Dynamics of Galaxies (D. Merritt)

I. Types of Dynamical Systems

Galaxies differ from the best-known dynamical systems (e.g. stars) in that the "particles" which make them up (mostly individual stars) interact via gravitation, a long-range force.

This means that the stars in a galaxy move in a potential that is determined by all the other stars, and hardly at all by the nearest neighbors, as in a gas.

There is therefore no simple analog to the "equation of state" of a gas.

To understand the dynamics of galaxies it is necessary to understand both the "microscopic" dynamics, i.e. individual stellar orbits, as well as the "macroscopic" dynamics, i.e. the way in which the orbits combine to produce the overall mass distribution.

Because this "self-consistency" problem is such a difficult one, only a handful of 'realistic' galaxy models exist.

II. Types of Galaxies

Galaxies come in two basic types:
Disk systems (spirals + SO's)
& Spheroidal systems (ellipticals and spiral bulges)

Disk galaxies are typically very thin ($b/a \lesssim 0.2$) and very round.
Most of the luminous matter is often distributed in a spiral pattern, which however only contains a small fraction of the total mass. It is nowadays believed that the spiral pattern is a transient phenomenon.
The luminosity is observed to fall off as:

$$I(r) \propto e^{-r/r_0}$$

outside of the bulge.

Spheroidal galaxies are essentially spherical ($0.3 \lesssim b/a \lesssim 1$).

They are essentially featureless, and contain very little gas.

The luminosity is observed to fall off as:

$$I(r) \propto r^{-n} \quad , \qquad 2 \lesssim n \lesssim 3 .$$

III. Dynamics of Disk Galaxies

The fact that disk galaxies are so flat and round suggests that most of the stellar orbits in the disk are essentially circular.
This conclusion is reinforced by observations of stars in the solar neighborhood, which typically have only a very small velocity relative to the sun, i.e., the dominant motion is circular streaming about the Galactic center.

The assumption of circular orbits allows us to infer the masses of external galaxies by measurements of their rotation curves, i.e. the orbital velocity as a function of radius in the disk.

This is most easily done in the case of spirals by measuring the Doppler shift of emission lines from HII regions lying in the disk. It may also be done using the 21 cm line of neutral hydrogen.

In a circular orbit, the centripetal acceleration is provided by the gravitational mass interior to the orbit, such that

$$\frac{v^2(r)}{r} \approx \frac{G M(r)}{r^2}$$

where $M(r)$ is the mass in the disk interior to r, and $V(r)$ is the rotational velocity at r. The relation is exact as written only if the mass interior to r is spherically distributed; if it is in a flattened disk, a factor slightly larger than one goes on the right hand side.

Measuring V as a function of r then gives the mass distribution, i.e.

$$M(r) \approx V^2 r / G .$$

It is instructive to consider first what general form one expects for the rotation curve.

at small radii, the matter making up the disk probably attains a nearly constant density (i.e. there is no central "spike"), so that:

$$\text{small } r: \quad M(r) \propto \int dr\, r^2 \rho(r) \propto \rho_0 \int dr\, r^2 \propto \rho_0 r^3$$

$$\text{and} \qquad V \propto r$$

This is called "solid body rotation", because $V \propto r$ implies a constant angular velocity.

at large radii, essentially all of the matter in the disk is interior to a given orbit, so that

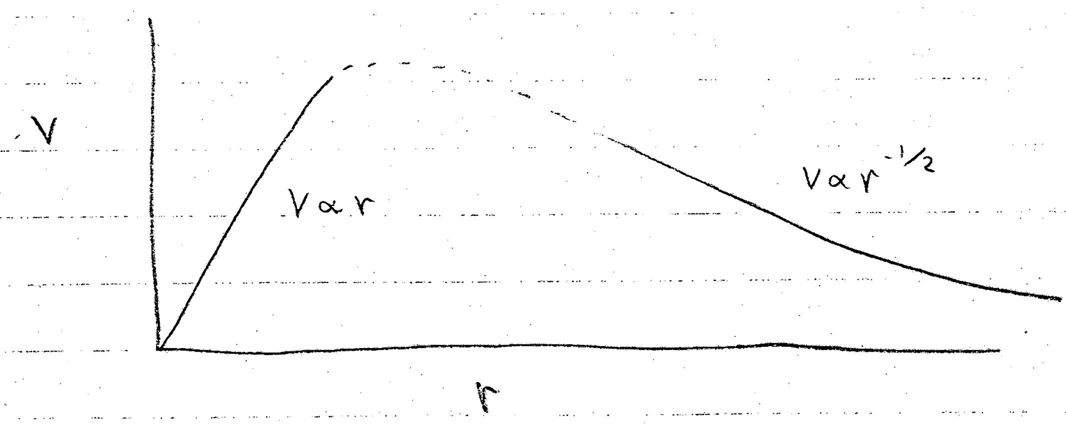
$$\text{large } r: \quad M(r) \approx M_{total}$$

and

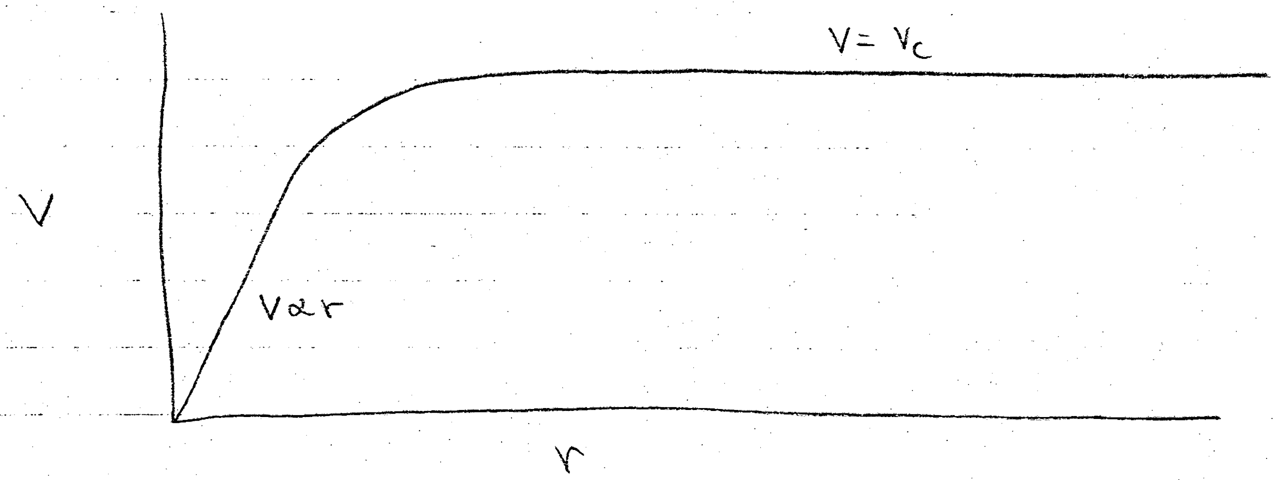
$$V \propto r^{-1/2}.$$

This is called "Keplerian rotation", since it is the same relation satisfied by planets orbiting about a central body.

We can therefore construct the expected form for the rotation curve:



Observations of dozens of spiral galaxies show the following characteristic form, however:



In other words, at large radii the rotation curves of observed spirals do not fall off, as they must if these galaxies have finite masses. Not a single spiral has yet been measured for which the rotation curve falls off as quickly as Keplerian.

We can derive the mass distribution in a galaxy with a flat rotation curve:

$$M(r) \simeq V^2 r / G$$
$$\propto r$$

so that the mass within radius r increases as r. The density is

$$M(r) \propto r \propto \int dr\, r^2 \rho(r)$$

$$\rho(r) \propto r^{-2}$$

The density goes to zero, but not quickly enough to keep the total mass from diverging.

The fact that the mass surrounding spiral galaxies appears to increase without limit, while the luminosity falls off exponentially, leads one to the conclusion that these systems are surrounded by huge halos of dark matter. This "massive halo hypothesis" is supported by an increasing amount of other data, although none is as compelling as the flat rotation curves

III Dynamics of Elliptical Galaxies

Until the early 70's, it was generally believed that elliptical galaxies were "supported" by rotation in a manner similar to disk galaxies; the fact that ellipticals are observed to be less flattened than spirals was thought to be due to the fact that they are rotating less slowly. But before 1975 no one had succeeded in measuring a rotation curve for an elliptical galaxy; the reason is that ellipticals contain no luminous HII regions from which emission line redshifts can be obtained.

In 1975, Bertola and Capaccioli published the first rotation curve for an elliptical, obtained from the Doppler shift of the absorption lines from stars in the galaxy. The principle result was that the galaxy rotated very slowly. Since then it has been confirmed that virtually all ellipticals rotate much more slowly than spiral galaxies.

The fact that rotation is unimportant in elliptical galaxies means that we cannot use the simple equation of the preceding section to derive their masses from rotation curves. Another way of saying this is that the general stellar orbit in an elliptical galaxy is not circular, so there is no simple relation between V and M(r).

A more general way of analyzing the dynamics of any self-gravitating system is through the Virial Theorem. The theorem states:

$$2T + W = 0$$

for any system in dynamical equilibrium; here T is the total amount of energy due to the motion of the bodies making up the system (the total kinetic energy), and W is the total potential energy resulting from the self-gravitation of the mass making up the system. (W is defined in the usual way to be negative). The theorem simply states that for a system to be in equilibrium (not expanding or contracting), there must be a balance between the tendency to fly apart (represented by T) and the tendency to collapse under its self-gravity (represented by W). It is a completely general theorem; it applies to systems of any size, regardless of the number of particles. For instance, for a planet of mass m orbiting a star of mass M, we have:

$$T = \frac{1}{2} m V^2 \quad ,$$

$$W = -GMm/r \, ,$$

$$2T + W = mV^2 - GMm/r = 0 \, ,$$

$$V^2 = GM/r \, ,$$

the same equation derived above.

In applying the Virial Theorem to entire galaxies, it is useful to make a distinction between motions that are "directed", e.g. the stellar streaming that comprises rotation in disk galaxies, and motions that are "random". An example of a system containing both types of motion is a free-streaming gas of non-zero temperature; the mean motion is in the direction of the streaming, and the random motion is that due to the thermal kinetic energy of the gas molecules.

The term "T" in the Virial Theorem includes both types of motions. For a system containing an ensemble of particles, we can write:

$$T = \frac{1}{2} \sum_i m_i v_i^2$$

$$= \frac{1}{2} \sum_i m_i \left[V_m^2 + V_R^2 \right]_i$$

where V_m is the mean velocity of all the particles making up the system, and V_R is the random velocity (measured with respect to the mean).

The Virial Theorem states that the sum of both types of motion must be great enough to balance gravity. It says nothing about the ratio of the two types of motion, however.

Define a parameter λ such that

$$\lambda = \frac{V_m}{V_R}$$

λ is a measure of how "hot" a system is, i.e. how much a system is dominated by "random" as opposed to "directed" motions. We have already seen that disk galaxies are dominated by rotation. In fact, measurements of stellar velocities with respect to the sun give $V_R \simeq 10$ km s^{-1}, while the rotation velocity is $V_m = 250$ km s^{-1}, so that for disk galaxies

$$\lambda_D \simeq 250/10 \simeq 25 \gg 1$$

Disk galaxies are cold systems; they are dominated by streaming motions.

For elliptical galaxies, rotation curve measurements in the mid 70's gave $0 \lesssim V_m \lesssim 50$ km s^{-1}. Random motions (measured by noting their broadening effect on stellar absorption line profile) were found to be much higher, $V_R \simeq 200$ km s^{-1}. Thus

$$\frac{0}{200} \lesssim \lambda_E \lesssim \frac{50}{200}$$

$$0 \lesssim \lambda_E \lesssim 0.25 \ll 1$$

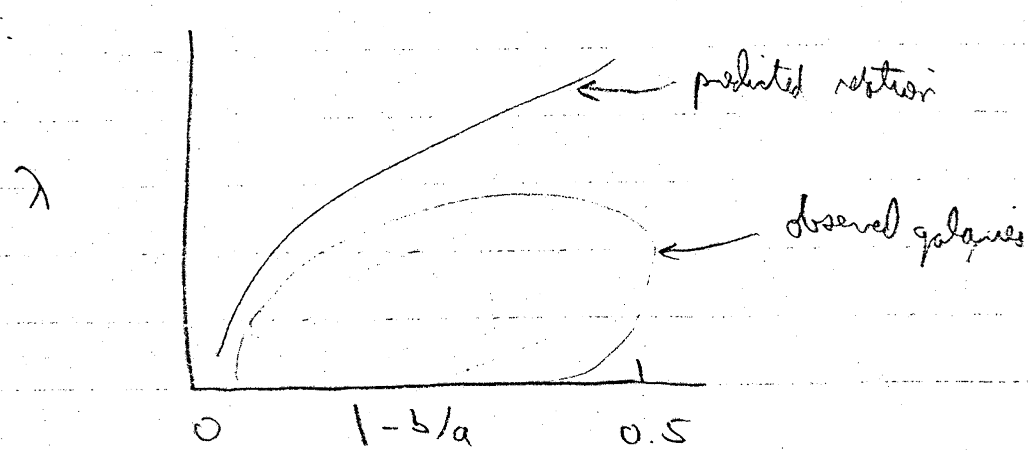
Ellipticals are apparently hot systems: they are dominated by chaotic motions.

That ellipticals are not "supported" by rotation may not seem surprising, given that they are not nearly as flat as disk galaxies. The surprising thing is that they appear to rotate too slowly to produce even the amount of flattening observed.

If you "spin up" a fluid body, like a star, it is easy to show that the body changes from a sphere to an oblate spheroid. The relation between the amount of spin and the amount of flattening is, for small spin,

$$\lambda^2 \approx \frac{8}{5}\left(1 - \frac{b}{a}\right)$$

where b/a is the ratio of minor to major axes of the oblate spheroid. For a "hot" system, $\lambda \approx 0$, and $b \approx a$, i.e. the system is spherical. For systems with little rotation, like ellipticals, this equation ought to predict the degree of flattening fairly well. In fact it does not, as shown in this diagram:



Not only do elliptical galaxies rotate slowly, they do so more slowly than expected from a simple fluid model, given their observed flattenings.

But galaxies are not fluids: because they are collisionless systems, there is not necessarily any tendency for them to revert to a spherical shape in the absence of distorting forces, as there is in a fluid body. That non-spherical galaxies can exist even in the absence of rotation was demonstrated in the late 70's by Binney and Schwarzschild.

Binney pointed out that, for a galaxy to be elongated in a certain direction, either it must be rotating, or there must somehow exist a greater degree of random internal motion in the direction of elongation. He called this "anisotropic pressure", and suggested that it accounted for the elongated shapes of ellipticals.

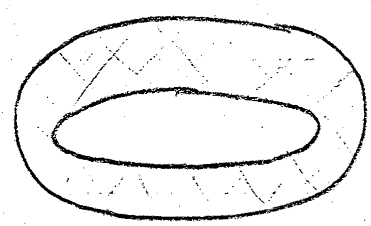
Schwarzschild showed how Binney's ideas could be used to construct a self-consistent model for a non-rotating elliptical galaxy. He did this in three steps:

First, he assumed some form for the mass distribution in his galaxy (roughly speaking, an ellipsoid with major axis ratios 1 : 5/4 : 2, and a power-law density falloff).

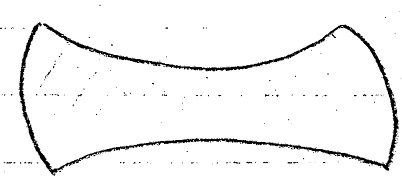
Second, he numerically computed a large (~500) number of stellar orbits in this potential.

Schwarzschild found that orbits in his model galaxy fell into two basic classes:

<u>Tube orbits</u>, which fill a roughly annular-shaped locus:



and <u>Box orbits</u>, which fill a box- or butterfly-shaped locus:



Note that tube orbits are basically the same orbits that exist in disk galaxies; they are characterized by motions in a given sense around the galaxy center, albeit with a somewhat larger random component. Box orbits are a new animal, however: their motions are primarily radial, and so there is no preferred sense of rotation; the mean velocity at every point in a box orbit is exactly zero.

Box orbits are clearly the sort which you need to "support" elliptical galaxies in the absence of rotation: in the terms used by Binney, they have a greater "pressure" along their long axis than along their short axis.

Once he had a large set of orbits, Schwarzschild completed his model construction by finding a subset of orbits which, when superposed, gave an overall mass distribution identical to that which he had assumed in his initial step. In order to do so he found that he needed tube as well as box orbits, although most of the contribution was from boxes.

The resulting model for a non-rotating elliptical galaxy is usually called the "Schwarzschild ellipsoid".

Note that the Schwarzschild ellipsoid is neither oblate nor prolate but triaxial, the most general possible shape which always looks elliptical in projection. It is now generally assumed that most elliptical galaxies are triaxial, since their absence of strong rotation eliminates any requirement that they be axially symmetric.

VI Dark Matter

It has already been pointed out that the mass of spiral galaxies as inferred from their rotation curves increases proportionately with radius, while the light falls off rapidly. This is an example of a very general phenomenon, usually called the "missing mass".

The missing mass is usually discussed in terms of the discrepancy between dynamically inferred mass, and photometrically determined light. The *mass-to-light ratio* of a system is usually defined to be

$$\frac{M}{L} \equiv \frac{\text{total mass in solar units, derived from the Virial Theorem}}{\text{total luminosity in solar units, in some wavelength range}}$$

Thus the sun has $M/L \equiv 1$. The typical star in the solar neighborhood has $M/L \simeq 2$; including the nonluminous material in the disk (eg molecular clouds) gives $M/L \simeq 5$. Elliptical galaxies have $M/L \simeq 10$ within their luminous parts. The difference is due primarily to the fact that ellipticals contain older (redder) stars than spirals.

By contrast, flat rotation curves of spirals give $M/L \gtrsim 50$.

It appears that larger systems have higher M/L's, and thus higher proportions of "dark matter". Dynamical systems bigger than galaxies exist; for instance, the Coma cluster is a roughly spherical distribution of $\gtrsim 10^3$ galaxies in a region of ~ 10 Mpc. Its mass may be determined from the Virial Theorem in the form

$$M = \frac{\langle v^2 \rangle \langle R \rangle}{G}$$

where $\langle v^2 \rangle$ is the mean square velocity of galaxies in the cluster, and $\langle R \rangle$ their mean separation. Analysis of this cluster and others gives

$$\frac{M}{L} \gtrsim 500 \quad ,$$

implying that virtually all of the matter that provides the gravitational binding in these systems is dark.

Studies like this show that the dark matter is <u>less</u> clustered than luminous matter, since the ratio of dark to light matter is greater in larger systems. An obvious assumption is that the dark matter comprising heavy halos is the same as that which binds clusters of galaxies; however, no one has managed to think up a convincing composition. Whatever the dark matter is, it has not been observed at any wavelength of the electromagnetic spectrum.

References

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