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GALACTIC HYDROGEN LINE STUDIES

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LECTURE NOTES

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21-cm line studies, largely by astronomers at Leiden and at CSIRO, have brought about a remarkable increase in our knowledge of galactic structure. We can here give only a brief summary of what has been done thus far. Emphasis will be placed on observational results and techniques of analysis, rather than on instrumentation, which is in itself a large and complex field.

We shall first review some of the basic equations necessary for the analysis of 21-cm line observations. Following this we will briefly discuss 21-cm line determinations of galactic rotation. Knowledge of the nature of the large scale motions of our galaxy is fundamental to all galactic 21-cm line studies. We will then take up some of the problems of galactic structure, more or less in the order of decreasing size. First, the gross, or large scale, structure of the galactic disk; second, the structure of the central region of the galaxy, within about three kilo-parsecs of the galactic center; third, local structure in the solar neighborhood; and finally, the relationship between interstellar hydrogen and regions or objects of special interest in the galaxy, such as interstellar dust, star clusters, and associations. Because this presentation will necessarily be rather sketchy, a fairly extensive bibliography has been included.

1. Basic Equations

1.1 We summarize here some of the basic equations used in the analysis of 21-cm line observations, without derivation. For a more detailed discussion of the analytical tools and techniques, the reader is referred to

Heeschen (1955) or to several of the Leiden papers in BAN Volumes 12 and 13.

The brightness temperature in the hydrogen line, $\Delta T(\nu)$, is given by

$$\Delta T(\nu) = T_K (1 - e^{-\tau_\nu}), \tag{1}$$

where T_K is the kinetic temperature of interstellar hydrogen and τ_ν is the optical depth in the line at frequency ν. This equation is strictly correct only if the excitation of the hydrogen line is by collisional processes, if T_K is constant along the line of sight, and if continuous absorption and emission processes can be neglected. If the line excitation is not by the collisional processes, T_K must be replaced by a temperature -- the so-called spin temperature -- which better expresses the relative population of the two hyperfine levels of the ground state of neutral hydrogen. In most cases encountered in galactic 21-cm line work, equation (1) is appropriate, the principal exception being in the case of absorption line studies.

The optical depth in the line is given by

$$\tau_\nu = 5.44 \times 10^{-14} \int_0^\infty \frac{n}{T_K} f(V)\,ds, \tag{2}$$

where n is the total number of hydrogen atoms per cm^3 and $f(V)$, with units cm^{-1} seconds, is a line shape function. This equation is generally valid, as long as most of the hydrogen atoms are in the ground state. If $f(V)$ and T_K are constant along the line of sight, equation (2) becomes

$$\tau_\nu = 5.44 \times 10^{-14} \frac{N_H}{T_K} f(V) \tag{3}$$

where $N = \int_0^\infty n\,ds$. The broadening of the hydrogen line, represented by $f(V)$, is in most cases due to random motions of the hydrogen clouds. Two types of function have been most commonly used for $f(V)$, a gaussian, and an exponential.

For the case $\tau << 1$ equations (1) and (2) reduce to

$$\Delta T(\upsilon) = \tau_\upsilon T_K = 5.44 \times 10^{-14} N_H f(V) \qquad (4)$$

We also have in this case

$$\int_0^\infty \Delta T(\upsilon)\, d\upsilon = 5.44 \times 10^{-14} N_H \int_{-0}^{+\infty} f(V)\, dV = 5.44 \times 10^{-14} N_H, \qquad (5)$$

that is, the area under a line profile is proportional to N_H, the surface density of neutral hydrogen.

2. Galactic Rotation

2.1 A knowledge of galactic rotation is essential for almost all galactic hydrogen line work, because it provides a means of determining the distance of an HI cloud or complex emitting 21-cm line radiation. Galactic rotation at distances from the center greater than the sun's distance cannot be determined directly from 21-cm line observations. In most 21-cm line work, a function $\omega(R)$ determined by Schmidt (1956) has been used. This law of galactic rotation was obtained from a model of mass distribution in the galaxy, which in turn is based in large part on values of $\omega(R)$ for $R < R_0$ determined from 21-cm line observations.

2.2 Galactic rotation in the inner region of the galaxy--at distances from the center less than the sun's distance--can be studied from 21-cm line observations. The radial velocity, relative to the local standard of rest, of a point distance R from the galactic center in a direction making an angle λ with the direction to the center, is, assuming circular galactic rotation:

$$V_r = R_o \left\{ \omega(R) - \omega_o \right\} \sin \lambda , \qquad (6)$$

where R_0 is the sun's distance from the center and ω is the angular velocity of rotation. For a given value of λ less than 90^o, as one moves away from the sun, V_r goes through a maximum (positive or negative depending on which side of the galaxy we look) and then changes signs for $R > R_0$. The maximum radial velocity occurs at the point where the radius vector from the galactic center is perpendicular to the line of sight. The distance from the center of the position of maximum velocity, is

$$R_m = R_0 \sin \lambda \quad (7)$$

2.3 $\omega(R)$ $(R < R_0)$ has been determined from hydrogen line observations, with the aid of equations (6) and (7), by Kwee, Muller and Westerhout (1954) and by Kerr (1962). The two sets of measurements, made in opposite halves of the galaxy, agree rather well, and in general the behaviour of the circular galactic rotation at distances of from about three to eight kpc. from the center is reasonably well established. At small distances from the center $\sin \lambda$ becomes small and $\omega(R)$ cannot be determined as reliable.

2.4 To determine $\omega(R)$ from 21-cm line observations, it is necessary that 1) R_0 and ω_0 are known. Errors in these parameters will produce both scale and zero point errors in the derived $\omega(R)$, but will not change its general form. 2) There is HI at, or very near, the point of maximum radial velocity along a given line of sight. 3) The motion of the gas is circular, with no radial motion and with no significant systematic peculiar motions in any particular region. 4) The random velocity of HI clouds is known, and can be allowed for in the analysis.

Probably none of these conditions is completely fulfilled, the result being to introduce some uncertainty in the derived $\omega(R)$. The effects of these uncertainties, and the possibility of large scale deviations from circular motion, will be discussed in later sections.

3. Large Scale Galactic Structure

3.1 A major objective of 21-cm line work is to provide a picture of the density distribution of neutral hydrogen in the plane of the galaxy. The starting point for all 21-cm line studies of galactic structure is the galactic rotation curve, $\omega(R)$. An observed line profile gives the radial velocities of major HI emitting regions in a particular direction. Distances to the HI regions are then obtained from the observed velocities and the adopted galactic rotation curve. The integrated area of a line profile, or of a feature within a line profile, is a measure of N_H. N_H can then be converted to HI density by estimating the extent of the HI complex from the shape of the line profile and the galactic rotation curve, or by adopting from other considerations an extent along the line of sight of, say, a spiral arm.

In practice, the procedure is complicated by a number of factors. Uncertainties in $\omega(R)$ obviously carry over directly into uncertainties in any derived neutral hydrogen distribution. Local systematic deviations from the general galactic rotation may cause emitting clouds to be placed at incorrect distances and thus somewhat distort the derived distribution. Any general, large scale, deviation from circular motion again distorts the distribution. Finally, the random motions within HI clouds, or cloud complexes, broadens the line profile and makes the true velocity distribution more difficult to determine. The effect of this broadening by random motions on the true velocity distribution in the line profile is similar to instrumental broadening, such as the broadening of the response of a point source by an antenna beam, or the broadening of a spectral line by the finite width of the slit, and can be treated in a similar fashion. Unfortunately, however, the magnitude of random cloud motions may differ from one region of sky to another, or along the line of sight in a given region. The average magnitude of these random motions is known reasonably well and can be allowed for in the analysis, but the variations mentioned above may again distort the derived gas distribution.

The inner region of the galaxy ($R < R_0$) presents an additional difficulty in analysis because of the ambiguity in radial velocity. Along any line of sight from the sun there are two regions, equidistant from the center, at which the gas will have the same radial velocity due to galactic rotation. Sorting out this ambiguity is difficult and uncertain.

3.2 Very extensive studies of the neutral hydrogen distribution in the plane of the galaxy have been made at Leiden (van de Hulst, Muller, and Oort, 1954; Westerhout, 1957), and at Sydney (Kerr, Hindman, and Gum, 1959). The Leiden observations cover roughly galactic longitudes 320° to 240° (old coordinates), while the Sydney workers concentrated on those longitudes unavailable to Leiden. Most of the analysis in the above references is limited to the outer region of the galaxy, $R > R_0$. Schmidt (1957) has made a detailed and complicated analysis of the gas distribution in the inner region, $R < R_0$.

From these studies there has come an excellent first picture of the large scale distribution of neutral hydrogen in the galaxy. The hydrogen is clearly distributed in lanes, or spiral arms, and a number of such arms can be recognized and followed for considerable distances around the center. The fact that such a picture of the distribution, with well-defined general features, is obtained indicates that the problems of interpretation mentioned in 3.1 are not enough to swamp the gross features of the hydrogen distribution.

On the other hand, the detailed distribution appears quite complex. Some of this apparent complexity is undoubtedly due to the factors mentioned above -- systematic deviations from general galactic rotation, variations in random motion, etc.

The current picture of neutral hydrogen distribution shows several features which suggest that not all of the assumptions which have been made in the analysis -- and which in the present state of our knowledge must be made -- are completely valid. The best defined "spiral arms" appear very nearly circular about the galactic center, rather than spiral. In the vicinity of the sun, where analysis is especially difficult because of the small radial velocities and resultant larger relative effect of random velocities, there is a suspicious symmetry with respect to the sun. Finally, the Leiden and Sydney portions of the distribution do not fit together as well as might be hoped.

In spite of these unresolved problems, the general picture of the neutral hydrogen distribution which has come from the work at Sydney and Leiden is a most important and remarkable contribution to the study of galactic structure. Further refinements should lead to better knowledge of random motions and of systematic deviations from circular galactic rotation.

3.3 Kerr (1962) has examined in some detail the fit between the Leiden and Sydney models. He believes that use of the Leiden circular velocity model leads to an implausible spiral structure in the southern Milky Way. He finds that one way to produce a better fit on the two sides of the galaxy is to postulate a radial expansion, in addition to the rotation, amounting to seven km/sec at the sun's distance. An alternate possibility, leading to similar results, is a local outward systematic motion of the sun and gas in the solar neighborhood of seven km/sec. There is at present no direct evidence of such motion, except the improvement it makes in the appearance of the derived neutral hydrogen distribution. It is an interesting possibility which needs further investigation.

Some local deviations from circular motion must certainly exist. Westerhout (1957) and others have pointed out regions in which systematic motions appear to be present. However, about the only way at present to distinguish systematic motions is by postulating that the distribution of neutral hydrogen is smooth in the galaxy and then looking for regions which, on the basis of the assumption of no systematic motions other than galactic rotation, seem to violate this postulate. There are unfortunately no reliable distance criteria for 21 centimeter line emitting regions, independent of the galactic rotation curve, which is itself derived from 21 centimeter line observations with the aid of the assumption we wish to test.

3.4 The 21 centimeter studies referred to above have shown that the bulk of the interstellar gas is confined to a very thin disk. The thickness of the disk, between points where the hydrogen density falls to half its maximum value, is on the order of 200 parsecs. The disk is also remarkably flat, except in the outermost regions. There, distortions of the disk have been noted, amounting in some cases to several hundred parsecs (Burke, 1957; Kerr, 1957; Westerhout, 1957). The distortion is of the form of a general bending down of the hydrogen layer in the outer region of the galaxy, in the quadrant centered around longitude 240°, and a similar upward bend in the opposite direction. Several explanations of this distortion have been proposed (above references, and Kahn and Woltjer, 1959) but no thoroughly convincing explanation has yet been presented.

The flatness of the hydrogen layer has been used, along with other radio and optical data, to make a new determination of the principal plane of the galaxy (Blaauw, Gum, Pawsey, and Westerhout, 1960). A new system of galactic coordinates based in large part on this determination of the HI plane, has been adopted by the IAU.

4. The Central Region of the Galaxy

4.1 The central part of the galaxy, within about 3 kiloparsecs of the galactic center, exhibits some interesting and complex features. In 1957 it was found (van Woerden, Rougoor and Oort, 1957) that a spiral arm about 3 kiloparsecs from the center was expanding with a velocity of some 50 kilometers per second, as well as taking part in the general galactic rotation. Rougoor and Oort (1960) have subsequently made an intensive investigation of the distribution and motion of neutral hydrogen in the central region, using emission and absorption line profiles obtained with the 82-foot telescope at Dwingeloo. They have developed the following picture. In the center of the galaxy is a disk of neutral hydrogen. The hydrogen density at the center must be very high, perhaps 1,000 atoms/cm^3 at a distance of 10 parsecs, and drop off rapidly with increasing distance from the center until there is practically no gas at a distance of 300 to 350 parsecs. From this distance to about 500 parsecs there appears to be very little neutral hydrogen. At 500 parsecs a ring of neutral hydrogen about 100 parsecs wide appears. The ring and central disk have a thickness of only about 80 parsecs, and both are rotating with high velocity -- the ring at some 265 kilometers per second, the disk at velocities up to about 220 kilometers per second near its edge. Neither the disk nor the ring appear to be expanding.

Between the sun and the center of the galaxy is a spiral arm, the so-called 3 kiloparsec arm, at about that distance from the galactic center. It has been traced over some 90° of galactocentric longitude. This arm participates in galactic rotation, with a circular velocity of about 200 kilometers per second. It is also expanding outward from the center with a velocity of 50 kilometers per second. The thickness of the arm is 120 parsecs -- only one-half the thickness of the neutral hydrogen layer further from the center. On the far side of the galactic center, gas appears to be streaming outward with velocities of 100 to 200 kilometers per second.

Rougoor and Oort compute that, from the observed rate of expansion, all gas in the central region should be removed in the order of 10^7 to 10^8 years if there is no replenishment. Since this time is much shorter than the age of the galaxy there must be replenishment of gas in the central region, probably by gas streaming into the nuclear region from the galactic halo. Pariiski (1961) has discussed a possible exchange of gas between the halo and the nuclear region of the galaxy.

4.2 Some other phenomena associated with this central region of the galaxy may have a bearing on the problem of neutral hydrogen distribution and motion. The intense continuum radio source Sagittarius A is generally considered to be at the center of the galaxy, primarily because its position coincides so closely with that of the center, and absorption studies place its distance at about that of the galactic center. Drake, (1959) investigated this source at 3.75 centimeter wavelength, with relatively high resolution. He found that it consists of two small diameter sources within 15 parsecs of the galactic center. In addition, he found two sources in the galactic plane symmetrically placed about 80 parsecs on either side of the center. Drake interprets these latter two sources as being the tangential points of a ring of continuum emission. All four of the continuum sources lie within the central disk of neutral hydrogen.

Westerhout (1958) has analyzed observations of the galactic continuum radiation to obtain a model of the distribution of ionized hydrogen in the galaxy.

He finds practically no ionized hydrogen within 3 kiloparsecs of the galactic center, and a strong, sharp peak in the ionized hydrogen density about 3.5 to 4 kiloparsecs from the center, just outside the rapidly expanding HI arm. The relationship, if any, between this ionized hydrogen concentration, the complex source Sagittarius A, and the complex distribution and motion of neutral hydrogen in the central hydrogen in the central region is not fully understood. Further investigations can be expected to lead to exciting results relating to the dynamics and driving forces of the galaxy.

5. Local Structure of Neutral Hydrogen

5.1 Studies of local structure in the solar neighborhood are difficult at low galactic latitudes because the radio velocity of galactic rotation is small and therefore doesn't provide much assistance in separating hydrogen emission regions at different distances. Most investigations of local structure have therefore utilized 21 centimeter line observations at intermediate and high galactic latitudes. Some of the questions we would like to have answered about the local structure of neutral hydrogen are:

1) What is the local distribution? Is there a cloud or cloud complex of neutral hydrogen around the sun?
2) What is the solar motion with respect to the nearby interstellar gas?
3) Are there local peculiar gas motions? What is the z-component of motion of the hydrogen above and below the plane in the galaxy in the solar neighborhood? Is there any evidence for gas streaming into or out of the galactic halo in the vicinity of the sun?

5.2 Heeschen and Lilley (1954) studied the neutral hydrogen distribution as a function of galactic latitude at the longitudes of the galactic center and anti-center. They found secondary maxima in the hydrogen distribution at an angle of about 20° to the plane of the galaxy which might be associated with the well-known Goulds belt phenomenon. Davies (1960) concluded from a more extensive study of 21-centimeter line observations out of the plane of the galaxy, that the sun is probably situated in a local cloud complex of neutral hydrogen, dust, and young stars. The so-called Goulds belt is one manifestation of this local system. The strong secondary concentrations of the neutral hydrogen observed at latitude $+20^{\circ}$ in the longitude of the galactic center and -20° in the longitude of the anti-center can be interpreted in other ways, however -- for example, as being associated with the well-known cloud complexes of Ophiuchus and Taurus, which might be completely unrelated -- and other groups have found no strong evidence for a local system of hydrogen. This question therefore must remain open.

5.3 The radial velocity of local hydrogen has been studied by various investigators. McGee and Murray (1961) find that in the galactic plane hydrogen is streaming away from the sun, in the directions of the center and the anti-center, with a velocity of about +6 kilometers per second, while perpendicular to the plane hydrogen is streaming into the solar region, from above and below, with about the same velocity. Erickson, Helfer, and Tatel (1959) find evidence that the local motion is in general circular, about the galactic center, with some complex non-circular components.

There is as yet, however, no clear picture of either the distribution or the motion of neutral hydrogen in the solar neighborhood. Nor has any full scale attempt yet been made to obtain the solar motion relative to local hydrogen. Lilley and Brouer are about to start a long term observing program aimed at this latter problem. A great deal more work on local structure is obviously needed.

5.4 Almost all workers agree on the thickness of the hydrogen layer in the solar neighborhood, about 200 parsecs, and on the density of neutral hydrogen in the solar neighborhood, about 0.5 atoms/cm^3. There is no agreement, however, on the position of the sun -- some investigators placing it 50 parsecs above the plane, others in, or slightly below, the plane.

6. Gas and Dust

6.1 The relationship between interstellar dust and neutral hydrogen has been studied by a number of investigators (Lilley 1955; Heeschen 1955; Bok, Lawrence, and Menon, 1955; Davies, 1956; and some of the Leiden papers previously referred to). Lilley (1955) was the first to show that in general a positive correlation exists between the amounts of neutral hydrogen and dust present in a given direction. He found an average gas to dust ratio of about 100. The correlation between gas and dust is only rough, however, and the gas to dust ratio appears to vary considerably from one region to another. Some of this may be due to observational difficulties, and to the difficulty of determining the gas and dust content along identical lengths of path in a given region. In the case of several heavily obscured regions, however, there definitely seems to be a lower 21-cm line intensity than one would predict from the gas to dust ration, and in some cases the intensity is even lower than that in neighboring unobscured regions. Attempts have been made to explain this by postulating that in heavily obscured regions atomic hydrogen is more easily converted to molecular hydrogen with the aid of the high concentration of dust, thereby reducing the atomic hydrogen content.

6.2 Davies (1956) made a detailed study of a region in Auriga in which the neutral hydrogen intensity was significantly lower than that in surrounding regions. He found that the low intensity HI "cloud" coincided in position with an obscuring dust cloud. From an analysis of the difference in hydrogen line profiles in neighboring regions and in the region of the cloud, Davies found that the kinetic temperature in the cloud was 60°K -- significantly lower than the average kinetic temperature of interstellar neutral hydrogen. He also deduced a very high neutral hydrogen density, of about 250 atoms/cm^3, and a gas to dust ratio of 300. Davies has found several other such "clouds" which yielded similar results. He postulates that the presence of molecular hydrogen in these clouds has cooled down the neutral hydrogen to well below the average temperature of the interstellar medium. This mechanism may help explain the apparent break-down of correlation between gas and dust in heavily obscured regions.

7. Correlation between Neutral Hydrogen and Stellar Associations and Clusters

7.1 Heeschen and Drake (1956) have made observations which suggest that there is a cloud or cloud complex, of neutral hydrogen associated with the Pleiades star cluster. Drake (1958) observed other clusters as well, and found evidence for neutral hydrogen associated with several of them. Helfer and Tatel (1959), on the other hand, interpreted their observations in the region of the Pleiades as indicating no association between neutral hydrogen and the star cluster. This question remains to be resolved.

7.2 A number of investigations have been made of the relation between neutral hydrogen and stellar OB associations (Howard, 1958; Kassim, 1961; Menon, 1956, 1958; Raimond, 1957; Dieter, 1960; Wade, 1957; Matthews, 1956). In some cases there clearly seems to be interstellar hydrogen associated with the stellar association. In other cases, there does not appear to be hydrogen connected with the stellar association, or observations by different observers are not in agreement. Thus no clear-cut general picture has yet emerged regarding the relationship between interstellar hydrogen and stellar associations.

7.3 The general procedure which has been used in attempts to relate interstellar hydrogen with stellar clusters or associations has been to look for an excess of hydrogen line emission at the position, and at the radial velocity of the cluster or association. This procedure is valid in principal, but runs into a number of difficulties in practice. Most clusters and associations are in or near the galactic plane, where there is generally a long path length in neutral hydrogen. The 21-cm line profiles are therefore usually rather complex, and only a small portion of the total hydrogen line emission observed from the region containing a stellar cluster or association may be expected to arise from hydrogen associated with the cluster or association. Separation of this component, if it exists, from the general interstellar field may be quite difficult.

8. Conclusion

8.1 It is clear that a great deal more work is required in the field of galactic 21-cm line studies. Refinement, and a general filling in of details, of the excellent picure of gross neutral hydrogen distribution developed by the Leiden and Sydney groups is required. Problems of the local structure and motion of neutral hydrogen, and of the relationships between HI and dust, stellar clusters and associations, and HII regions are essentially barely touched thus far. Much remains to be learned also of the small scale structure of neutral hydrogen, that is, studies of the interstellar cloud structure.

The physical conditions of interstellar gas, especially temperature and random motions, need much additional study. It has generally been assumed in hydrogen line work that the kinetic temperature of hydrogen is, on the average, constant, with a value of about 125°K. The principal evidence in favor of this assumption is the fact that brightness temperatures in the hydrogen line up to about this value, but never above it, have been observed. Davies has shown, however, that, in some regions at least, considerably lower temperatures may prevail. Further studies of random motions in clouds, or cloud complexes, and of the small scale cloud structure of the interstellar medium, are important. Knowledge of these parameters is important to almost all other galactic hydrogen line work, because they effectively smear out, or distort, the true systematic velocity distribution of interstellar hydrogen.

Most of the problems require high resolution in both frequency and direction, and very stable receivers. The question of whether there is a detectable amount of neutral hydrogen in the galactic halo remains open also. Here a small telescope might be useful, but the receiver must be extremely sensitive and stable.

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REFERENCES

Blaauw, A., Gum, C.S., Pawsey, J.L., and Westerhout, G. 1960, MN 121, 123.

Bok, B.J., Lawrence, R.S., and Menon, T.K., 1955, PASP 67, 108.

Burke, B.F., 1957, A.J. 62, 90.

Davies, R.D., 1956, MN 116, 443.

Davies, R.D., 1960, MN 120, 483.

Dieter, N.A., 1960, Ap J 132, 49.

Drake, F.D., 1959, Nat. Rad. Ast. Obs. Ann. Rpt.

Drake, F.D., 1958, Thesis, Harvard Univ.

Erickson,W.C., Helfer, H.L., and Tatel, H.E., 1959, IAU Symp. No. 9, pp. 390.

Gum, C.S., Kerr, F.J., and Westerhout, G., 1960, MN 121, 132.

Heeschen, D.S., and Drake, F.D., 1956, A.J. 61, 5.

Heeschen, D.S., and Lilley, A.E., 1954, Publ. Nat. Acad. Sci. 40, 1095.

Heeschen, D.S., 1955, Ap J 121, 569.

Helfer, H.L., and Tatel, H.E., 1959, Ap J 129, 565.

Howard, W.E. III, 1958, A.J. 63, 50.

Hulst, H.C. van de, Muller, C.A., and Oort, J.H., 1954, BAN 12, 117, No. 452.

Kahn, F.D., and Woltjer, L., 1959, Ap J 130, 705.

Kassim, M.A., 1961, Ap J 133, 821.

Kerr, F. J., 1957, A.J. 62, 93.

Kerr, F. J., 1957, MN 123, 327.

Kerr, F.J., Hindman, J.V., and Gum, C.S., 1959, Aust. J. Phys. 12, 270.

Kwee, K.K., Muller, C.A., and Westerhout, G., 1954, BAN 12, 211, No. 458.

Lilley, A.E., 1955, Ap J 121, 559.

Mathews, R.T., 1956, Thesis, Harvard Univ.

Menon, T.K., 1956, A.J. 61, 9.

Menon, T.K., 1956, Ap J 127, 28.

McGee, R.X., and Murray, J.D., 1961, Aust. J. Phys. 14, 260.

Pariiski, Yu. N., Soc. Ast. A.J., 5, 280, 1961.

Raimond, E., 1957, BAN 13, 269, No. 475.

Rougoor, G.W., and Oort, J.H., 1960, Proc. Nat. Acad. Sci. 46, 1.

Schmidt, M., 1956, BAN 13, 15, No. 468.

Schmidt, M., 1957, BAN 13, 247, No. 475.

Wade, C.M., 1957, A.J. 62, 148.

Westerhout, G., 1957, BAN 13, 201, No. 475.

Westerhout, G., 1958, BAN 14, 215, No. 488.

van Woerden, H., Rougoor, W., and Oort, J., 1957, C.R. 244, 1691.