\dot{R}}{R_0} = H_0 \left[1 - 2q_0 + 2q_0 \frac{R_0}{R}\right]^{1/2}$$

Invert and integrate

$$t = \frac{1}{H_0} \int_0^{R/R_0} \left[1 - 2q_0 + \frac{2q_0}{x}\right]^{-1/2} dx$$

In particular, the present age of the universe is

$$t_0 = \frac{1}{H_0} \int_0^1 \left[1 - 2q_0 + \frac{2q_0}{x}\right]^{-1/2} dx$$

$$t_0 < \frac{1}{H_0} \quad \text{for } q_0 > 0.$$

Results of integration

① If $q_0 = 1/2$:

$$t = \frac{1}{H_0} \int_0^{R/R_0} \sqrt{x}\, dx = \frac{1}{H_0} \frac{2}{3} \left(\frac{R(t)}{R_0}\right)^{3/2}$$

$$\text{or} \quad \frac{R(t)}{R_0} = \left(\frac{3}{2} H_0 t\right)^{2/3}$$

"Einstein-de Sitter universe"

② If $q_0 > \frac{1}{2}$ define θ by $\frac{R(t)}{R_0} = \frac{q_0}{2q_0 - 1}(1 - \cos\theta)$

then

$$t = \frac{1}{H_0} \int_0^\theta \frac{q_0}{(2q_0-1)^{3/2}} (1 - \cos\theta')\, d\theta'$$

$$H_0 t = \frac{q_0}{(2q_0-1)^{3/2}} (\theta - \sin\theta), \text{ equation of a cycloid.}$$

③ If $q_0 < \frac{1}{2}$ define ψ by $\frac{R(t)}{R_0} = \frac{q_0}{1-2q_0}(\cosh\psi - 1)$

then

$$t = \frac{1}{H_0} \int_0^\psi \frac{q_0}{(1-2q_0)^{3/2}} (\cosh\psi' - 1)\, d\psi'$$

$$H_0 t = \frac{q_0}{(1-2q_0)^{3/2}} (\sinh\psi - \psi)$$

Use the small angle relations

$$\cos\theta \simeq 1 - \frac{\theta^2}{2} \quad \text{and} \quad \cosh\psi \simeq 1 + \frac{\psi^2}{2}$$
$$\sin\theta \simeq \theta - \frac{\theta^3}{6} \qquad \sinh\psi \simeq \psi + \frac{\psi^3}{6}$$

to show that
$$\frac{R(t)}{R_0} \simeq \left[\frac{3}{2}\sqrt{2q_0}\,(H_0 t)\right]^{2/3}$$ for all q_0 at early times

Thus $R(t)$ behaves similarly for all models, at early times.

Integration of $R(t)$ in the Pressure-Dominated Era

In early universe, pressure is given by relativistic particles and photons, so $p = \frac{1}{3}\rho$ and $\left(\frac{\rho}{\rho_0}\right) = \left(\frac{R_0}{R}\right)^4$

Neglecting curvature in the $\dot{R}$ equation,

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2$$
$$= \frac{8\pi G\rho_0}{3}\left(\frac{R_0}{R}\right)^4 R^2$$
$$R\dot{R} = \left(\frac{8\pi G\rho_0}{3}\right)^{1/2} R_0^2$$

Solution $$R(t) = \left(\frac{32\pi G\rho_0}{3}\right)^{1/4} R_0\, t^{1/2}$$

or $$t = \left(\frac{3}{32\pi G\rho}\right)^{1/2}.$$

Distances and Redshifts.

If the universe is homogeneous and isotropic, the line element used for measuring intervals in space and/or time is the Robertson-Walker metric:

$$d\tau^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right\}$$

where τ is proper time or distance (light signals propagate with $\Delta\tau = 0$)

r, θ, ϕ are <u>comoving</u> cosmic coordinates

t is cosmic standard time

$R(t)$ is an <u>arbitrary</u> function of t, called "the radius of the universe" or "the cosmic scale factor"

k is curvature constant, $= \pm 1$ or 0

Redshift:

Use a coordinate system centered on us, so that we are at $r = 0$, and a distant galaxy is at $r = r_1$ (θ and ϕ ignorable). The equation of motion of a light wave is

$$d\tau^2 = 0 = dt^2 - R^2(t)\,\frac{dr^2}{1-kr^2}.$$

Light which leaves r_1 at time t_1 arrives at $r=0$ at time t_0 given by

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \left\{\begin{matrix} \sin^{-1} r_1 \\ r_1 \\ \sinh^{-1} r_1 \end{matrix}\right\} \equiv f(r_1)$$

Next wave crest leaves at $t_1 + \delta t_1$, arrives $t_0 + \delta t_0$

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{R(t)} = f(r_1) \text{ as above.}$$

$R(t)$ is constant over light period, so

$$\frac{\delta t_0}{R(t_0)} = \frac{\delta t_1}{R(t_1)}.$$

Since $\lambda_0 = \frac{1}{\nu_0} = \delta t_0$, $\frac{\lambda_0}{\lambda_1} = \frac{R(t_0)}{R(t_1)}$

<u>Define the redshift</u> $z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{R(t_0)}{R(t_1)} - 1.$

Notice: nothing has been said about <u>velocities</u>. These shifts arise from the change in scale of the universe and are <u>not</u>, properly speaking, Döppler (velocity) shifts.

If the universe is expanding, $R(t_0) > R(t_1)$ and $z > 0$ (redshift). If the universe were contracting, $R(t_0) < R(t_1)$ and $z < 0$ (blueshift).

For small distances r_1, the cosmological redshift is identical to a velocity shift, provided by velocity we mean rate of change of <u>proper distance</u>.

The proper distance measured between $r=0$ and $r=r_1$ is the distance measured at the same cosmic time (impossible to do!)

$$d = R(t_0) \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}}$$

for <u>small</u> r_1, $\quad d \simeq R(t_0)\, r_1$

and $\quad v_{rad} = \dot{d} \simeq \dot{R}(t_0)\, r_1$

likewise for <u>small</u> $t_0 - t_1$, $\displaystyle\int_{t_1}^{t_0} \frac{dt}{R(t)} \simeq \frac{t_0 - t_1}{R(t_0)} \simeq r_1$

then, using $R(t_1) \simeq R(t_0) - \dot{R}(t_0)\,(t_0 - t_1)$,

we have
$$z = \frac{R(t_0)}{R(t_1)} - 1 \simeq \frac{\dot{R}(t_0)\,(t_0 - t_1)}{R(t_0)}$$
$$\simeq \dot{R}(t_0)\, r_1$$
$$\simeq v_{rad}$$

This ($z \simeq v_{rad}$) is not valid at large distances because

(1) distance itself is an ill-defined concept when the universe is changing in scale over cosmic time. It is possible to define 4 different measures of distance which differ from each other and from the proper distance.

(2) the effects of curvature make the approximations used above invalid.

Distance measures:

1. Absolute luminosity compared to apparent luminosity
2. True diameter compared to apparent (angular) diameter
3. Parallax
4. Proper motion

These all differ from one another for distances $> 10^9$ light years

We will derive only the most useful, the luminosity distance d_L
we define

$$d_L \equiv \left(\frac{L}{4\pi l}\right)^{1/2}$$

where L is absolute luminosity, and l is apparent luminosity, using Newton's inverse square law.

$$l = \text{apparent luminosity} = \frac{\text{Power received by telescope}}{\text{Aperture of telescope}}$$

$$\text{Power} = \frac{\text{number of photons} \times \text{energy of photons}}{\text{unit time}}$$

The fraction of all isotropically emitted photons that strike a telescope mirror of aperture A is $\dfrac{A}{4\pi R_0^2(t_0)\, r_1^2}$

Each photon has energy $h\nu_1$, redshifted to $h\nu_1\left(\frac{R(t_1)}{R(t_0)}\right)$
Time interval between photons is dilated to $\delta t_1\left(\frac{R(t_0)}{R(t_1)}\right)$

So power received is
$$P = \frac{L\left(\frac{A}{4\pi R(t_0)^2 r_1^2}\right)\cdot\left(\frac{R(t_1)}{R(t_0)}\right)}{\left(\frac{R(t_0)}{R(t_1)}\right)}$$

$$l = \frac{P}{A} = \frac{L}{4\pi}\,\frac{R^2(t_1)}{R^4(t_0)\, r_1^2}$$

$$d_L = R^2(t_0)\,\frac{r_1}{R(t_1)}$$

$$= r_1 R(t_0)\,(1+z).$$

Observational cosmology uses relation between d_L and z to determine H_0 and q_0. This can be demonstrated by expanding r_1 and d_L as ~~a~~ Taylor series in z, using definitions of H_0 and q_0

The Hot Big Bang Model ("Standard Model")

At early times $R(t)$ was very small and $\rho \propto R^{-3}$ or $\rho \propto R^{-4}$ was very large, so matter and radiation were presumably in thermal equilibrium at a high temperature. As the universe expanded, matter and radiation cooled. When the temperature reached $T \sim 4000°K$, roughly the ionization temperature of hydrogen, free electrons joined protons for the first time (although it is called "recombination"), and the opacity of cosmic matter dropped sharply, and ~~[illegible]~~ matter and radiation were decoupled.

The presently observed microwave background radiation, discovered in 1965, is the redshifted image of the universe at the time of decoupling, called the "surface of last scattering". Its present temperature is approximately 2.7 °K, and we can deduce the redshift from the Wien displacement law for black body radiation:

$$\lambda_{max} T \simeq \text{constant}$$

$$1+z = \frac{\lambda_0}{\lambda_1} = \frac{T_1}{T_0} \simeq \frac{4000}{2.7} \simeq 1500.$$

$z \sim 1500$ is the maximum possible observable redshift, since the universe was opaque at earlier epochs.

From thermodynamic considerations, the blackbody character of the background radiation must be preserved by the expansion of the universe, if there is no interaction between matter and radiation. This tells us how the temperature of the universe depends on the expansion.

The Wien displacement law $\lambda_{max} T = \text{constant}$ requires

$$\frac{T}{T_0} = \frac{R_0}{R} .$$

We can derive this more rigorously from the energy conservation equation

$$\frac{d}{dR}(\rho R^3) = -3 p R^2$$

The total energy density is

$$\rho = nm + \frac{1}{(\gamma-1)} nkT + aT^4$$

The total pressure is

$$p = nkT + \tfrac{1}{3} aT^4$$

Then we have

$$\frac{d}{dR}\left[nmR^3 + \frac{nkTR^3}{\gamma-1} + aT^4R^3\right] = -3nkTR^2 - aT^4R^2$$

We also have particle conservation $\frac{d}{dR}(nmR^3) = 0.$

then $$\frac{d}{dR}\left[\frac{nkTR^3}{\gamma-1} + aT^4R^3\right] = -3nkTR^2 - aT^4R^2$$

get $$\frac{R}{T}\frac{dT}{dR} = \frac{-3nkR^2\left(\frac{\gamma}{\gamma-1}\right) - 4aT^3R^2}{nkR^2\left(\frac{1}{\gamma-1}\right) - 4aT^3R^2}$$

define $\sigma = \frac{4aT^3}{3nk}$: "dimensionless photon entropy", roughly the ratio of the number of photons per gas particle, approximately conserved.

Then $$\frac{R}{T}\frac{dT}{dR} = -\left(\frac{\frac{\gamma}{\gamma-1} + \sigma}{\frac{1}{3(\gamma-1)} + \sigma}\right)$$

In the present universe $\sigma \simeq 2.4 \times 10^8 \gg 1.$

so $$\frac{R}{T}\frac{dT}{dR} \simeq -1 \quad \text{and } T \propto R^{-1}.$$

Scenario of the Early Universe

The existence of the microwave background, and the fact that $T \propto \frac{1}{R}$ for most of the history of the universe, enable us to study the physics of the early universe, which turns out to be simpler than we have a right to expect. The only complication is that at very early times there are a multitude of particles interacting with each other, contributing to the energy density and pressure in different ways, and decaying at different times, and keeping track of them is a necessary bookkeeping chore. The recent (1976) discovery of the tauon and its associated neutrino has made many books on cosmology obsolete, but only trivially so: a constant in the formulas for energy density must be changed from $\frac{9}{2}$ to $\frac{43}{8}$, for example. Cosmologists must keep track of developments in particle physics, and the equations of state depend strongly on what particles are playing the game. Further, at temperatures greater than $10^{12}K$ or $t < 10^{-4}$ sec, the mean interparticle distances are less than the particle wavelengths, and no equation of state is available.

Most discussions of the early universe begin, then, at $t \sim 10^{-4}$ sec. At this point most of the protons and antiprotons, neutrons and antineutrons have annihilated each other, leaving a small (~ 1 part in 10^{8}) residue which will eventually become stars, planets, and people. Muons annihilate before $T \sim 1.3 \times 10^{11} K$, and neutrinos and antineutrinos decouple from the radiation field, leaving only electrons, positrons, and photons in thermal equilibrium. Finally the electrons and positrons annihilate each other at $5 \times 10^{9} K$ ($t \sim 4$ sec), so that only photons and neutrinos remain in free expansion, at slightly different temperatures. Except for the very small residue leftover from all the annihilation, the role of matter in the universe is over after the first 4 seconds.

Concentrating now on the residue: free interchange between neutrons and protons ceases when the electrons annihilate, leaving the proton to neutron ratio frozen at ~5:1. The neutrons then fuse with the protons to make about 25% He and a few heavier nuclei. The gas is ionized, and remains so with its temperature locked to the radiation temperature until $T \sim 4000°K$. At about the same time (maybe a little before or after, depending on ρ_0/ρ_c), the energy density of photons and neutrinos (declining as R^{-4}) drops below the energy density of matter (declining as R^{-3}) and the universe becomes dominated by the matter "residue" (the "dust-filled universe").

As indicated before, the expansion of the pressure dominated universe is

$$R(t) \propto t^{1/2}$$

$$\text{or } t \propto \rho_\gamma^{-1/2}$$

$$\text{so } \rho_\gamma \propto t^{-2} \text{ - while } \rho_m \propto R^{-3} \propto t^{-3/2}$$

When the universe is matter dominated,

$$R(t) \propto t^{2/3}$$

$$\rho_\gamma \propto R^{-4} \propto t^{-8/3} \text{ while } \rho_m \propto R^{-3} \propto t^{-2}$$

Helium Synthesis.

A simple calculation which yields the right answer!

Two parts

(1) Calculate the neutron/proton abundance ratio as a function of time

(2) Assume all neutrons go into helium atoms at the appropriate time.

Part (1). The reactions involved are

$$n + \nu \leftrightarrow p + e^-$$
$$n + e^+ \leftrightarrow p + \bar{\nu}$$
$$n \leftrightarrow p + e^- + \bar{\nu}$$

From the rate equations one can calculate the net rates $\lambda(p \to n)$ and $\lambda(n \to p)$, but this isn't really necessary. Instead define $Q \equiv m_n - m_p = 1.293\ MeV$ $(\simeq 1.5 \times 10^{10} K)$

Initially, $kT >> Q$, and the neutron ratio X_n has its equilibrium value

$$X_n \simeq \frac{\lambda(p \to n)}{\lambda(p \to n) + \lambda(n \to p)}$$

~~which~~ with $\dfrac{\lambda(p \to n)}{\lambda(n \to p)} \simeq \exp\left(\dfrac{-Q}{kT}\right)$

So that $X_n \simeq \dfrac{1}{1 + e^{Q/kT}}$

$\simeq 1/2$, drops slowly as temperature falls to $\sim 10^{10}K$, then drops rapidly.

Electrons and positrons annihilate at $T \sim 9\times10^9\,K$, so the neutron / proton ratio is frozen at the value N_n

$$N_n = X_n\,(9\times10^9 K) \simeq 0.16.$$

The only reaction left which influences the neutron abundance is the free neutron decay, with

$$\lambda^{-1}(n \rightarrow p + e^- + \bar{\nu}) = 1013\,sec.$$

After this time the neutron abundance is

$$X_n = N_n \exp\left(\frac{-t}{1013 sec}\right).$$

Photodissociation of deuterium is preventing nucleosynthesis from occurring until $T \lesssim 10^9 K$ or $t \sim 200 sec$. At that point deuterium can last long enough to be made into helium, and the reaction proceeds until all available neutrons are used up.

Part (2) The neutron abundance at $t_{nuc} \sim 200 sec$ is

$$X_n = N_n \exp\left(\frac{-200}{1013}\right)$$
$$\sim 0.16 \times 0.8 = 0.13$$

The helium abundance by weight, Y, is twice the neutron abundance since there are 4 nucleons for every two neutrons in helium, so the helium abundance is

$$Y = 2X_n = 0.26$$

Which is the same as the abundance calculated through more careful analysis, and is in agreement with observations.