Radio Telescope Theory – Jim Ulvestad

Radio telescopes detect radiation at frequencies in the range from a few MHz (ionospheric cutoff) to several hundred GHz (water vapor in atmosphere, transition to IR band). This range corresponds to wavelengths from ~100 meters to ~1 millimeter. A variety of astronomical sources radiate at these frequencies by various mechanisms, such as synchrotron radiation and 21-cm line emission. Today, I will ignore all radiation processes and concentrate on the telescope specifications that are important in the detection and measurement of radio emission. I will restrict myself to the basics of single, filled aperture antennas. A few of these basics have been discussed in an earlier lecture by John Findlay. Interferometers will be discussed by Larry D'Addario, and Alan Bridle will talk more specifically about the VLA.

(1) The Principle of Reciprocity

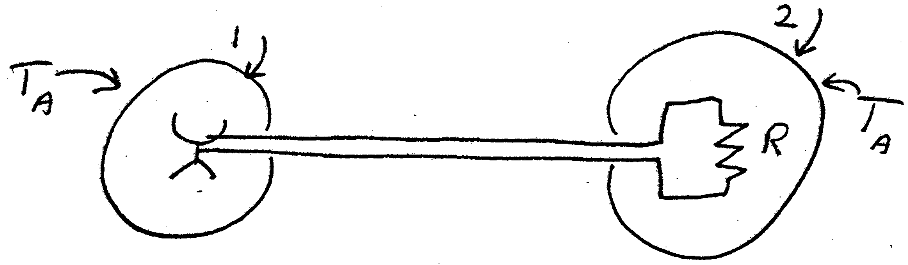
Assertion: Random formulae derived for transmission of power from a given antenna are identical when the antenna is considered as a receiver rather than a transmitter of power.

I.e., suppose antenna A generates a power P_A, and we use antenna B at a distance R_{AB} to measure a power P_B. Now, turn off the transmitter at A and allow antenna B to broadcast a power P_B. Assuming no changes in the antennas, electronics, etc., antenna A will measure a noise power P_A, identical to the power it transmitted in the first instance.

② Temperatures and Brightnesses

ⓐ Nyquist's Theorem

Consider the following system: An antenna is connected to a resistor by means of a circuit consisting of loss-free transmission lines. The antenna and resistor are enclosed in independent black bodies maintained at temperature T_A.



Blackbody 1 will radiate a certain noise power that is absorbed by the telescope, transmitted along the line to the resistance, and heats the resistance. Similarly, the terminals of the resistor will have a noise power that is conducted to the antenna and radiated to blackbody 1. Since the resistor and blackbody 1 both have a temperature T_A, the net energy exchanged is zero. Thus the power radiated by the antenna is the same as the background noise of the resistor at T_A; i.e., the power radiated by the antenna can be thought of in terms of the temperature of the equivalent resistor:

$$P_\nu = k\,T_A \quad \text{watts Hz}^{-1} \qquad (1.)$$

where $T_A \equiv$ antenna temperature.

(b) More on Brightness and Temperature

Let B_ν be the surface brightness of a radio source, measured in watts m^{-2} Hz^{-1} $ster^{-1}$. In general, B_ν is a function of direction; i.e. $B_\nu \to B_\nu(\theta, \phi)$. For a blackbody, recall the Planck law

$$B_\nu(\theta,\phi) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT_B(\theta,\phi)} - 1\right)^{-1}. \qquad (2)$$

In radio astronomy, the Rayleigh-Jeans approximation often holds -- $h\nu \ll kT$. Expanding the exponential then gives

$$B_\nu(\theta,\phi) \approx \frac{2kT_B(\theta,\phi)}{c^2}\nu^2 = \frac{2kT_B(\theta,\phi)}{\lambda^2}. \qquad (3)$$

This approximation is not very good in the high frequency (mm wave) regime. Formula (2) or (3) defines the <u>brightness temperature</u>, $T_B(\theta,\phi)$.

The <u>flux density</u> of a radio source is defined to be the source brightness integrated over solid angle:

$$S_\nu = \iint_{4\pi} B_\nu(\theta,\phi)\, d\Omega. \qquad (4)$$

The units of S_ν are watts m^{-2} Hz^{-1}. However, cosmic radio sources are incredibly weak, so S_ν is normally expressed in <u>Janskys</u> or mJy, where

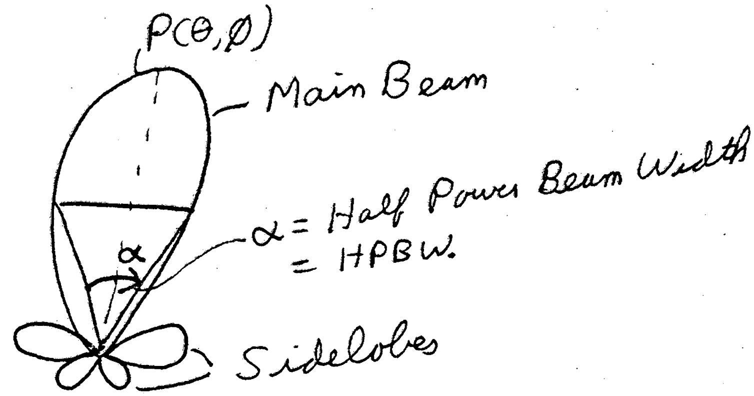
$$10^3 \text{ mJy} = 1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}.$$

Note that an older usage is the <u>flux unit</u>,

$$1 \text{ flux unit} = 1 \text{ Jy}.$$

3 Antenna Parameters

Suppose an antenna has a power pattern $P(\theta, \phi)$ which is normalized to unity in the direction of maximum response. A polar plot of such a pattern is given below:



The half power beam width (HPBW) is

$$HPBW \approx 1.4\ \lambda/d \quad , \qquad (5)$$

where λ is the wavelength, d the antenna diameter.

The beam solid angle Ω_A is the solid angle that would be sampled by an antenna of uniform (maximum) response within Ω_A and no response outside Ω_A:

$$\Omega_A = \iint_{4\pi} P(\theta, \phi)\, d\Omega . \qquad (6)$$

Similarly, one can define a main beam solid angle

$$\Omega_M = \iint_{\text{MAIN BEAM}} P(\theta, \phi)\, d\Omega . \qquad (7)$$

The beam efficiency ϵ_M measures how well the antenna restricts its power to the primary direction

$$\epsilon_M = \frac{\Omega_M}{\Omega_A} . \qquad (8)$$

In real telescopes, $\epsilon_M \approx 0.75 \pm 0.15$.

An antenna will have a certain physical area or aperture size. However, because of imperfections in the surface and shadowing by mechanical structures (among other things), the entire physical area of the antenna is not used. Instead, the antenna has an effective area A_e which varies with direction (and wavelength). If the maximum effective area is A_e, the directional dependence is

$$A_e(\theta,\phi) = A_e \, P(\theta,\phi) . \qquad (9)$$

For an antenna of physical area A, the aperture efficiency ϵ_a is defined by

$$A_e = \epsilon_a A . \qquad (10)$$

In general, $\epsilon_a = 0.6 \pm 0.1$.

The noise power measured at the antenna is then

$$P_\nu = \frac{1}{2} \iint_{4\pi} B_\nu(\theta,\phi) A_e(\theta,\phi) \, d\Omega = \qquad (11)$$

$$= \frac{1}{2} A_e \iint_{4\pi} B_\nu(\theta,\phi) \, P(\theta,\phi) \, d\Omega$$

(Factor of $\frac{1}{2}$ is due to sensitivity to only 1 polarization).

(4) Gain, Directivity

The antenna directivity, D, measures how well the antenna concentrates its response in a certain direction. It is defined as the ratio of the maximum power (i.e., power in the direction of maximum response) to the total power (= average power / 4π):

$$D = \frac{P(\theta,\phi)_{max}}{\frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\,d\Omega} = \qquad (12)$$

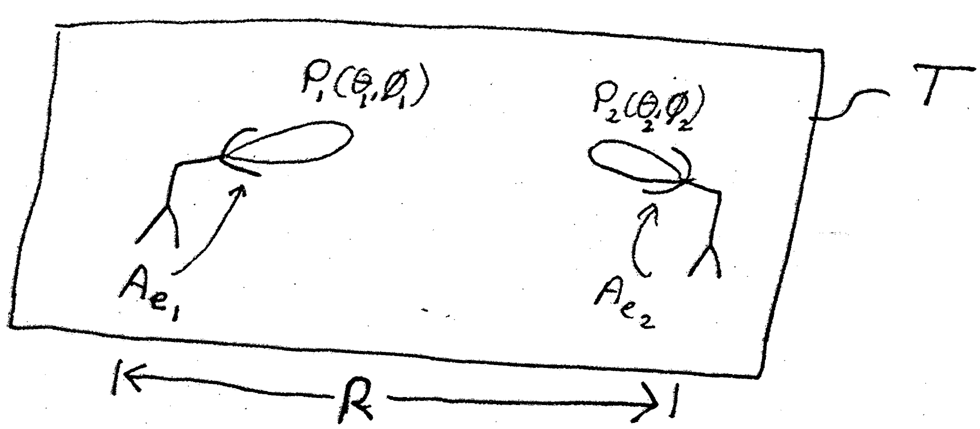
$$= \frac{4\pi}{\iint_{4\pi} P(\theta,\phi)\,d\Omega} \quad \text{if } P(\theta,\phi)_{max} \equiv 1 .$$

$$\therefore \quad \boxed{D = \frac{4\pi}{\Omega_A}.} \qquad (13)$$

The directive gain $D(\theta,\phi)$ is the gain in an arbitrary direction (θ,ϕ):

$$D(\theta,\phi) = D\,P(\theta,\phi) \qquad (14)$$

Finally, we note that the ratio D/A_e is the same for every antenna. To sketch this proof, consider two antennas in an isothermal enclosure separated by a large distance R:



The power from antenna i received by antenna j is

$$P_{i\to j} = P_i(\theta_i, \phi_i)\,\Omega_{A_j}(\theta_j, \phi_j)$$

$$= P_i(\theta_i, \phi_i)\,\frac{A_{e_j}(\theta_j, \phi_j)}{4\pi R^2} \qquad (15)$$

Within the isothermal enclosure, $P_{1\to 2} = P_{2\to 1}$

$$\Rightarrow \quad \frac{P_1(\theta_1, \phi_1)}{A_{e_1}(\theta_1, \phi_1)} = \frac{P_2(\theta_2, \phi_2)}{A_{e_2}(\theta_2, \phi_2)} \qquad (16)$$

$$= \frac{P_{1\,MAX}}{A_{e_1}} = \frac{P_{2\,MAX}}{A_{e_2}}$$

But $P_{i_{max}} \propto D_i$

$$\therefore \quad \frac{D_1}{A_{e_1}} = \frac{D_2}{A_{e_2}} = \text{constant for all antennas.} \qquad (17)$$

Since $D = \frac{4\pi}{\Omega_A}$, this is equivalent to saying that $A_e \Omega_A = \text{constant}$.

E.g., for a dipole, $A_e = \frac{3}{8\pi}\lambda^2$, $D = \frac{3}{2}$

$$\Rightarrow \boxed{A_e \Omega_A = \lambda^2 \text{ for all antennas} \qquad (18)}$$

Given an effective area at a given wavelength, the telescope beam size can then be calculated.

5 Minimum Detectable Signal

Detection of cosmic radio sources is usually complicated by the fact that the antenna temperature due to the source is usually much less than the noise temperature due to the telescope and its receivers. The telescope thus measures a system temperature T_{sys} where

$$T_{sys} = T_{noise} + T_A. \qquad (19)$$

$$(T_{noise} >> T_A)$$

The procedure for measuring T_A is to alternately point the telescope on and off source (often this is accomplished by switching in the electronics rather than by physically pointing the telescope in a different direction. T_A is then found from

$$T_{sys}\,(\text{on source}) - T_{sys}\,(\text{off source}) = T_A. \qquad (20)$$

But T_A is only detectable if $T_A \approx 5\,\Delta T_{sys}$, where ΔT_{sys} is the uncertainty in T_{sys}. Thus we must compute ΔT_{sys}.

PROCEDURE

Suppose N independent measurements of T_{sys} are made. Using Poisson statistics (i.e., random walk in T_{sys}-space),

$$\frac{\Delta T_{sys}}{T_{sys}} \approx \frac{K_T}{\sqrt{N}}. \qquad (21)$$

K_T is a constant depending on the telescope properties and is of order unity.

Assume that the observing system has a bandwidth $\Delta\nu$. The minimum integration time necessary for a single independent record is $\Delta t = 1/\Delta\nu$. Why? Recall the Heisenberg uncertainty principle. If we wish to restrict our measurement within the energy range ΔE, we must integrate for Δt, where $\Delta E \Delta t \simeq h$. Since $\Delta E = h\Delta\nu$, we have $\Delta\nu\Delta t \simeq 1$, or $\Delta t = 1/\Delta\nu$. The number of integrations, N, is then

$$N = \frac{\tau}{\Delta t} = \tau\Delta\nu , \qquad (22)$$

where τ is the total integration time.

$$\therefore \quad \Delta T_{sys} \approx \frac{K_T \, T_{sys}}{\sqrt{\tau\Delta\nu}} \qquad (23)$$

Formula (23) is the <u>radiometer equation</u>.

The minimum detectable signal is

$T_A \approx 5\,\Delta T_{sys}$.

But $P_\nu = k T_A \approx \frac{1}{2} A_e S_\nu$

$$\therefore \boxed{S_{\nu_{min}} \approx \frac{2 k T_{A,min}}{A_e} \approx \frac{10 k K_T T_{sys}}{A_e\sqrt{\tau\Delta\nu}}} . \qquad (24)$$

Example

Consider the 140-foot telescope at 18 cm. It has an effective area of ~ 787 m^2. Suppose we are observing a source with the Mark II VLBI system, having a 2 MHz bandwidth. The system temperature is 50K and we wish to integrate coherently for 13 minutes. How weak a source can we detect?

Answer

Using (24),

$$S_{\nu_{min}} \approx \frac{10\,(1.38\times10^{-23})\,(1)\,(50)}{787\sqrt{780 \times 2\times10^{6}}}$$

$$\approx 2.2\times10^{-28}\ W m^{-2} Hz^{-1} = 22\ mJy.$$

Now, switch to the Mark III VLBI system, in which data can be recorded in a 56 MHz bandwidth. The minimum detectable flux density is lowered by a factor of $\sqrt{56/2}$ to ~ 4.2 mJy.

In real life, we'll do well to get within a factor of ~ 2 of these numbers, especially with the Mark III system.

References: Christensen + Högbom; Kraus; Steinberg + Lequeux