Radio Telescopes – Jim Ulvestad 6/22/84

This lecture will not be concerned with the physical properties of radio telescopes, such as their surfaces, mounts, subreflectors, receivers, and so on. I will assume that you know enough optics to realize what a Cassegrain system is, but I will discuss no detailed designs. (Among other reasons, this is because I'm not competent to do so.) Next week, Sandy Weinreb will discuss receivers in detail and will probably summarize what happens to the signal from a radio source once it is focused by the antenna.

The major purpose of this lecture will be one of definition and theory. As in most specialized fields, a considerable amount of jargon has become associated with radio astronomy, making it incomprehensible to those who haven't been initiated into the discipline. My goal today is to acquaint you with the many definitions used to described the signal acquisition characteristics of a telescope and to relate quantities measured by the telescope to the radio properties of the sky or celestial sphere.

"Radio" telescopes detect radiation at frequencies ranging from a few MHz (ionospheric cutoff) to several hundred GHz (water vapor in the atmosphere, transition to IR band). This range corresponds to wavelengths between ~100 meters and ~1 millimeter. Astronomical sources radiate at these frequencies by a variety of mechanisms, such as synchrotron processes, bremsstrahlung, and 21-cm line emission. Steve Reynolds will discuss some continuum radiation processes next week. Larry D'Addario will describe interferometers in a later lecture, but today's lecture will be restricted to single, filled aperture antennas.

The Principle of Reciprocity

Assertion: Random formulae derived for transmission of signals from a given antenna are identical when the antenna is considered as a receiver rather than a transmitter of power.

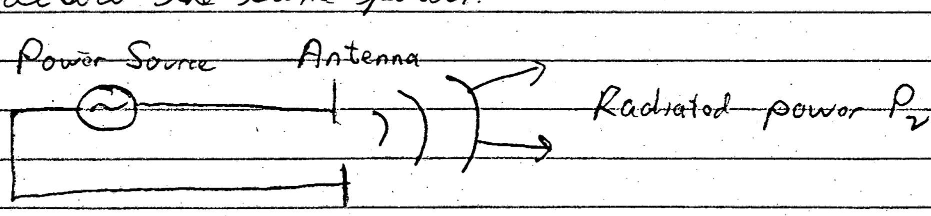
In other words, let antenna A generate a power P_A to be measured by antenna B at a distance R_{AB}. The measured power is P_B. Now, allow antenna B to transmit a power P_B. Antenna A will measure a noise power P_A, identical to the power it transmitted in the first case.

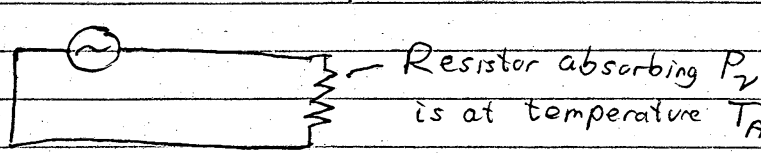
Temperatures and Brightness

Let an antenna be connected to an energy source by means of lossless transmission lines. This antenna will radiate a power P_ν, in watts per Hertz. One can replace the antenna in the circuit by a resistor that just absorbs the power P_ν. This resistor will then be raised to the temperature T_A. T_A is the antenna temperature, which is related to the power by Nyquist's formula,

$$P_\nu = k T_A \quad \text{watts Hz}^{-1}.$$

Thus the power radiated by the antenna can be thought of in terms of the temperature of the resistor that would absorb the same power.





(from Steinberg and Lequex, page 29).

Brightness Temperature and Flux Density

Let I_ν be the specific intensity (sometimes called surface brightness) of a radio source, measured in Watts m^{-2} Hz^{-1} $ster^{-1}$. In general, I_ν is a function of direction; i.e., $I_\nu \rightarrow I_\nu(\theta,\phi)$. Planck's blackbody law is

$$I_\nu(\theta,\phi) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT_B(\theta,\phi)} - 1\right)^{-1}.$$

At long radio wavelengths, the Rayleigh-Jeans approximation often holds; i.e. $h\nu \ll kT$. Expanding the exponential then gives

$$I_\nu(\theta,\phi) \simeq \frac{2kT_B(\theta,\phi)}{c^2}\nu^2 = \frac{2kT_B(\theta,\phi)}{\lambda^2}.$$

This approximation usually does not work in the high frequency (mm wave) regime. Depending on the appropriate wavelength, one of the above formulas defines the brightness temperature, $T_B(\theta,\phi)$.

Note that the brightness temperature is, in general, not the same as the antenna temperature. The relation between the two depends on the response and beam size of the telescope, which integrates over some region $(\delta\theta, \delta\phi)$ of the sky. If $P(\theta,\phi)$ is the power pattern of an antenna, normalized to unity in the direction of maximum response, we have

$$T_A = \frac{\iint_{4\pi} d\Omega\, P(\theta,\phi)\, T_B(\theta,\phi)}{\iint_{4\pi} d\Omega\, P(\theta,\phi)}.$$

For limiting cases, see the 1978 lecture notes, "Intro. to Radio Astronomy Jargon," p. 10, by Lee J. Rickard.

The flux density of a radio source is defined to be the specific intensity (brightness) integrated over solid angle:

$$S_\nu = \iint_{4\pi} I_\nu(\theta, \phi)\, d\Omega .$$

Note that S_ν is tiny for cosmic radio sources. The usual units for S_ν are Janskys, where

$$1\ \mathrm{Jy} = 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}},$$
$$1\ \mathrm{mJy} = 10^{-3}\ \mathrm{Jy}.$$

An older piece of jargon is the flux unit (f.u.),

$$1\ \mathrm{f.u.} = 1\ \mathrm{Jy}.$$

For a source whose size is larger than the primary response of the telescope, the main beam does not sample the entire radio source. This is particularly true of an interferometer, whose primary beam may be less than an arcsecond in size. Radio astronomers often use a weird unit of brightness which may be called

Jy per beam, or
Jy per clean beam.

Sometimes, they will also use

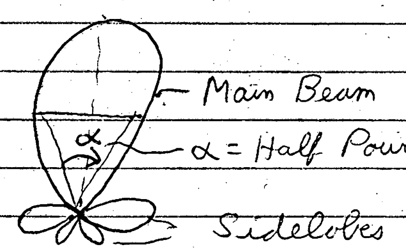
Jy per square arcsecond.

These units are essentially defined by integrating $I_\nu(\theta, \phi)$ over the relevant area: For example the brightness in Jy/beam is found from the integral

$$\iint_{\substack{\text{main}\\ \text{beam}\\ \ll 4\pi}} I_\nu(\theta, \phi)\, P(\theta, \phi)\, d\Omega .$$

Antenna Parameters

I've been using the antenna power pattern, $P(\theta,\phi)$. A polar plot of such a pattern is given below:



The half power beam width (HPBW) is

$$HPBW \approx 1.4\,\lambda/d,$$

where λ is the wavelength, d is the antenna diameter.

The beam solid angle, Ω_A, is the solid angle that would be sampled by an antenna of uniform (maximum) response within Ω_A and no response outside Ω_A:

$$\Omega_A = \iint_{4\pi} P(\theta,\phi)\,d\Omega.$$

The main beam solid angle is defined similarly as the effective solid angle sampled by the main beam:

$$\Omega_M = \iint_{\substack{\text{Main}\\\text{Beam}}} P(\theta,\phi)\,d\Omega.$$

The beam efficiency, ϵ_M, measures how well the antenna restricts its power to the main beam:

$$\epsilon_M = \frac{\Omega_M}{\Omega_A}.$$

In real life, $\epsilon_M \approx 0.75 \pm 0.15$.

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An antenna has a certain physical area or aperture size. Because of surface irregularities, blockage by subreflectors and feeds, spillover at the edges, and other similar imperfections, the entire physical area is not used at maximum efficiency. The antenna has an effective area, A_e, which varies with direction and wavelength. If the maximum effective area is A_e, the directional dependence is

$$A_e(\theta,\phi) = A_e P(\theta,\phi).$$

For an antenna of physical area A, the aperture efficiency ϵ_a is defined by

$$A_e = \epsilon_a A.$$

At centimeter wavelengths, we generally have $\epsilon_a \approx 0.7 \pm 0.1$. Depending on the telescope used, ϵ_a is often much less at 1 and 2 cm and at millimeter wavelengths.

The noise power measured by the antenna is

$$P_\nu = \frac{1}{2} \iint_{4\pi} I_\nu(\theta,\phi) A_e(\theta,\phi)\, d\Omega$$

$$= \frac{1}{2} A_e \iint_{4\pi} I_\nu(\theta,\phi) P(\theta,\phi)\, d\Omega.$$

The factor of 1/2 is because of the fact that a single receiver is sensitive to only 1 polarization.

Gain, Directivity

The word "gain" is used to describe many different concepts, and depends on the context. An antenna's directivity, D, measures how well the antenna response is concentrated in a given direction. It is defined as the ratio of maximum power to ~~total~~ average power (~~average~~ total power $/4\pi$)

$$D = \frac{P(\theta,\phi)_{max}}{\frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\,d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} P(\theta,\phi)\,d\Omega} \quad \text{for } P(\theta,\phi)_{max} \equiv 1.$$

$$\therefore \boxed{D = \frac{4\pi}{\Omega_A}}$$

The directive gain, $D(\theta,\phi)$ is the directivity in a given direction:

$$D(\theta,\phi) = D\,P(\theta,\phi).$$

It can be shown that D/A_e is constant for all antennas (see my lecture last year, or *Radio Astronomy*, by Kraus).

Since $D = \frac{4\pi}{\Omega_A}$, then $\boxed{A_e \Omega_A = \text{constant.}}$

For example, a dipole has $A_e = \frac{3}{8\pi}\lambda^2$, $D = 3/2$

$$\Rightarrow \boxed{A_e \Omega_A = \lambda^2 \text{ for all antennas.}}$$

Given an effective area of a telescope working at a wavelength λ, the telescope beam size can then be calculated from this formula.

Detection of Signals

Detection of cosmic radio sources is difficult because the antenna temperature due to the source is usually much less than the noise temperature due to the telescope and its receivers. The antenna measures a system temperature, T_{sys}

$$T_{sys} = T_{noise} + T_A. \qquad (T_{noise} \gg T_A)$$

The procedure for measuring T_A is to alternately point the telescope on and off source (usually accomplished by switching in the electronics rather than moving the telescope). T_A is then found from

$$T_A = T_{sys}\,(\text{on source}) - T_{sys}\,(\text{off source}).$$

T_A is only detectable and believable if $T_A \gtrsim 5\,\Delta T_{sys}$, where ΔT_{sys} is the uncertainty in T_{sys}.

To compute ΔT_{sys}, suppose we make N independent measurements of T_{sys}. Assuming Poisson statistics (i.e., random walk about the mean in T_{sys}-space),

$$\frac{\Delta T_{sys}}{T_{sys}} \approx \frac{K_T}{\sqrt{N}},$$

where K_T is a constant depending on the telescope and is of order unity.

Let the observing system have a bandwidth $\Delta\nu$. The minimum integration time necessary for a single independent ~~sep~~ record is $\Delta t = 1/\Delta\nu$. Why? This is a fundamental limit imposed by the Heisenberg uncertainty principle. If we restrict the measurements within the energy range ΔE, we must integrate for Δt, where $\Delta E \Delta t \simeq h$. But $\Delta E = h\Delta\nu$, so we easily see that $\Delta\nu\Delta t \approx 1$.

If τ is the total integration time, the number of independent integrations is

$$N = \frac{\tau}{\Delta t} = \tau \Delta\nu.$$

Therefore $\boxed{\Delta T_{sys} \approx \frac{K_T T_{sys}}{\sqrt{\tau \Delta\nu}}}$

This important result is the radiometry equation

Now, the minimum detectable signal is $T_A \approx 5 \Delta T_{sys}$.
But $P_\nu = k T_A \approx \frac{1}{2} A_e S_\nu$.

$$\Rightarrow \boxed{S_{\nu_{min}} \approx \frac{2 k T_{A,min}}{A_e} \approx \frac{10 k K_T T_{sys}}{A_e \sqrt{\tau \Delta\nu}}}.$$

Example

Consider the 140 foot telescope at 18 cm, where it has an effective area of ~ 787 m². Suppose we are observing a source with the Mark II VLBI system, having a 2 MHz bandwidth. The system temperature is 50 K and we wish to integrate coherently for 13 minutes. Plugging into the above formula gives

$$S_{\nu_{min}} \approx 2.2 \times 10^{-28}\ W m^{-2} Hz^{-1} = 22\ mJy.$$

For a bandwidth of 56 MHz (Mark III VLBI), $S_{\nu_{min}}$ is reduced by $\sqrt{56/2}$ to ~ 4.2 mJy.

References

Kraus, Radio Astronomy;
Christensen and Högbom, Radiotelescopes;
Steinberg and Lequex, Radio Astronomy.