Quasi-Optical Techniques for mm Astronomy

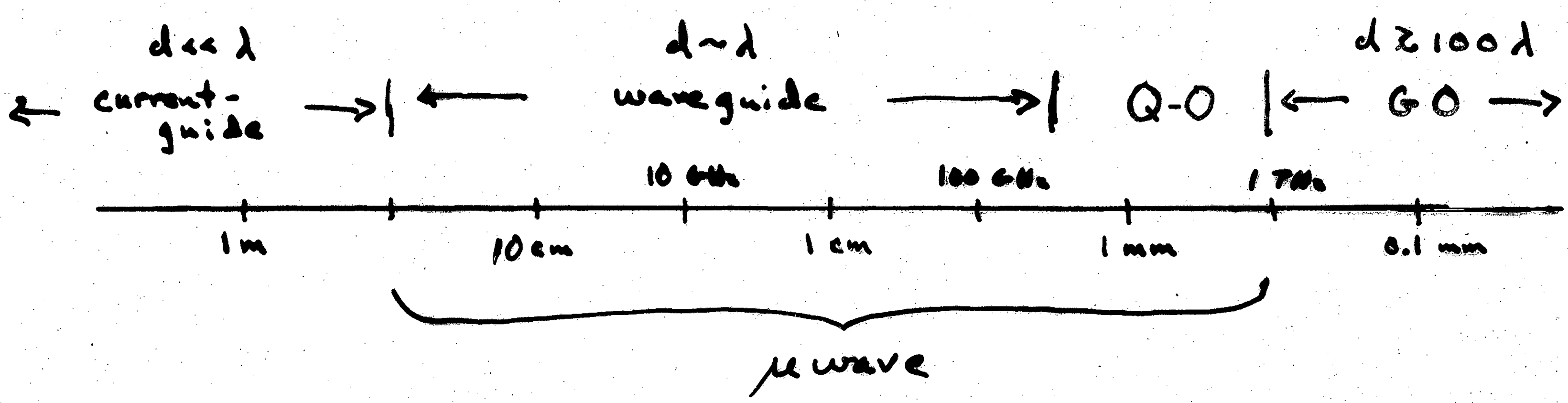
Buddy Martin
7/28/84

Refs:

Goldsmith, P.F. 1982, IR + mm Waves, ed. K.J. Button, Vol. 6, ch. 5.

Martin, D. H. + Lesurf, J. 1978, IR Physics 18 405.

Quasi-optical techniques are used to guide EM radiation at wavelengths ~ 1 mm



long wavelengths : e.g. transmission line

moderate " : currents radiate; guide Fields with waveguide

short " : (a) waveguide attenuation severe

(b) can't construct guides with $d \lesssim 1$ mm

so let waves propogate freely betw/ focusing elements

But must pay attention to wave nature of radiation:

eg often work in near field;

coherent beam has finite width + can't be truncated

Gaussian beams

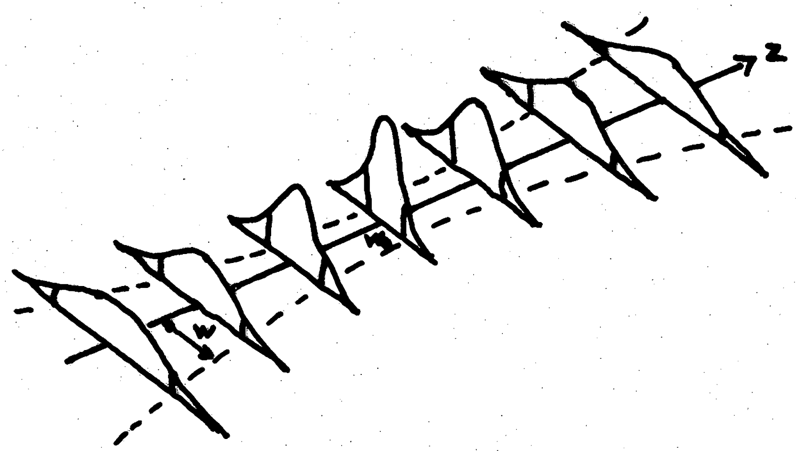
Describe rad which is fairly directed, not too different from plane wave → wave whose E-field is Gaussian in plane ⊥ propogation.

E-field $\psi = u(x,y,z)\, e^{-ikz}$ u varies slowly with z

wave eqn $\nabla^2\psi + k^2\psi = 0 \rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ik\frac{\partial u}{\partial z} = 0$

Different solns ≡ dif. beam-modes of propogation

Fundamental mode:



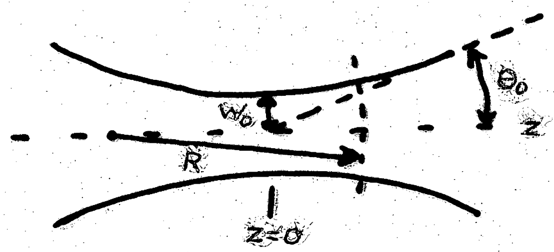
$$\psi = \frac{w_0}{w}\exp\left(\frac{-r^2}{w^2}\right)e^{-i(kz-\varphi)}\exp\left(-i\frac{kr^2}{2R}\right)$$

beam radius $w(z) = w_0\left(1+\hat{z}^2\right)^{1/2}$ (hyperbola, $\theta_0 = \frac{\lambda}{\pi w_0}$)

rad. of curv. $R(z) = z\left(1+\hat{z}^{-2}\right)$

extra phase $\varphi(z) = \arctan\hat{z}$

where $\hat{z} \equiv \frac{\lambda z}{\pi w_0^2}$



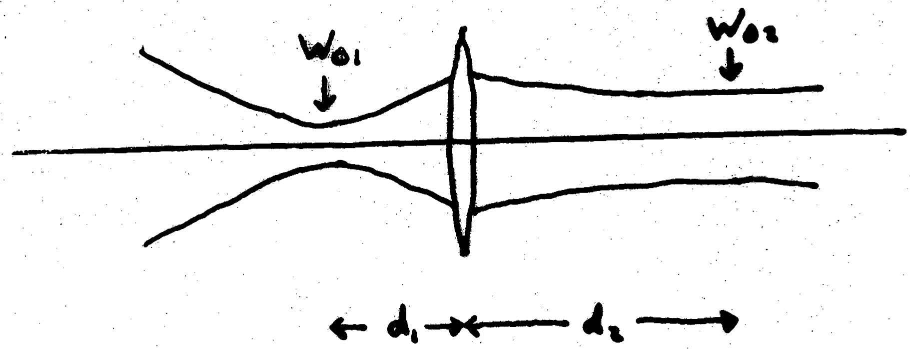
Note $R(0) = \infty$ (plane wave) while for large z $R(z) \approx z$. R_{min} occurs at $z = \frac{\pi w_0^2}{\lambda}$, natural division between near + far field

Practical concern: Can we make detector respond to fundamental Gaussian mode? Yes (at least at longer λ), scalar feeds produce symmetric pattern with low sidelobes, fit well by Gaussian down to 20 dB level

<u>Focusing</u>: lenses lossy but retain symmetry
mirrors efficient but distort beams

(truncation: require aperture radius $a \geq 2w$
$P(2w) = 3\times10^{-4} P(0) \rightarrow 3\%$ increase in far-field beamwidth)

Wave fronts spherical, so lens/mirror changes R as for light ray.

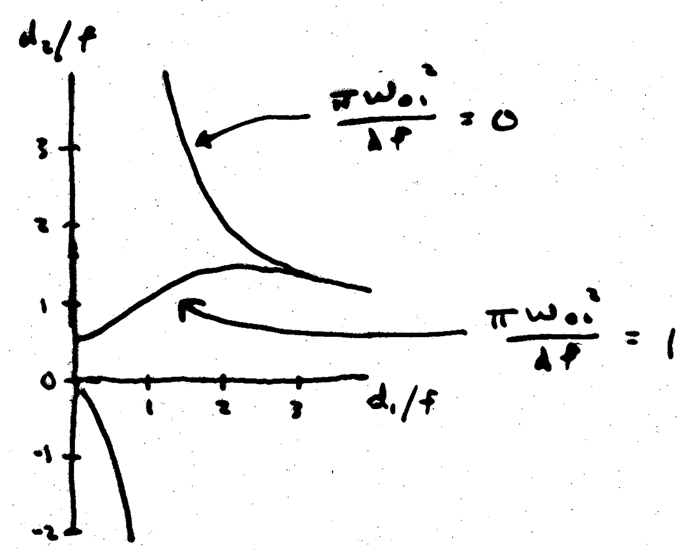


$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{f} \quad \text{and} \quad w_1(d_1) = w_2(d_2) \quad \Longrightarrow$$

$$\left(\frac{w_{02}}{w_{01}}\right)^2 = \frac{1}{\left[(d_1/f) - 1\right]^2 + \left(\pi w_{01}^2/\lambda f\right)^2}$$

$$\frac{d_2}{f} = 1 + \frac{(d_1/f) - 1}{\left[(d_1/f) - 1\right]^2 + \left(\pi w_{01}^2/\lambda f\right)^2}$$

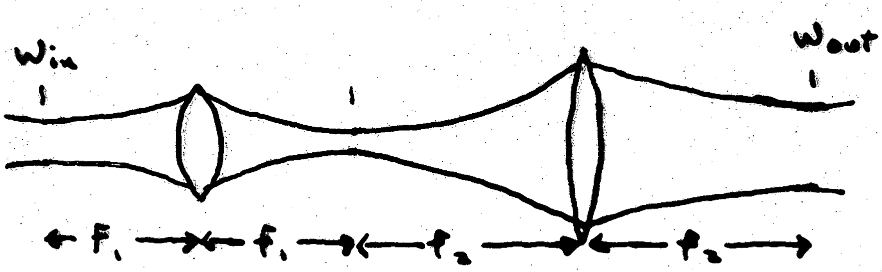
Waist locations differ from GO due to 2nd term in denom.



$d_1 = f \Rightarrow d_2 = f, \quad \frac{w_{02}}{w_{01}} = \frac{\lambda f}{\pi w_{01}^2}$

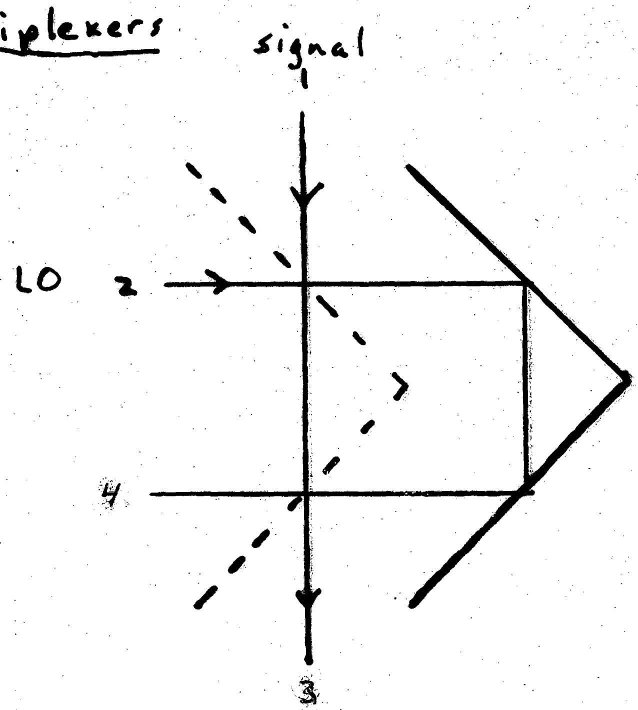
waist location indep of λ but radius not.

<u>Gaussian beam telescope</u>

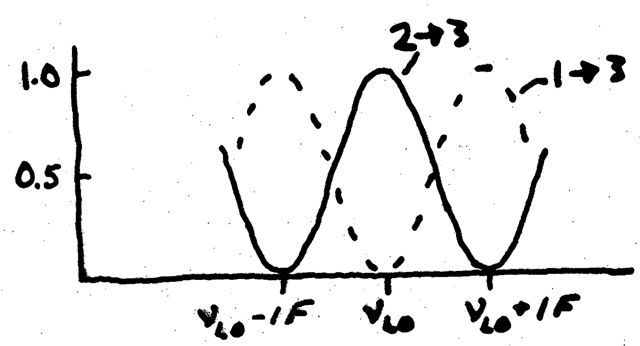


$\frac{w_{out}}{w_{in}} = \frac{f_2}{f_1}$ i.e. waist location and radius indep of λ

<u>Diplexers</u>



If reflection coef $R = 0.5$ and path difference $\Delta = (2K-1)(\lambda_{IF}/2)$ then transmission to port 3 is:



<u>Beam coupling</u>: Above depends on combination of phases only. In fact 1 beam grows relative to other, so have dif. amplitude distribs.

Power coupling coef. betw 2 Gaussian beams

$$K = |c|^2 \quad \text{where} \quad c = \int_0^\infty \int_0^{2\pi} d\phi \, dr \, r \, \psi_1 \psi_2^*$$

If ψ_1 + ψ_2 are same beam with waist displaced by Δ,

$$K_{12} = \left[1 + \left(\frac{\lambda \Delta}{2\pi w_0^2}\right)^2\right]^{-1} \simeq 1 - \left(\frac{\pi \Theta_0^2 \Delta}{2\lambda}\right)^2$$

e.g. $\Delta = \frac{\lambda_{IF}}{2} = 10$ cm, $\Theta_0 = 2°$

$$\text{loss} = 0.04 \left(\frac{\lambda}{mm}\right)^{-2}$$

Regain some of this by optimizing path halfway between 2 paths through diplexer.

But may do much worse if size of mirrors requires $\Delta = 3\lambda_{IF}/2$

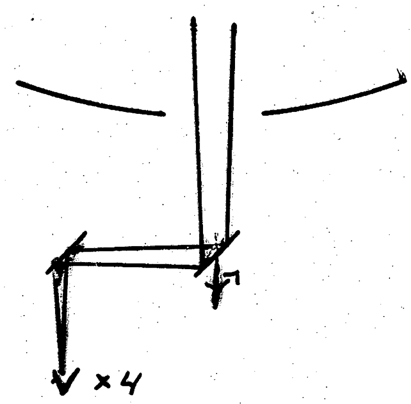
Fast beam-switcher

Beam-switch to subtract atmosphere + receiver noise. For 12 m continuum obs, low-freq. variations → imperfect subtraction ~ 3× theoretical noise if switching freq $\nu_s \lesssim 10$ Hz

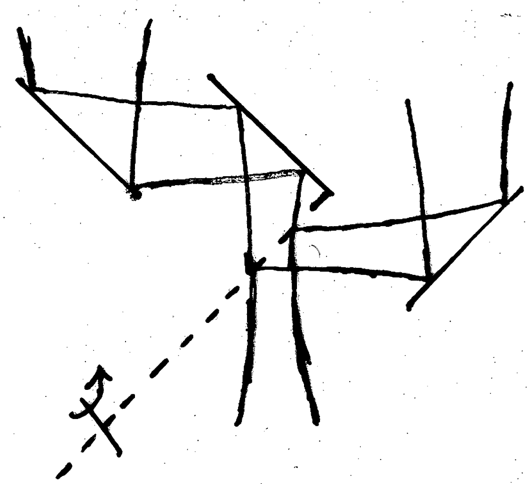
Currently wobble 2-ft subreflector at 7 Hz

Need small chopper wheel at > 30 Hz

current 12 m optics



Want to chop between 2 focal positions like:

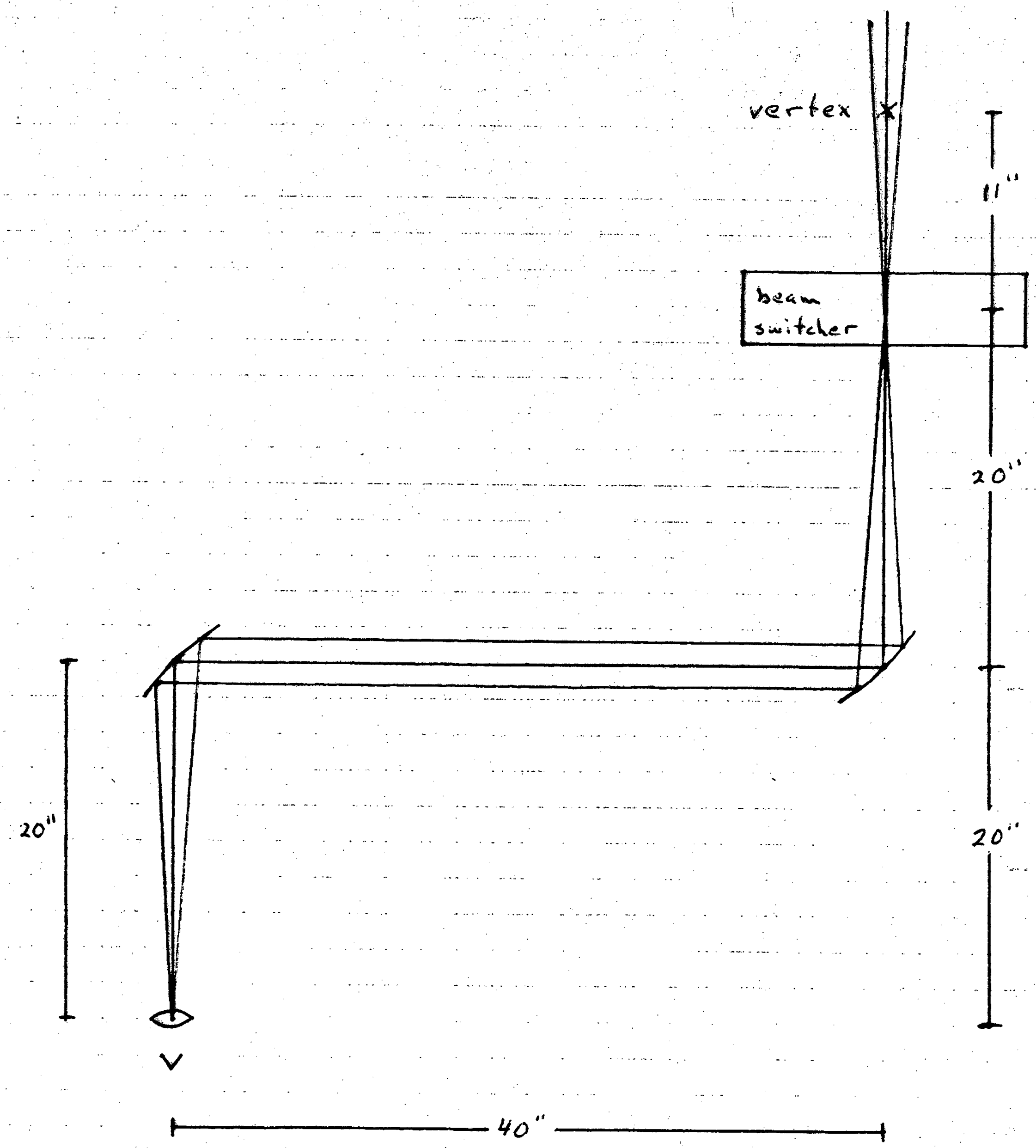


But (a) require chopper radius ≥ 3× aperture

(b) large chopper at 30 Hz rattles and maims

∴ Must have narrow beamwaist at chopper

Focusing optics for beam switcher



Scale 10:1

Mirrors are identical sections of paraboloid with F = 10"

surface given by $\rho(\theta) = \frac{2F}{1+\cos\theta}$

mirror centered at $\theta = 90°$

angular radius of mirror as seen from focus $\simeq 11°$

Works as GO system as $\lambda \to 0$, as Q-O system at $\lambda \sim 1$ mm

λ/mm	w_{01}	w_{mirror}	w_{02} (between 2 mirrors)
1	8	22	20
3	24	31	20

<u>Problems</u> due to loss of axisymmetry

1) Curvature of wavefront varies across mirror
2) Power distribution distorted by mirror
3) Cross-polarization

All 3 more serious for large offset angle and large ang. diam. of mirror. (1) more serious in near field. (3) negligible for such small mirrors.

Take account of (1) + (2) by numerically evaluating coupling between 2 Gaussian beams, one each side of mirror.

Results (for f = 20")

λ/mm	w_{01}	K
1	8	.999
3	24	.996
10	80	.877