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NRAO Summer Student Lecture 840708

Interferometers and Synthesis Telescopes

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An interferometer is a device for measuring the interference, or correlation, between two spatially separated signals. In the case of electromagnetic waves, the signals can be obtained from two separated antennas. In this way, certain information about the radiation field can be obtained which is not available from either antenna separately.

In radio astronomy, the main use of interferometers is now as the building blocks of synthesis telescopes, such as the VLA and the VLBA. These instruments operate by measuring the correlations of signals picked up at a wide variety of antenna spacings. From this information, it is possible to synthesize a detailed picture of the brightness distribution of a distant source.

In the present lecture, we'll develop the basic theory of synthesis telescopes. A great many details will be left out, but suggestions for further study will be given for those who are interested.

I. Mutual Coherence of Radiation Field: No Antennas.

Let's first consider some properties of the kind of electromagnetic (EM) field we are trying to measure. This will have to do with the field alone, undisturbed by any measuring devices such as antennas.

In a region of space which is free of sources and moreover is very far from all sources[1] of EM waves, it can be shown from Maxwell's equations that the EM field is describable as a collection of plane waves, where the waves are propagating in all possible directions. Since the electric and magnetic fields of plane waves are simply related, it is sufficient to consider only one of them; here we choose the electric field, $\vec{e}(\vec{r},t)$ where $\vec{r}$ is position in space and t is time. For the most part, celestial sources are very far from the regions where we build our telescopes, so the relevance of this picture is apparent.

We now restrict attention to a single frequency, f, and wavelength $\lambda = c/f$ (monochromatic case). This unrealistic restriction will be relaxed later. We can then write the electric field in terms of its complex amplitude:

$$e(\vec{r},t) = \mathrm{Re}\left\{ \vec{E}(\vec{r})\, e^{j2\pi ft} \right\} \tag{1}$$

where $\vec{E}$ is complex. We are now ready to calculate the interference of the fields at two separated points in our region, $\vec{r}_1$ and $\vec{r}_2$. Let

$$\vec{E}_1 = \vec{E}(\vec{r}_1), \quad \vec{E}_2 = \vec{E}(\vec{r}_2), \quad \vec{B} = \vec{r}_2 - \vec{r}_1,$$

and let $b(\vec{s})d\Omega(\vec{s})$ be the flux density in a plane wave from direction of unit vector $\vec{s}$, where the source subtends solid

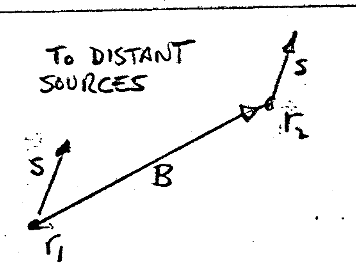

Fig 1: Geometry

[1] More precisely, we are in the "far field" of sources if their distances are >> D^2/λ where D is the size of our region of space and λ is the wavelength. Exercise: What celestial objects are not in the far field of the VLA?

angle $d\Omega$. If the contribution of the latter wave to the field at the first point is $\delta\vec{E}_1$, then its contribution at the second point is $\delta\vec{E}_2 = \delta\vec{E}_1 e^{-j2\pi f\vec{B}\cdot\vec{s}/c}$. That is, the propagation of a monochromatic plane wave involves only a change of phase. The correlation is then

$$\delta\vec{E}_1 \cdot \delta\vec{E}_2^* = |\delta E_1|^2 e^{-j2\pi f\vec{B}\cdot\vec{s}/c} \tag{2}$$

$$= b(\vec{s})\, d\Omega\, e^{-j2\pi f\vec{B}\cdot\vec{s}/c}. \tag{3}$$

If we now assume that the radiation from each direction $\vec{s}$ is independent of that from any other direction, then we can integrate (3) over all directions and obtain the total correlation:

$$\boxed{\vec{E}_1\cdot\vec{E}_2^* = \int_{\text{all } |\vec{s}|=1} b(\vec{s})\, e^{-j2\pi f\vec{B}\cdot\vec{s}/c}\, d\Omega(\vec{s})} \tag{4}$$

MUTUAL COHERENCE EQUATION.

Our ultimate goal is to recover the brightness distribution $b(\vec{s})$, which is our picture of the sky. If we can measure the quantity on the left side, perhaps we can solve the integral equation. We'll see that under certain circumstances the right side becomes a Fourier integral, in which case the solution is known: the inverse Fourier integral.

II. Measuring the Field: Insert Some Antennas.

To sample the field without disturbing it much, let's insert a small dipole antenna of length $h \ll \lambda$. The voltage induced at its terminals (across a matched load) will be approximately $\boxed{v(t) = \vec{h}\cdot\vec{e}(t)}$ where $|\vec{h}| = h$ and the orientation (5) of the antenna is included in $\vec{h}$. This is a very general antenna response equation, and works even if the antenna is not a short dipole; an "effective height" $\vec{h}$ can be determined for any antenna. $|\vec{h}|^2$ is the <u>effective area</u> which you encountered in an earlier lecture.

In terms of complex amplitudes, we can write

$$\begin{aligned} \delta V_1 &= \vec{H}_1 \cdot \delta\vec{E}_1 \\ \delta V_2 &= \vec{H}_2 \cdot \delta\vec{E}_2 \end{aligned} \qquad (6)$$

for the voltage amplitudes at two antennas located at $\vec{r}_1$ and $\vec{r}_2$. Here H_1, H_2 are called the <u>complex vector effective heights</u>.

Once we have the signals as voltages at the antenna terminals, we can amplify them, transmit them to the same place, and measure their correlation:

$$\delta V_1 \delta V_2^* = (\vec{H}_1 \cdot \delta\vec{E}_1)(\vec{H}_2 \cdot \delta\vec{E}_2) = \delta\vec{E}_1 \, \underset{\sim}{H} \, \delta\vec{E}_2 \qquad (7)$$

where $\underset{\sim}{H}$ is a tensor formed from $\vec{H}_1, \vec{H}_2$. To keep this from getting too complicated (if not so already!), let's make some simplifying assumptions: suppose the antennas are identical, so $\vec{H}_1 = \vec{H}_2 = \vec{H}$, and suppose that each antenna's polarization[2] is matched to that of the incoming waves for each direction $\vec{s}$. Then $\underset{\sim}{H}$ becomes the effective area of one antenna, $A(\vec{s})$, and (7) simplifies to

$$\delta\vec{E}_1 \, \underset{\sim}{H} \, \delta\vec{E}_2 = (\delta\vec{E}_1 \cdot \delta\vec{E}_2) \, A(\vec{s}) \qquad (8)$$

[2]Discussion of polarization is beyond the scope of the present lecture

(Notice that $\underset{\sim}{H}$ and A are functions of direction $\hat{s}$; this is because antennas are generally more sensitive in some directions than others.) Using eqn. (3) and integrating over all directions gives

$$\boxed{V(\vec{B}) = V_1 V_2^* = \int_{\text{all } |\vec{s}|=1} b(\vec{s})\, A(\vec{s})\, e^{-j 2\pi f \vec{B}\cdot\vec{s}/c}\, d\Omega(\vec{s})} \qquad (8)$$

MONOCHROMATIC INTERFEROMETER EQUATION.

The LHS is known as the complex visibility, and I've shown explicitly that it depends on the antenna spacing $\vec{B}$. It also depends on frequency, f.

III. Synthesis

Re-write (8) as

$$V(\vec{B}) = \int b'(\vec{s})\, e^{-j 2\pi f \vec{B}\cdot\vec{s}/c}\, d\Omega(\vec{s}) \qquad (9)$$

where b' = bA is called the modified brightness of the source. Well-designed antennas will have a smooth main beam large enough to cover a source of interest; within this region, the modified brightness looks about the same as the source brightness. However, only the modified brightness can be determined from measurements of $V(\vec{B})$, as (9) shows.

To synthesize an image or estimate of b', first write (9) in rectangular coordinates:

$$V(u,v,w) = \iint_{\ell^2+m^2+n^2=1} b'(\ell,m)\, e^{-j(2\pi f/c)(u\ell+vm+wn)}\, \frac{d\ell\, dm}{n} . \qquad (10)$$

The components (l,m,n) of $\vec{s}$ are direction cosines, and we have taken l and m as the independent variables. Now consider the case w = 0; i.e., let the visibility be measured only in the (u,v) plane. Then

$$V(u,v,0) = \iint\limits_{\ell^2+m^2 \le 1} \frac{b'(\ell,m)}{\sqrt{1-\ell^2-m^2}} \, e^{-j(2\pi f/c)(u\ell + vm)} \, d\ell \, dm \quad . \qquad (11)$$

The latter is a two-dimensional Fourier integral with conjugate variables ($u/\lambda, v/\lambda$) and (l,m). The functions

$$V(u,v,0) \quad \text{and} \quad \frac{b'(\ell,m)}{\sqrt{1-\ell^2-m^2}}$$

are thus a F.T. pair and we can solve for b' as

$$\boxed{b'(\ell,m) = \sqrt{1-\ell^2-m^2}\left(\frac{f}{c}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(u,v,0)\, e^{+j(2\pi f/c)(u\ell+vm)}\, du\, dv} \qquad (12)$$

FOURIER SYNTHESIS EQUATION

Apparently, then, if enough measurements of the visibility V(u,v,0) can be made, then we can compute - or synthesize[3] - the brightness of the sky b'(l,m) at frequency f.

[3] In the literature, the process is often called "aperture synthesis" and it is said that one synthesizes the aperture of a large telescope with which to observe the source. The latter is incorrect; modern synthesis telescopes have <u>no</u> equivalent aperture.

IV. A Few Annoying Details

A. <u>We can't measure everywhere we want</u>. Obviously, it would take forever to measure V for $-\infty < (u,v) < +\infty$. Instead, practical instruments take a finite set of samples $\{V(u_i,v_i),\ i = 1,\ldots,N\}$. Various "synthesized maps" are then possible, depending on how one interpolates and extrapolates from the available samples. Several reasonable schemes are in standard use. For telescopes like the VLA, N can be quite large, $\sim 10^6$.

B. <u>We can't measure anywhere we want</u>. It is hard to make all the measurements in the $w = 0$ plane. If we're stuck on earth, then even if we have a flat piece of ground, the earth's rotation will keep the antennas from remaining coplanar. However, if the source is sufficiently small in angular size, then an approximate correction can be made: let the (u,v) plane be normal to the direction to the center of the source; then if $w \neq 0$ on a particular baseline, delay the signal from antenna 2 by w/c before correlating. This gives, very nearly, the correlation that would be obtained for $w = 0$.

C. <u>We can't measure just one frequency</u>. For most celestial signals, as the bandwidth goes to zero, so does the signal strength. So our receivers and correlators must cover a band of frequencies. Generally, each frequency behaves independently and one obtains the correct result by integrating (8)-(10) over the range of frequencies observed. For small bandwidths, $b'(\vec{s})$ will be nearly constant, and the main effect is in the exponential factor. This can be neglected if $\Delta f\ \vec{B}\bullet\vec{s}/c << 1$ for all $\vec{s}$ of significance, and the latter inequality holds in many practical cases.

D. <u>We can't make any measurement perfectly</u>. Of course, all measurements are corrupted by noise. In addition, the measurements involve lots of equipment which must be stable and well calibrated, and deviations from this can cause errors. The way in which these errors propagate into maps and the kind of defects they cause are complicated subjects which could take whole lectures to explore. For further study, see:

[1] Synthesis Mapping: Proceedings of the NRAO-VLA Workshop held at Socorro, NM, June 1982, ed. by A. R. Thompson and L. R. D'Addario (NRAO: Green Bank) QB479.2/595.

[2] Fomalont, E. B. and M. C. H. Wright, "Interferometry and aperture synthesis," in Galactic and Extragalactic Radio Astronomy, pp. 256-290, Verschuur and Kellermann, eds., 1974. QB475.U54