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Use of Gasdynamics Codes in Astronomy.

1) Where in astronomy can one apply gascodes?

Since most objects studied by astronomers are gasseous in nature, you can essentially apply gascodes to all theoretical astronomical problems, ranging from stellar interiours, supernovae, stellar winds, star formation, HII regions, general interstellar medium, galaxies to cosmology. In fact, different types of codes, depending on the particular properties of the problem, have been used in almost all of these problems.

2) What sets astronomical applications apart from other applications?

In astronomical gasses much of their behaviour is determined by how they interact with their environment, through the so-called source terms. These are terms in the conservation equations for mass, momentum and energy that describe the rate of change of these conserved quantities due to external effects, such as ~~forces~~ and gravitational forces and ~~heating~~ radiative heating. In many cases a precise balance between different ~~source~~ terms may be attained, leading to long term stability.

Examples: stars - here generally a precise balance between heating, heatloss, gravitation and internal pressure ~~are~~ is attained

variable stars - generally pulsations can be treated as a perterbation on the usual hydrostatic equilibrium model. Here is an example of an object where ~~all~~ the relevant time scales for the various processes are very different, making it neccessary to use different techniques to study different aspects.

super novae - similar to the above in that one method is used to construct the pre-supernova

	star, and another method to compute the actual explosion (Arnett; Colgate; Johnstone and Yahil ...)
Interstellar medium	A large variety of interesting processes can be studied using gasdynamical codes, such as development of HII regions, cloud-cloud collisions, cloud collapse, interaction with stars etc. A large number of authors have published in this field. One of the large problems here often is the time-evolution of the radiation field, which interacts strongly with the gas.
Large scale gas flow in galaxies.	An important simplifying assumption that is often made in this field has to do with the relative short time scale for radiative processes compared to the other relevant time scales - it is generally assumed that the ISM is isothermal on the scales relevant here. Another assumption that has been made generally is that the gas is only a small fraction of the total mass, so that its selfgravity is not important. (except in recent work by Huntley).

It will be clear from the above examples that in some cases the source terms and their associated time scales complicate matters considerably, whereas in other cases they will simplify matters by acting as a stabilizing influence on very short time scales.

3) What type of codes are being used?

Of course this cannot be an exhaustive discussion of the methods used - if only because I am not familiar with the majority. The algorithm(s) selected to compute a certain model naturally must depend on the expected properties of the result. In stars, where mechanical and radiative time scales are generally much shorter than the evolutionary time scale, models will be based on radiative (energy transport) and hydrostatic equilibrium. (Schwarzschild, Structure and Evolution of the Stars).

In many cases where radiative processes are important iterative methods are needed because of the ~~very~~ short time scales on which equilibrium may be reached even in non-LTE situations.

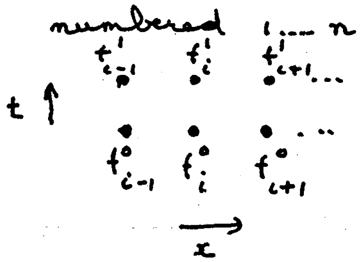
~~Thus~~ In a way the codes used for the calculation of gas flows in galaxies are the simplest, since we have to keep track of relatively few different quantities, and because the signal speed in the medium is low - namely the gas velocity + the sound speed. This allows us to use so called "explicit" codes, where the state of the gas at a certain point ~~is dependent only on~~ at the new time level depends only on the state of the gas at relatively few points at the earlier time level. The complicating factor is that generally a rather large number of points (e.g. 80 by 80) have to be followed over a long time (e.g. 1000 steps), so that updating each point should take as little time as possible, but yet needs to be done accurately.

An example of types of codes:

Lets look at the equation $\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = 0$ (v constant)

f is then a conserved quantity that is transported to the right with a velocity v. We can now find various numerical procedures that solve this equation. All algorithms discussed here will be ~~explicit~~ Eulerian - i.e. our grid is fixed and is not comoving with f.

1) Nodal point approach: we know f on a regular grid of points numbered $i \dots n$, spaced Δx apart at time level zero (f_i^0)



and we are looking for the time advanced f_i^1's at time Δt. Say $\alpha = -\frac{\Delta t}{\Delta x} V$. ~~$\alpha$ must be ≤ 1 for~~

1a) Explicit ($|\alpha| \leq 1$)

i) $f_i^1 - f_i^0 = \frac{\alpha}{2}(f_{i+1}^0 - f_{i-1}^0)$ central difference

ii) $f_i^1 - f_i^0 = \alpha(f_i^0 - f_{i-1}^0)$ upwind (exact for $\alpha = -1$)

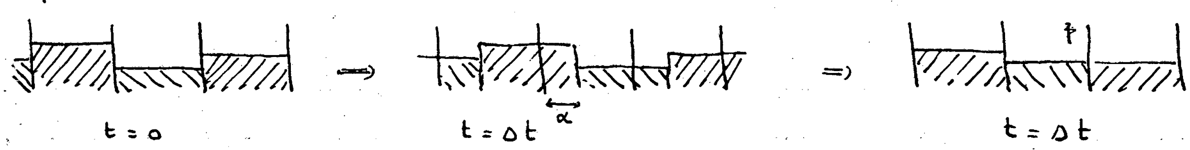
iii) $f_i^1 - f_i^0 = \frac{\alpha}{2}(f_{i+1}^0 - f_{i-1}^0 - \alpha(2f_i^0 - f_{i-1}^0 - f_{i+1}^0))$ second order

1 b) Implicit. $f_i^1 - f_i^0 = \frac{\alpha}{4}\left(f^0_{i+1} + f^1_{i+1} - f^0_{i-1} - f^1_{i-1}\right)$ second order accurate

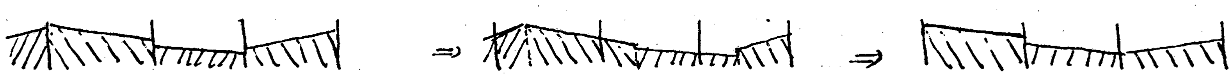
For implicit methods the constraints on α are generally less severe. The big disadvantage is that advanced time-level quantities occur on the right hand side of the equations, in this case leading to a tridiagonal set of equations. Much more complicated, especially for a full set of conservation equations

2) Control volume methods: we will assume that our grid is a grid of cells, each containing a certain amount of f (fluid). This representation is more pleasant if you know that you are working with conserved quantities.

Explicit:

a) 

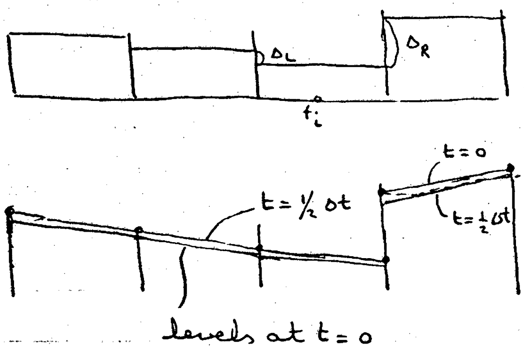
Same as upstream in this case, leads to Godunov's method in gasdynamics. 1^{st} order accurate. Also beamscheme and other flux-splitting methods.

b) 

Second order accurate method, leads to Muscle (Van Leer, Woodward, later also Hancock, and Van Albada)

c) PPM (using parabolas, Woodward) third order accurate.

In these higher order accurate methods the real challenge is to treat the slopes correctly. In the original formulation of Muscle the slopes were stored and updated for all the zones together with the total amount of each of the conserved quantities. The actual transport was evaluated as accurately as possible. A simplification proposed by Hancock states that the boundary values at $\frac{1}{2}\Delta t$ (which are needed to calculate the transport to second order accuracy) can be obtained for each zone separately by using the slopes in those zones. The slopes can be obtained by comparing with the adjoining zones. An improved algorithm for this was proposed by Van Albada. In easy steps



Find $s_i^o = \dfrac{\Delta_L(\Delta_R^2+\epsilon^2) + \Delta_R(\Delta_L^2+\epsilon^2)}{\Delta x(\Delta_R^2 + \Delta_L^2 + 2\epsilon^2)}$

and advance to half time

find boundary values.
use upwind centered method to find transport. Add to values at t=0 to find those at t = Δt.

For gasdynamics the appropriate upwind method seems to be a flux splitting method formulated by Van Leer.

General: Roache, P. J., 1976, "Computational Fluid Dynamics" (Hermosa Publishers, Albuquerque).

General: Richtmyer & Morton, 1967, "Difference Methods for Initial Value Problems" (Interscience, N.Y.)

Muscl: Van Leer, 1979, Journal of Computational Physics 32, 101

Beamscheme: Sanders and Prendergast, 1974, ApJ 188, 489