NATIONAL RADIO ASTRONOMY OBSERVATORY

VLA ANTENNA MEMORANDUM NO. 3

REPORT ON THE SPECIFICATION OF THE SERVO ASSEMBLY FOR THE VERY LARGE ARRAY (VLA) PROTOTYPE ANTENNA

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Introduction

This report is supplementary information to the specifications for the control system of the VLA prototype. It contains a summary of work done to establish the specifications and other detail considerations involved in establishing the specifications.

The work started with a study of the system to establish the state of the technology involved. This included a review of the literature on design methods and parametric values such as wind gust properties. A continuation of this work and a study of the Rohr Corporation work [1] for the VLA constituted a second phase. A third phase included the writing of the prototype specification and this report.

Many parts of the specification were taken from the Rohr Corporation report. However, there are some exceptions both in the final recommendations and the details of arriving at similar conclusions. Most of these exceptions were made necessary by factors such as the need to define a final configuration, insufficient time to establish agreement with their recommendations, and to arbitrary differences in form. Detail calculations are not repeated if there is substantial agreement with the Rohr report.

The following major subject areas are considered:

1) Modeling
2) Wind gust phenomena
3) Wind gust induced errors
4) Parameter studies and components

SECTION I

Modeling

Several models of varying complexity have been analyzed in an attempt to obtain a realistic description of the system. The simplest useful model neglects gear box stiffness, antenna equivalent spring and damping constants, and soil dynamics. It serves the purpose of establishing the system gain constant and other parameters and an approximation of tracking errors.

The modeling starts by writing a transfer function for a comparitor amplifier, drive motor, gear box and the antenna represented by a simple inertial load. These are the minimum essential components for a system to steer an antenna in either azimuth or elevation. This analysis yields the open loop transfer function

$$G(s) = \frac{K_v}{s(\tau_m s+1)} \tag{1.1}$$

where: K_v = motor CEMF constant
T_m = motor time constant (defined in Eq. 1.22)

The next step is to consider errors which result from the tracking inputs

$$E = \sin^{-1}[\sin\delta\sin\phi+\cos\delta\cos\phi\cos H] \tag{1.2}$$

$$\dot{E} = -\dot{H}\,\frac{\cos\delta\cos\phi\sin H}{\cos E} \tag{1.3}$$

$$\ddot{E} = -\dot{H}\,\dot{A}\,\cos\phi\cos A \tag{1.4}$$

$$A = \cos^{-1}\ \frac{\sin\delta - \sin\phi\sin E}{\cos\phi\cos E} \tag{1.5}$$

$$\dot{A} = -\dot{H}\,(\sin\phi - \cos\phi\tan E\cos A) \tag{1.6}$$

$$\ddot{A} = -\dot{H}\cos\phi\ [\dot{A}\ \sin A\ \tan E - \dot{E}\sec^2 E\ \cos A] \tag{1.7}$$

where

E = elevation angle

A = azimuth angle

ϕ = station latitude

δ = declination angle

H = hour angle

The rate of change of hour angle is a constant dependent on the rotation of the earth. It is, $\dot{H} = 15\ °/\mathrm{hr}$.

Higher order derivatives of E and A were not calculated because the dynamic error series required to use them is very time consuming to derive. It will be assumed that the error series converges rapidly so that errors due to higher derivatives of the input may be neglected. Other designers [1,2] implicitly make this assumption and it is believed that the higher order terms are negligible. It could be validated by simulation.

The dynamic error series is

$$e(t) = C_0 r(t) + C_1 r'(t) + C_2 r''(t) + \ldots\ldots \tag{1.8}$$

where

$$C_i = \int_0^\infty (-\tau)^i W_e(\tau)\,d\tau \qquad i = 0,\ 1,\ 2,\ \ldots\ldots \tag{1.9}$$

and

e = system error

r = system input (Eqs. 1.2 through 1.7)

W_e = system error weighting function

τ = time variable

It can be shown [3] that

$$C_i = \left[\frac{d^i}{ds^i} \; W_e(s) \right]_{s = 0} \tag{1.10}$$

If two poles and two zeros are introduced into Eq. 1.1 for compensation, the transfer function becomes

$$G(s) = \frac{K_v(\tau_1 s+1)}{s(\tau_2 s+1)(\tau_3 s+1)} \tag{1.11}$$

One of the zeros cancels the pole due to the motor at $1/\tau_m$. It can be shown that

$$C_0 = 0 \tag{1.12}$$

$$C_1 = \frac{1}{K_v} \tag{1.13}$$

$$C_2 = \frac{(2\tau_2+2\tau_3)K_v-3(1+\tau_1 K_v)}{K_v^{\,2}} \tag{1.14}$$

The worst case of the inputs occur for the azimuth servo. This is when E = 87.5°, which is as close to the zenith as required by the specification. A site in the southwestern part of the country is assumed. The values are

$\dot{A} = 4.92\ ^\circ/\text{min}$

$= 1.43 \times 10^{-3}$ rad/sec

$\ddot{A} = 0.266\ ^\circ/\text{min}^2$

$= 1.29 \times 10^{-6}$ rad/sec^2

Using the above analysis it can be shown that even if a relatively large tracking error (e.g. 5 seconds) is allowed, such a large value of K_v is required that the system is, in effect a type 2 system after compensation. Therefore, an integrator will be made a basic part of the system to reduce tracking error. The first realistic open loop model (transfer function) for the system becomes

$$G(s) = \frac{K_a(\tau_2 s+1)}{s^2(\tau_1 s+1)} \tag{1.15}$$

as shown in Fig. 1.1. The pole $(1/\tau_m)$ due to the motor and antenna inertia has been cancelled out by a zero at $1/\tau_m$ in the compensation network. This is illustrated by Fig. 1.2.

The block diagram of Fig. 1.2 and the parameters it contains are established by the analysis which follows. The torque generated by the servo motor at the output of the gear train is

$$T = nK_T i_a \tag{1.16}$$

where

T is torque

n is the gear train speed ratio; the gear train input divided by the output

K_T is the servo motor torque constant

i_a is the servo motor armature current

K_a is the gain constant (i.e., the value of the low frequency minus two slope segment at one rad/sec on the asymptotic alpha Bode diagram)

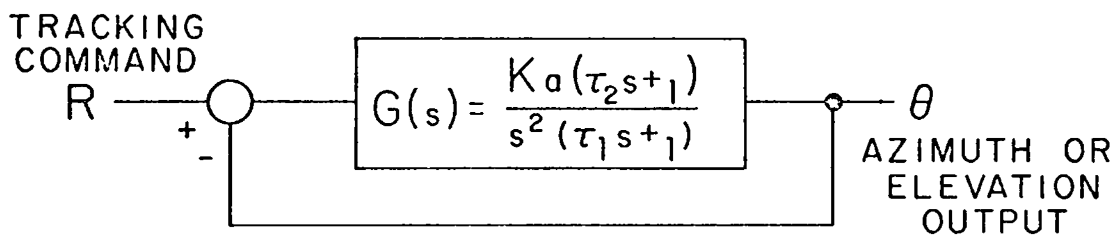

Fig. 1.1 Elementary model of servo.

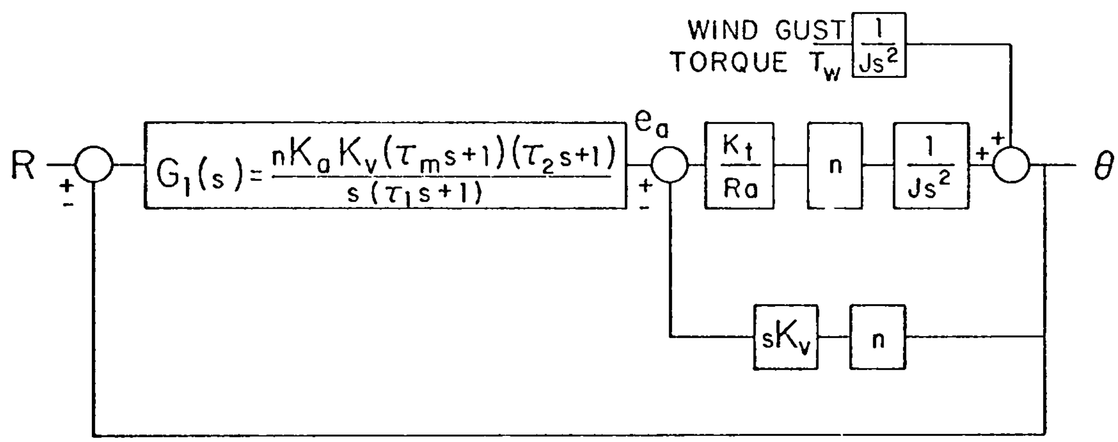

Fig. 1.2 Elementary model of servo including wind gust disturbances.

Rohr [1] established a value of

$$n = 31{,}500$$

which is specified for the prototype. The Rohr report also establishes the superiority of using an electrical drive system.

The inertial load is equal to the sum of the disturbance torque and the generated torque, i.e.

$$T_W + nK_T i_a = J\ddot{\Theta} \tag{1.17}$$

where

J is the inertia of the antenna, gear train and servo motor referred to the gear train output

Θ is output position, either azimuth or elevation

T_W is the wind gust torque

This relationship assumes negligible linear friction, a reasonable assumption for the orders of magnitude involved.

The servo motor input voltage is

$$e_a = i_a R_a + nK_v \dot{\Theta} \tag{1.18}$$

where

e_a is the servo motor input voltage

R_a is the servo motor armature resistance

K_v is the servo motor back emf constant

The armature current from Eq. 1.18 is

$$i_a(s) = \frac{e_a(s) - nsK_v\theta(s)}{R_a} \tag{1.19}$$

This defines a portion of the block diagram. From Eq. 1.17

$$\theta(s) = \frac{nK_T i_a(s)+T_w(s)}{Js^2} \tag{1.20}$$

This equation defines the remainder of the block diagram associated with the output part of the system.

Equations 1.18 and 1.19 may be combined by elimination of i_a and letting T_w be zero to yield

$$\frac{\theta(s)}{e_a(s)} = \frac{1/K_v n}{s(\frac{JR_a}{K_v K_T n^2} s + 1)} \tag{1.21}$$

where it is convenient to define

$$\tau_m = \frac{JR_a}{K_v K_T n^2} \tag{1.22}$$

The value of τ_m will be in the neighborhood of 0.005 seconds.

The transfer function relating the wind gust torque and antenna position is required to make the error calculations. It may be obtained from Fig. 1.2 by noting that

$$\theta(s) = \frac{T_w(s)}{Js^2} - \frac{n^2 K_v K_T}{R_a Js} \theta(s) - \frac{nG_1(s)K_T}{R_a Js^2} \theta(s)$$

$$= \frac{1/J}{s + \frac{n^2 K_v K_T}{R_a J} s + \frac{nK_T}{R_a J} G_1(s)} T(s) \tag{1.23}$$

Using G_1 from Fig. 1.2 and τ_m from Eq. 1.22 the transfer function becomes

$$\frac{\theta(s)}{T(s)} = \frac{[1/\tau_2 J]\ [\tau_2 s^2+s]}{s^4+\left[\frac{1}{\tau_2}+\frac{1}{\tau_m}\right]s^3+\frac{1}{\tau_2}\left[\frac{1}{\tau_m}+K_a\tau_1\right]s^2+\frac{K_a}{\tau_2}\left[\frac{\tau_1+\tau_m}{\tau_m}\right]s+\frac{K_a}{\tau_m\tau_2}} \tag{1.24}$$

The values of the parameters in Eq. 1.24 are established by the error and relative stability considerations. It can be shown that the coefficients in Eq. 1.8 for the type 2 model of Eq. 1.15 are

$$\begin{aligned} C_0 &= 0 \\ C_1 &= 0 \\ C_2 &= \frac{1}{K_a} \end{aligned} \tag{1.25}$$

Equations 1.8 and 1.25 establish the tracking error as

$$e = \frac{a_0}{K_a} \tag{1.26}$$

where a_0 is $\ddot{A}$.

The cost is negligible for making the tracking error small. Thus, the system gain will be established on the basis of a 0.25 second error which is 1.21×10^{-6} radians. From Eq. 1.26

$$\begin{aligned} K_a &= \frac{1.29 \times 10^{-6}}{1.21 \times 10^{-6}} \\ &= 1.1 \end{aligned}$$

The relative stability will arbitrarily be defined as a phase margin of 45°. This value is based on experience and judgment. This is a commonly used industrial value. It can be shown [3] that the optimum value of crossover frequency, ω_c, is

$$\omega_c = \frac{1}{\sqrt{\tau_1 \tau_2}} \tag{1.28}$$

Crossover frequency is the frequency at which the open loop transfer function has the magnitude 1.0. A practical value for ω_c will probably be in the range of 1 - 3 rad/sec. A Bode attenuation diagram is shown in Fig. 1.3 for a value of $\omega_c = 1.7$. The constants for Eq. 1.24 are

$$\omega_c = 1.7 \text{ rad/sec} \quad \tau_1 = 1.47 \text{ sec} \quad \tau_2 = 0.235 \text{ sec} \quad K_a = 1.15 \tag{1.29}$$

From the geometry of the diagram in Fig. 1.3 it is apparent that

$$\tau_1 = \frac{2.5}{\omega_c} \tag{1.30}$$

$$K_a = \frac{2.5}{\tau_1^{\,2}} \tag{1.31}$$

$$\tau_2 = \frac{1}{\tau_1 \omega_c^{\,2}} \tag{1.32}$$

if the relative stability is to be kept constant while ω_c (the bandwidth) is varied. Note that K_a can vary over a wide range and yield an acceptable error. A lower bound on K_a is in the range of 0.2.

It is interesting to note at this point that the error due to relatively low frequency wind fluctuations is negligible. Assume that the wind results in a step function of torque

$$T_w(s) = \frac{|T_w|}{s} \tag{1.33}$$

According to the final value theorm

$$\theta(\infty) = \lim_{s \to 0} s \frac{|T_w|}{s} \frac{\theta(s)}{T(s)} \tag{1.34}$$

$$= 0$$

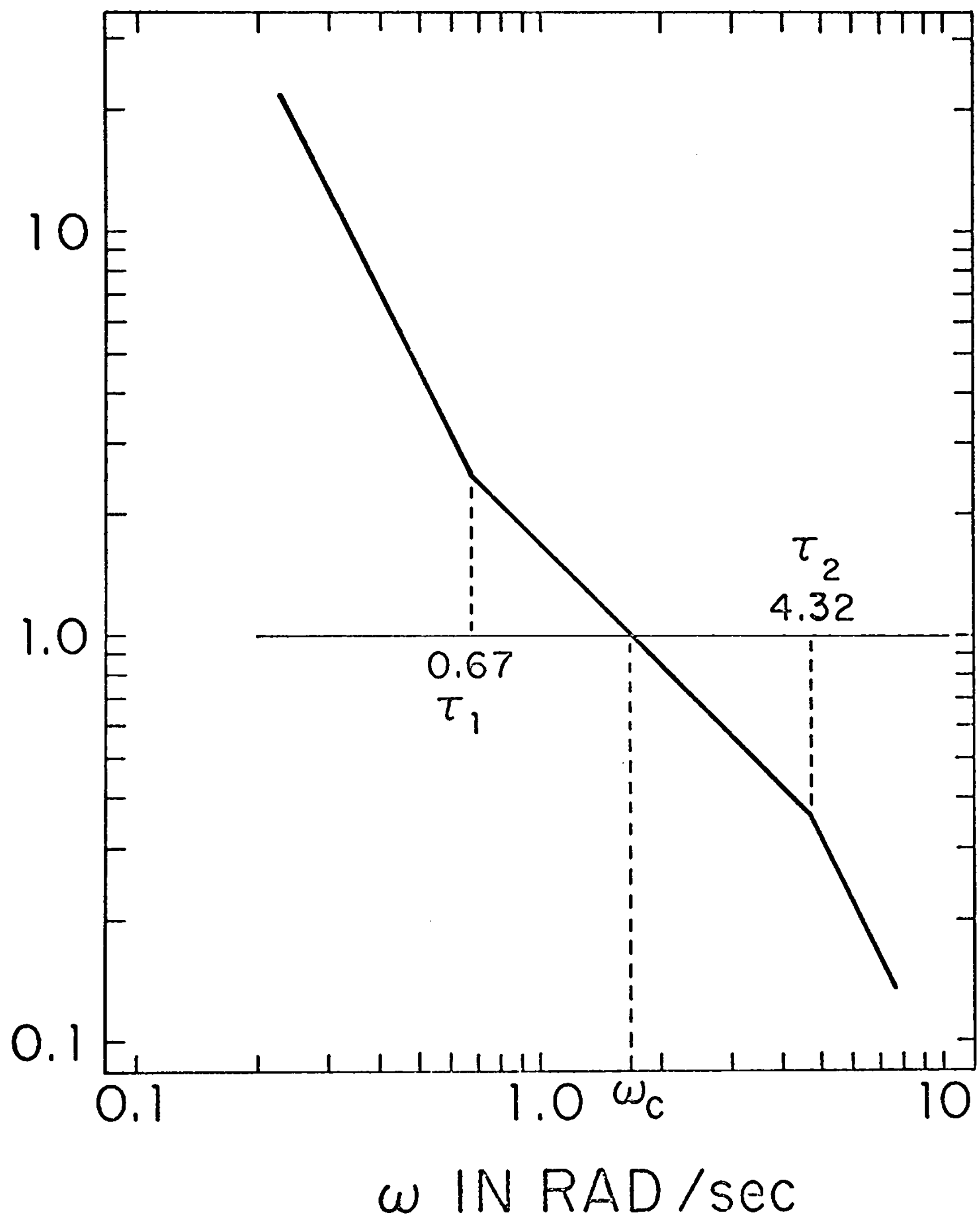

Fig. 1.3 Compensated asymptotic open loop Bode attenuation diagram.

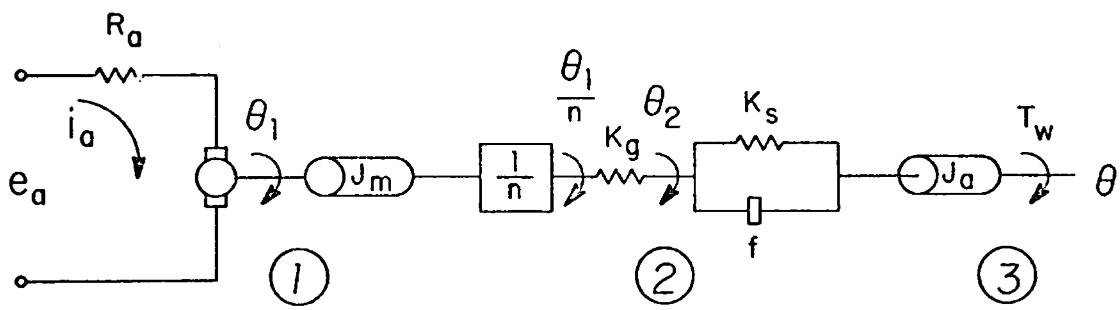

Fig. 1.4 Schematic of drive section containing spring and damping elements.

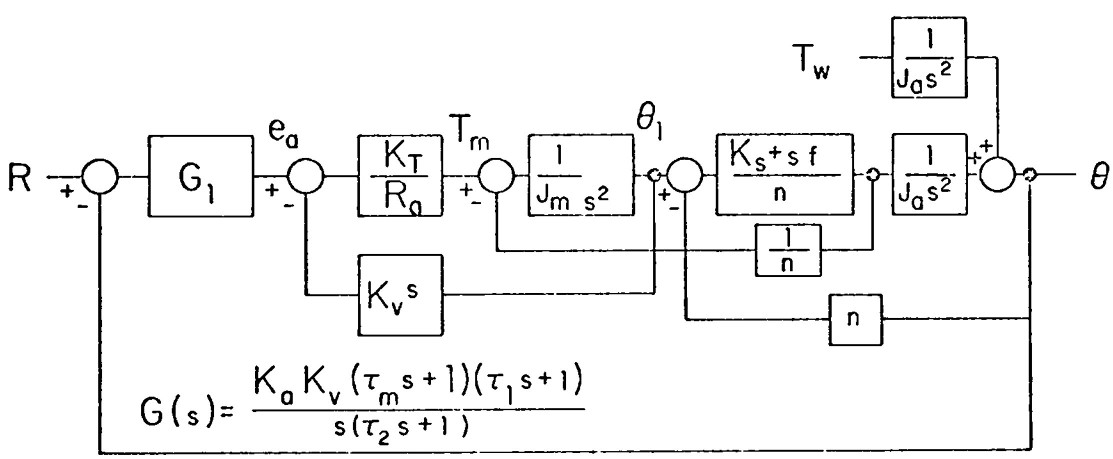

Fig. 1.5 Block diagram for model containing antenna spring constant and damper elements.

Equation 1.24 and other results of the analysis are used later in the I_5 calculations to predict wind gust errors.

Model Containing a Resonance Due to the Antenna

Modification of the previous model by including equivalent spring and damping constants for the antenna makes the model more difficult to use in calculations but analytical results should represent more closely the actual system.

A schematic of the system is shown in Fig. 1.4 for the motor, gear box and antenna. This drive section may be described by the equations

$$e_a(s) = i_a(s)R_a + K_v s\Theta_1 \tag{1.35}$$

$$\tau_m(s) = K_T i_a(s) \tag{1.36}$$

$$\tau_m(s) - \left[\Theta_1(s) - n\Theta(s)\right]\frac{\left(K_s + fs\right)}{n^2} = J_m s^2 \Theta_1(s) \tag{1.37}$$

$$T_w(s) + \left[\theta_1(s) - n\Theta(s)\right]\frac{\left(K_s + sf\right)}{n} = J_a s^2 \theta(s) \tag{1.38}$$

A block diagram which represents these equations is shown in Fig. 1.5.

The transfer function from wind torque disturbances to antenna position is

$$\frac{\theta(s)}{T_w(s)} = \frac{m_4 s^4 + m_3 s^3 + m_2 s^2 + m_1 s}{n_6 s^6 + n_5 s^5 + n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0} \tag{1.39}$$

where

$$m_1 = \frac{K_s}{n^2 J_m \tau_2}$$

$$m_2 = \frac{K_s}{n^2 J_m} + \frac{1}{\tau_2 \tau_m} + \frac{K_s \tau_f}{n^2 J_m \tau_2}$$

$$m_3 = \frac{1}{\tau_m} + \frac{K_s \tau_f}{n^2 J_m} + \frac{1}{\tau_2}$$

$$m_4 = 1.0 \tag{1.40}$$

$$n_0 = \frac{K_a K_s}{n\ \tau_m \tau_2}$$

$$n_1 = \frac{K_a K_s \tau_f}{n\ \tau_m \tau_2} + \frac{K_a K_s (\tau_1 + \tau_m)}{n\ \tau_m \tau_2}$$

$$n_2 = \frac{K_s}{\tau_m \tau_2} + \frac{K_a K_s \tau_f (\tau_1 + \tau_m)}{n\ \tau_2 \tau_m} + \frac{K_a \tau_1 K_s}{n\ \tau_2}$$

$$n_3 = \frac{K_s \tau_f}{\tau_m \tau_2} + \frac{(\tau_2 + \tau_m) K_s}{\tau_m \tau_2} + \frac{K_a \tau_1 \tau_f K_s}{n\ \tau_2} + \frac{J_a K_s}{n^2 \tau_2 J_m}$$

$$n_4 = \frac{J_a K_s}{J_m \tau_2} \left[\frac{J_m}{\tau_m K_s} + \frac{\tau_m + \tau_2}{n^2} \right] + \frac{(\tau_2 + \tau_m)\ K_s \tau_f}{\tau_m \tau_2} + K_s$$

$$n_5 = \frac{J_a K_s}{J_m \tau_2} \left[\frac{(\tau_m + \tau_2) J_m}{\tau_m K_s} + \tau_2 \tau_m \right] + K_s \tau_f$$

$$n_6 = J_a \tag{1.41}$$

The constant τ_f and τ_m are defined as

$$\tau_f = \frac{f}{K_s} \qquad \tau_m = \frac{J_m R_a}{K_T K_v} \tag{1.42}$$

and G_1 is defined in Fig.1.5. The following values for the azimuth axis are from Rohr's report [1]:

$$K_s = 2.84 \times 10^8 \text{ ft \#/rad}$$
$$J_a = 1.8 \times 10^6 \text{ ft \# sec}^2$$
$$n = 31{,}500 \tag{1.43}$$

The value of f has been calculated by assuming a damping factor of ζ=0.02. The resulting value of f is

$$f = 9.05 \times 10^5 \text{ ft \#/rad/sec} \tag{1.44}$$

The validity of this assumption is established in the section on wind gust error calculations where the model is used.

Model Which Includes the Gear Train Spring Constant and Tachometer Feedback

The gear train spring constant will be in the neighborhood of $K_g = 10^9$ ft #/rad. Approximate calculations indicate that this parameter may not be negligible, especially if tachometer feedback is required to obtain sufficient system output stiffness to reduce wind gust disturbance error to an acceptable value.

The tachometer has been assumed to be at the output of the gear train so that it can help compensate for gear train windup.

A schematic of the drive components is shown in Fig. 1.6. The dynamic equations which describe the drive section are

$$e_a(s) = i_a(s)R_a + K_v s\Theta_1(s) \tag{1.45}$$

$$T_m(s) - \frac{\left[\Theta_1(s) - n\Theta_2(s)\right] K_g}{n^2} = J_m s^2 \Theta_1(s) \tag{1.46}$$

$$T_w(s) + \left[\theta_2(s) - \theta(s)\right] (K_s + sf) = J_a s^2 \theta(s) \tag{1.47}$$

$$- K_g \left[\theta_2(s) - \frac{\theta_1(s)}{n}\right] - (K_s + sf) \left[\theta_2(s) - \theta(s)\right] = 0 \tag{1.48}$$

A block diagram which represents these equations and the other parts of the model is shown in Fig. 1.7.

The transfer function from T_w to θ has been written and incorporated in a computer program which makes error calculations due to wind gusts. It is not included here because it is rather large. Copies of the program are available.

All modeling has assumed that soil dynamics are negligible. This means that poles due to the pedestal foundation and soil acting together must be at higher frequencies than locked rotor poles.

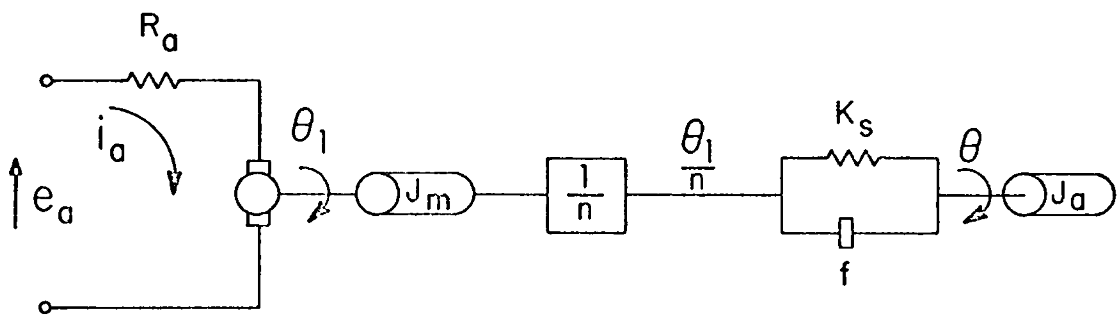

Fig. 1.6 Schematic of drive components including the gear train spring constant.

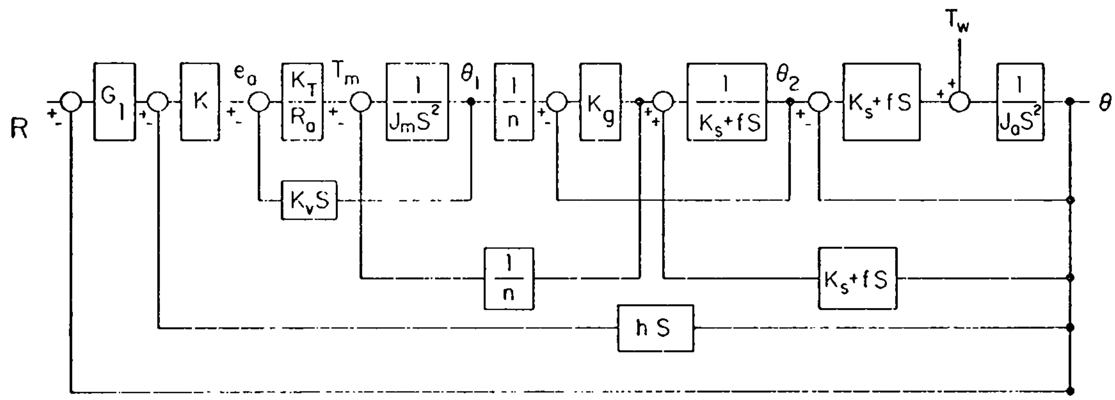

Fig. 1.7 Servo model consisting of gear train spring characteristic and tachometer.

SECTION II

Wind Gust Error

The random error caused by wind gusting is predicted to be one of the largest items in the control system error budget. For this reason, error calculations have been made to predict system error performance as accurately as available data will allow. Slowly varying wind fluctuations cause negligible servo error because the positioning control system can produce correcting torques, as demonstrated by Eq. 1.34 if the fluctuation frequencies are low compared to the servo system bandwidth. Thus, gusting is the primary wind problem for conditions when the average wind velocity is below the maximum allowable operational value used in the design.

Pointing error caused by gusting is described statistically. The mean squared value of error is compatible with the wind gust description and other system errors so it will be used here.

The three major problem areas involved in making gust error calculations are:

(1) Wind description
(2) Antenna wind resistance characteristics
(3) Control system model

The modeling problems were considered in the first section, and the other two problems are considered in this section.

A reference for the theory which follows is Newton, et al [4]. The best data found in a literature search on wind gust characteristics is by Titus [5] even though it is not certain that it is directly applicable. Certain questions concerning gusting errors have not been treated adequately in the literature. Site data is being taken to verify the analyses and it is anticipated that additional data will be taken on the prototype antenna.

Wind Gust Descriotion and its Effect on the Antenna

A suggested description [4] for the wind velocity function using an autocorrelation function is

$$\phi_{vv}(\tau) = \overline{|V_1^2(t)|}\ e^{-\omega_g|\tau|} \tag{2.1}$$

where

$\overline{V_1^2(t)}$ is the mean squared value of the wind fluctuation about the average (or mean) component, V_o

ω_g is the break frequency in the sprectrum in rad/sec

τ is a time variable in seconds

By definition [4], the power density spectrum is the Fourier transform of the autocorrelation function divided by 2π. From the autocorrelation function of Eq. 2.1

$$\phi_{uu}(s) = \frac{\overline{|V_1^2(t)|}}{2\pi}\ \frac{\omega_g}{(-s^2+\omega_g^2)} \tag{2.2}$$

The use of this form for the power-desnsity spectrum was experimentally justified by the relatively complete experimental study by Titus [5] at the Stump Neck Annex of the Indian Head Naval Power Factory in Maryland for the wind gust torque induced in a particular antenna. Fig. 2.1 is page 23 of that study. It is typical

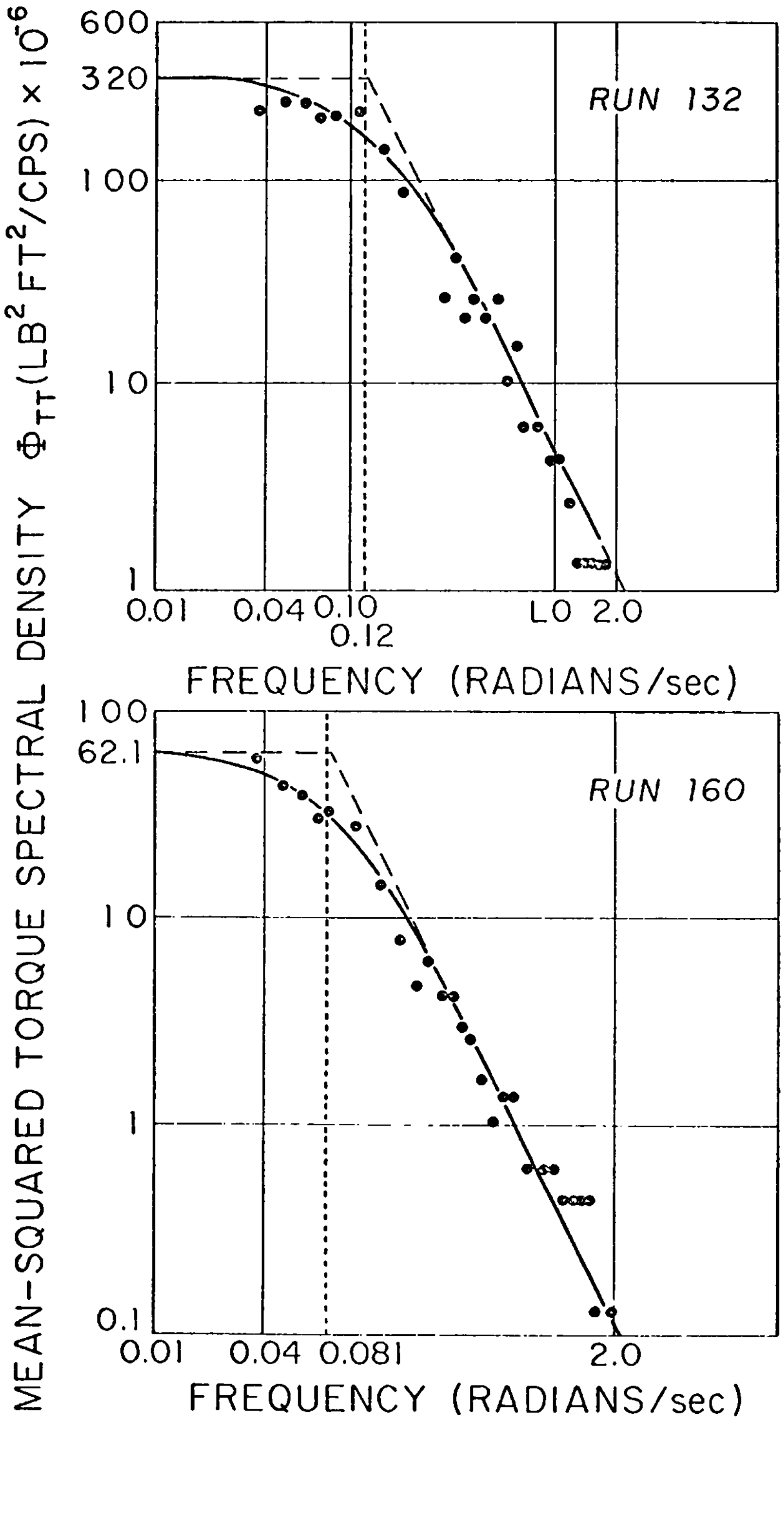

Fig. 2.1 Log-log plots of torque spectral density vs frequency.

of results from 60 different 33-minute runs of data they took with a variety of wind conditions and antenna elevation orientations. They present actual (measured) torque data on a 60-ft antenna (Kennedy). Torque and wind gusts are related below.

Note that the low frequency value of the power-density spectrum function from Eq. 2.2 is

$$\phi_{uu}(o) = \lim_{s \to o} \Phi_{uu}(s) = \frac{\overline{V_1^2(t)}}{2\pi\omega_g} \qquad (2.3)$$

and that the high frequency value decreases as the inverse square of frequency, which results in a slope of -2 on a log-log graph.

Wind torque, T, in ft-lb is proportional to the square of wind velocity, i.e.

$$T(t) = C_w V^2(t) \qquad (2.4)$$

The constant C_w depends on a number of factors as shown in the following. The total instantaneous wind component is V(t).

The dynamic pressure, P, developed by the wind on an antenna is

$$P = C_D \rho \, V^2(t) \qquad (2.5)$$

in lbs/ft where

C_D is the drag coefficient

ρ is the density of air - 2.38 x 10 slugs/ft for standard air at sea level

The torque can also be expressed as

$$T(t) = \frac{\pi}{4} D^2 \sigma L P(t) \tag{2.6}$$

where

D is the antenna diameter - 82 ft.

σ is the surface porosity factor

L is an equivalent lever arm for the antenna wind gusts

The torque will depend on the direction of the wind relative to the antenna. This factor could be included in the drag coefficient or one of the other terms, but it will not be included because only the worst case will be used for the calculations.

From Eqs. 2.5 and 2.6

$$T(t) = \frac{\pi}{4} D^2 \sigma L C_D \rho V^2(t) \tag{2.7}$$

From Eqs. 2.4 and 2.7 it is apparent that

$$C_w = \frac{\pi}{4} D^2 \sigma L C_D \rho \tag{2.8}$$

The value of C_w will be taken from the Rohr study as 200 lb sec^2/ft.

Let the instantaneous wind velocity V(t) be defined as

$$V(t) = V_o + V_1(t) \tag{2.9}$$

where V_o is the average, (or mean) wind velocity. $V_1(t)$ is the instantaneous variation of wind velocity relative to V_o. From Eqs. 2.4 and

and 2.9

$$T(t) = C_w\,[V_o^2+2V_oV_1(t)+V_1^2(t)] \tag{2.10}$$

In calculating the mean value of Eq. 2.10 the middle term integrates to zero because $V_1(t)$ by definition will be negative and positive the same amounts. Thus, the mean torque $\overline{T(t)}$ is

$$\overline{T(t)} = C_w\,[V_o^2+\overline{V_1^2(t)}] \tag{2.11}$$

The instantaneous deviation of the torque, $T_1(t)$, from its mean value is found from Eqs. 2.10 and 2.11 to be

$$T_1(t) = C_w[2V_oV_1(t) - \overline{V_1^2(t)} + V_1^2(t)] \tag{2.12}$$

Data from the proposed site will probably indicate that the average wind velocity component, V_o, will be large enough compared to the rms value of the variations during the periods when gusting will produce maximum error that the mean squared and squared value of the instantaneous gusts may be neglected. This assumption results in the approximate relationship

$$T_w(t) = 2C_wV_0V_1(t) \tag{2.13}$$

where T_w is the approximate value of T_1.

According to Eq. 2.13, the instantaneous torque fluctuation about the average is proportional to a constant $2C_wV_0$ and the wind fluctuation. The autocorrelation function of the torque, $T_{vv}(\tau)$, according to Eqs. 2.1 and 2.13 will be

$$T_{vv}(\tau) = (2C_w V_0)^2 \, \overline{[V_i^2(t)]} \, e^{-\omega_g |\tau|} \tag{2.14}$$

The definition of the power spectrum density function is

$$\Phi_{uu}(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-s\tau} T_{vv}(\tau) d\tau \tag{2.15}$$

Thus

$$\Phi_{uu}(s) = \frac{(2C_w V_0)^2 \, \overline{[V_i^2(t)]}}{2\pi} \int_{-\infty}^{\infty} e^{-s\tau} e^{-\omega_g |\tau|} d\tau$$

$$= \frac{\beta^2}{2\pi} \int_{-\infty}^{0} e^{-(\omega_g - s)\tau} d\tau + \int_{0}^{\infty} e^{-(\omega_g + s)\tau} d\tau \tag{2.16}$$

where

$$\beta = 2C_w V_0 \left[\overline{V_i^2(t)} \right]^{1/2} \tag{2.17}$$

Eq. 2.16 yields

$$\Phi_{uu} = \frac{\beta^2}{\pi} \frac{\omega_g}{-s^2 + \omega_g^2} \tag{2.18}$$

This is a description of the antenna torque due to wind fluctuations about the average value of wind which will be used to make tracking error calculations.

<u>Data required for Wind Gust Error Calculations</u>

The only wind data available were recorded at a proposed site at the rate of 3 in/hr chart speed. These data give some idea of the V_o to be encountered and an approximate value for the relative magnitude of $\overline{V_1^2(t)}$ These data are adequate for preliminary calculations but additional data are needed to make final decisions.

It seems reasonable to assume that the relationship between V_o and $V_1^2(t)$ of the wind will remain essentially constant (on a log-log plot) in view of the results obtained by Titus [4] in Fig. 2.2 which is one of Titus's curves. These data indicate that β is proportional to the fourth power of V_o. Thus, a relatively small quantity of data from the site should be adequate and this need not be taken at V_o maximum, although data in the range of V_o maximum would be optimum.

The parameters required from the wind data are:

1) ω_g - the pole associated with the power-density spectrum
2) a confirmation of the autocorrelation function which has been assumed to have the form of Eq. 2.1
3) a confirmation of the relationship between $V_1^2(t)$ and V_o so that $V_1^2(t)$ for the maximum allowable V_o may be extrapolated

An ω_g in the neighborhood of 0.12 rad/sec is anticipated in view of the Titus data. This means that it would be desirable to read data points on the recordings that are separated by approximately 2 seconds. A chart recording speed of approximately 1/30 in/sec should be adequate for wind recording. However, it is recommended that some preliminary data be taken and used as a basis for setting the

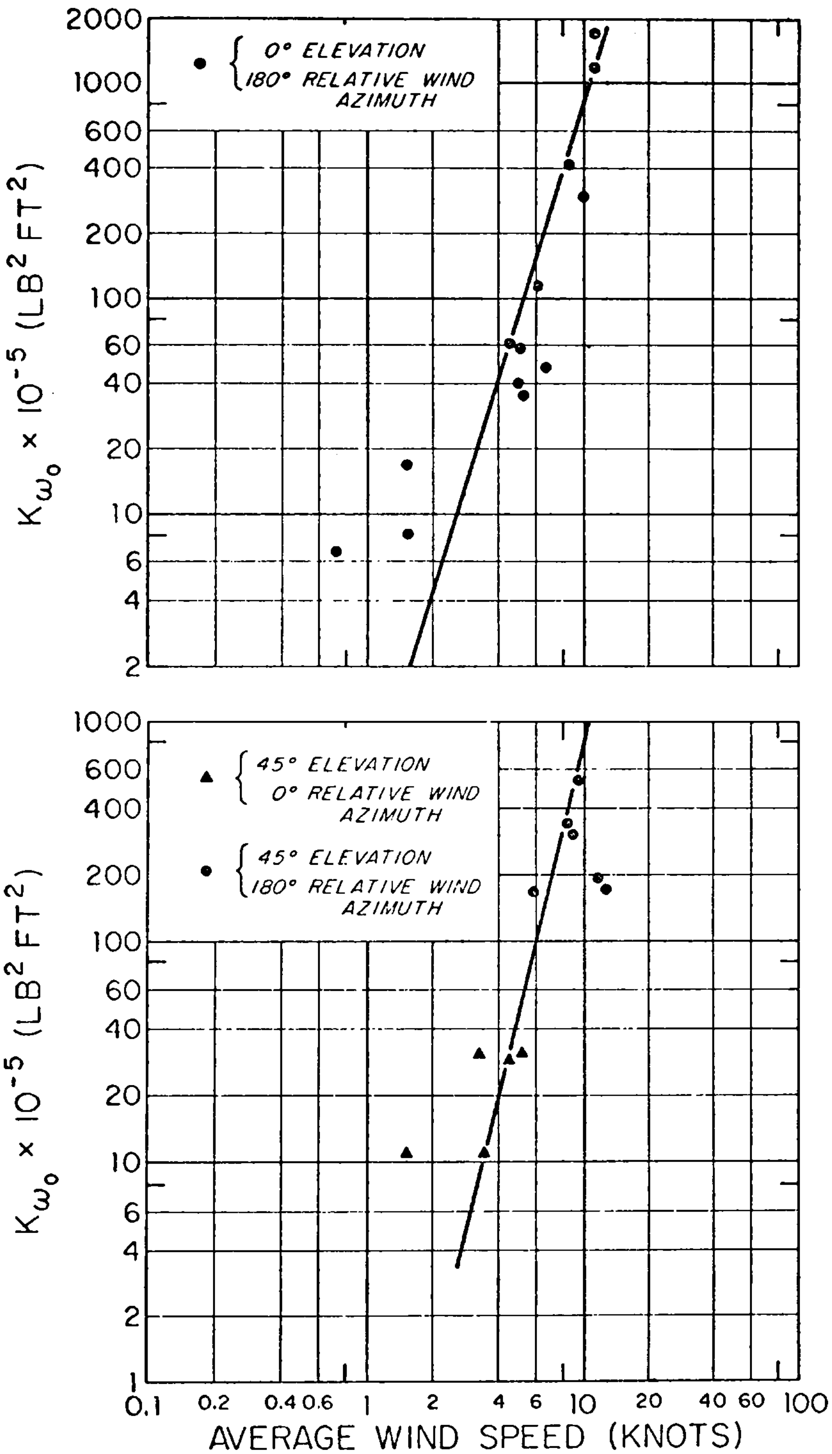

Fig. 2.2 Relationship between V_o and $\overline{V_1^2(t)}$ (Titus [1]).

chart speed. These rough estimates were made using Eq. 2.1. If $\omega_g = 0.12$, $\tau = 1/0.12$ is the time value for the calculation of $\Phi_{vv}(\tau)$ at the one time constant point. Two second values of τ will allow calculations of a point near the maximum of the autocorrelation function.

As stated above, the quantity of data is not important because the wind fluctuations can probably be assumed to be stationary and ergodic. Certainly, for the time periods used in calculating the response of the control system it is adequate to make these assumptions. Three minute samples of data will probably suffice and it is desirable to obtain data for several different V_o values.

SECTION III

Error Calculations

It can be shown [4] that system error, e, is

$$\overline{e^2(t)} = \frac{1}{j}\int_{-j\infty}^{j\infty} \Phi_{ee}(s)ds \tag{3.1}$$

by writing the inverse transform expression for the output auto-correlation function due to a wind gust torque and then taking the limit as time approaches zero. In Eq. 3.1

$$\Phi_{ee}(s) = \Phi_{uu}H_n(s)H_n(-s) \tag{3.2}$$

where

$$H_n(s) = \frac{\Theta(s)}{T(s)} \tag{3.3}$$

is defined by Eqs. 1.24 or 1.39, and Φ_{uu} is defined by Eq. 2.18.

Integrals of the form

$$I_n = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{c(s)c(-s)}{d(s)d(-s)}\,ds \tag{3.4}$$

where

$$\begin{aligned} c(s) &= c_{n-1}s^{n-1} + \ldots\ldots +c_o \\ d(s) &= d_ns^n + \ldots\ldots +d_o \end{aligned} \tag{3.5}$$

can be evaluated using available solutions [4]. As an example, from Eqs. 2.18 and 1.24

$$\frac{c(s)}{d(s)} = \frac{\left(1/\tau_2 J\right)\left(\tau_2 s^2+s\right)}{s^5 + As^4 + Bs^3 + Cs^2 + Ds + E} \tag{3.6}$$

where

$$A = \frac{1}{\tau_2} + \frac{1}{\tau_m} + \omega_g$$

$$B = \left[\left(\frac{1}{\tau_2} + \frac{1}{\tau_m}\right)\omega_g + \frac{1}{\tau_2}\left(\frac{1}{\tau_m} + K_a\tau_1\right)\right]$$

$$C = \left[\frac{\omega_g}{\tau_2}\left(\frac{1}{\tau_m} + K_a\tau_1\right) + \frac{K_a}{\tau_2}\left(\frac{\tau_1+\tau_m}{\tau_m}\right)\right]$$

$$D = \left[\frac{\omega_g K_a}{\tau_2}\left(\frac{\tau_1+\tau_m}{\tau_m}\right) + \frac{K_a}{\tau_m\tau_2}\right]$$

$$E = \frac{\omega_g K_a}{\tau_m\tau_2} \tag{3.7}$$

This analysis is for the model which does not include antenna damping and the gear box spring constant.

From Eqs. 3.5 and 3.7

$$c_0 = 0$$

$$c_1 = 1$$

$$c_2 = \tau_2$$

$$c_3 = 0$$

$$c_4 = 0$$

$$d_0 = \frac{\omega_g K_a}{\tau_m \tau_2}$$

$$d_1 = \frac{\omega_g K_a}{\tau_2} \left[\frac{\tau_1+\tau_m}{\tau_m}\right] + \frac{K_a}{\tau_m \tau_2}$$

$$d_2 = \frac{\omega_g}{\tau_2} \left[\frac{1}{\tau_m} + K_a \tau_1 \right] + \frac{K_a}{\tau_2} \left[\frac{\tau_1+\tau_m}{\tau_m}\right]$$

$$d_3 = \left[\frac{1}{\tau_2} + \frac{1}{\tau_m}\right] \omega_g + \frac{1}{\tau_2} \left[\frac{1}{\tau_m} + K_a \tau_1 \right]$$

$$d_4 = \frac{1}{\tau_2} + \frac{1}{\tau_m} + \omega_g$$

$$d_5 = 1 \tag{3.8}$$

The value of I from [4] is

$$I_5 = \frac{1}{2\Delta_5} \left[c_4^2 m_0 + (c_3^2 - 2c_2 c_4)\, m_1 + (c_2^2 - 2c_1 c_3 + 2c_0 c_4) m_2 + (c_1^2 - 2c_0 c_2) m_3 + c_0^2 m_4 \right] \tag{3.9}$$

where

$$m_0 = \frac{1}{d_5} (d_3 m_1 - d_1 m_2)$$

$$m_1 = -d_0 d_3 + d_1 d_2$$

$$m_2 = -d_0 d_5 + d_1 d_4$$

$$m_3 = \frac{1}{d_0} (d_2 m_2 - d_4 m_1)$$

$$m_4 = \frac{1}{d_0} (d_2 m_3 - d_4 m_2)$$

$$\Delta_5 = d_0 (d_1 m_4 - d_3 m_3 + d_5 m_2) \tag{3.10}$$

The final solution of Eq. 3.1 is

$$\overline{e^2(t)} = \frac{2\pi\beta^2\omega_g}{\pi\tau_2^2 J^2} I_5$$

$$= \frac{2\beta^2\omega_g}{\tau_2^2 J^2} I_5 \tag{3.11}$$

where the factor of 2π difference in Eqs. 3.1 and 3.4 has been taken into account.

The wind gust error study can now be made using a computer to vary suitable parameters.

SECTION IV

Parameter Studies and Components

The errors due to wind gusts is a function of the servo output stiffness. The stiffness is the reciprocal of H_n defined in Eq. 3.3. Stiffness for the model of Fig. 1.2 defined by Eq. 1.24, which neglects the antenna damping and gear train stiffness, is plotted in Fig. 4.1 for different values of servo bandwidth where bandwidth is defined as the crossover frequency on the Bode open loop attenuation diagram (e.g., see ω_c in Fig. 1.3). At relatively low and high frequencies the stiffness depends on servo action. At relatively high frequencies stiffness depends on antenna inertia. Note that the smallest values of stiffness occur at wind gust frequencies in the neighborhood of ω_c where the combination of servo action and antenna inertia are least effective. One can readily see in Fig. 4.1 the role that bandwidth plays in wind gust error.

The 3σ (three times the square root of the value from Eq. 3.11) value of error for the model of Fig. 1.2 is shown in Fig. 4.2. The system does not contain tachometer feedback in this model to stiffen the system at frequencies near ω_c. (A high quality servo motor [GE Frame 3105] has a motor time constant of 0.005 seconds.)

The inclusion of a spring constant and damping in the antenna (model defined by Eq. 1.39 and Fig. 1.5) has a 3σ error shown in Fig. 4.3. Note that the inclusion of antenna damping and spring constant increases the 3σ value of error by 600% at ω_c=1.5.

Fig. 4.4 shows the 3σ error for the model of Fig. 1.7 which includes the gearbox spring constant. It was assumed that the low frequency characteristics of the wind remained constant as ω_g was varied. From Eq. 2.18 it is apparent that the low frequency value of the torque varies inversely with ω_g and would make the error increase as ω_g is decreased if the low frequency value of torque is not held constant.

A value of ω_g=0.36 rad/sec were assumed for the final calculations because this is the upper limit of the Titus report data and because Rohr uses this value (the maximum value of V_o divided by the antenna diameter).

Fig. 4.5 shows the effect of varying the tachometer and motor time constant. A tachometer is not justified for stiffening the system if a relatively good servo motor is used. However, the tachometer is needed for rate control and possibly for stablizing the system. Location of the tachometer on the drive motor will be adequate for these purposes.

Fig. 4.6 shows that the assumed damping factor of f=0.02 is adequate from an error point of view. The error is essentially independent of f over the range of values one would anticipate for f.

Fig. 4.7 shows the effect of gear box spring constant. The spring constant, K_g, is unimportant if $K_g \geq 0.7 \times 10^8$ ft-lb/rad. The actual value is expected to be 1×10^9.

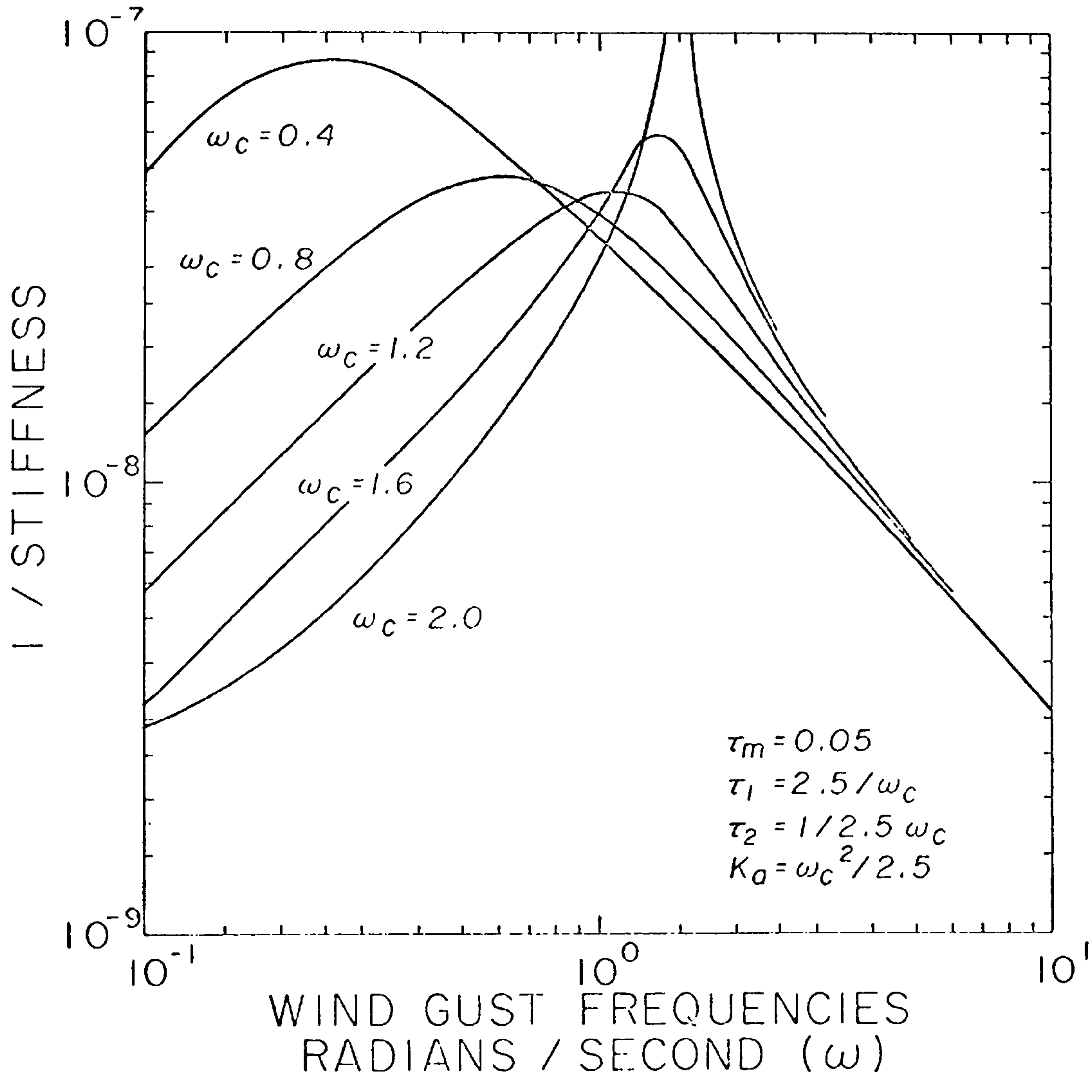

Fig. 4.1 Servo stiffness for different servo cross over frequencies.

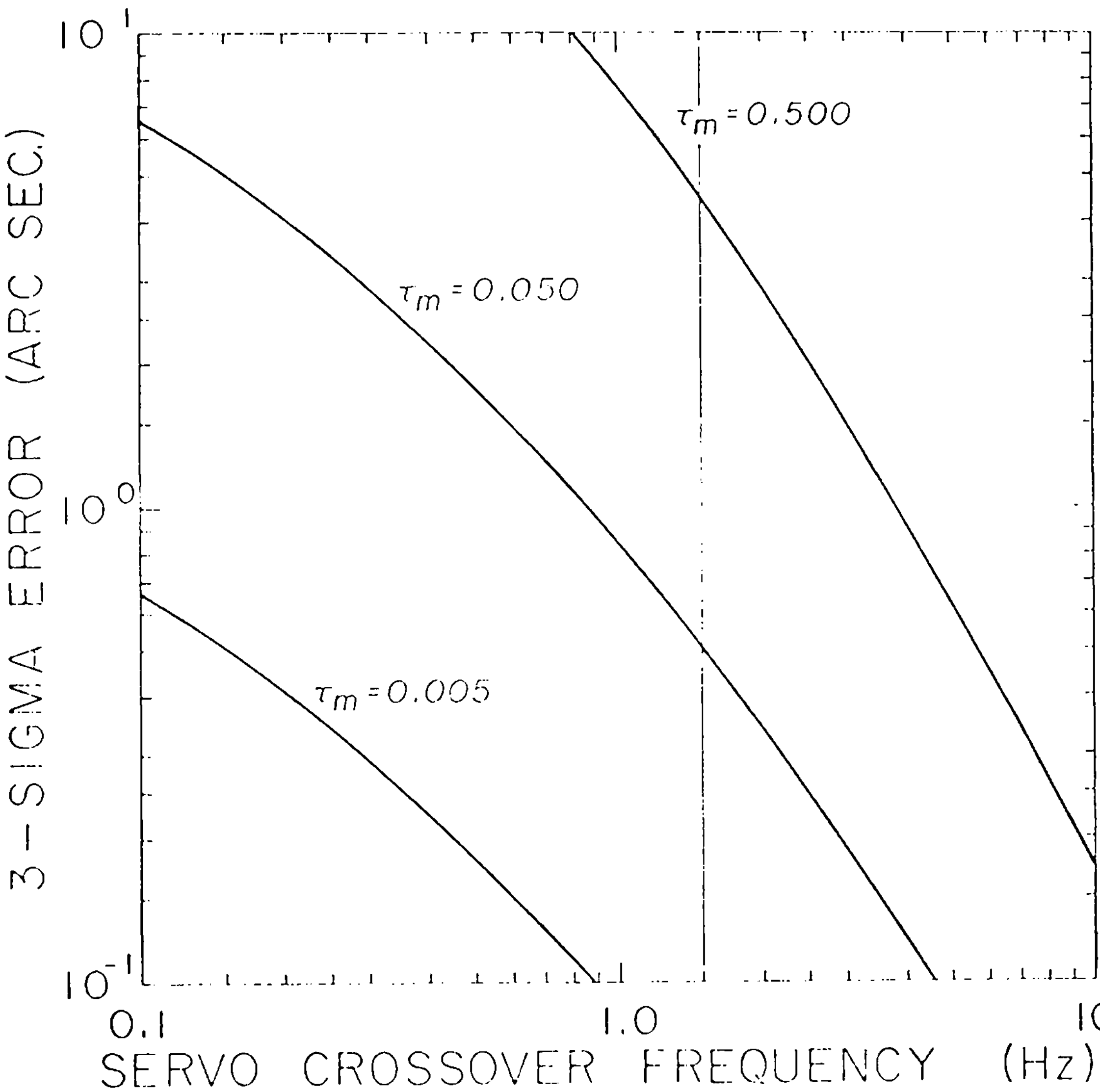

Fig. 4.2 Error for different values of servo motor time constant.

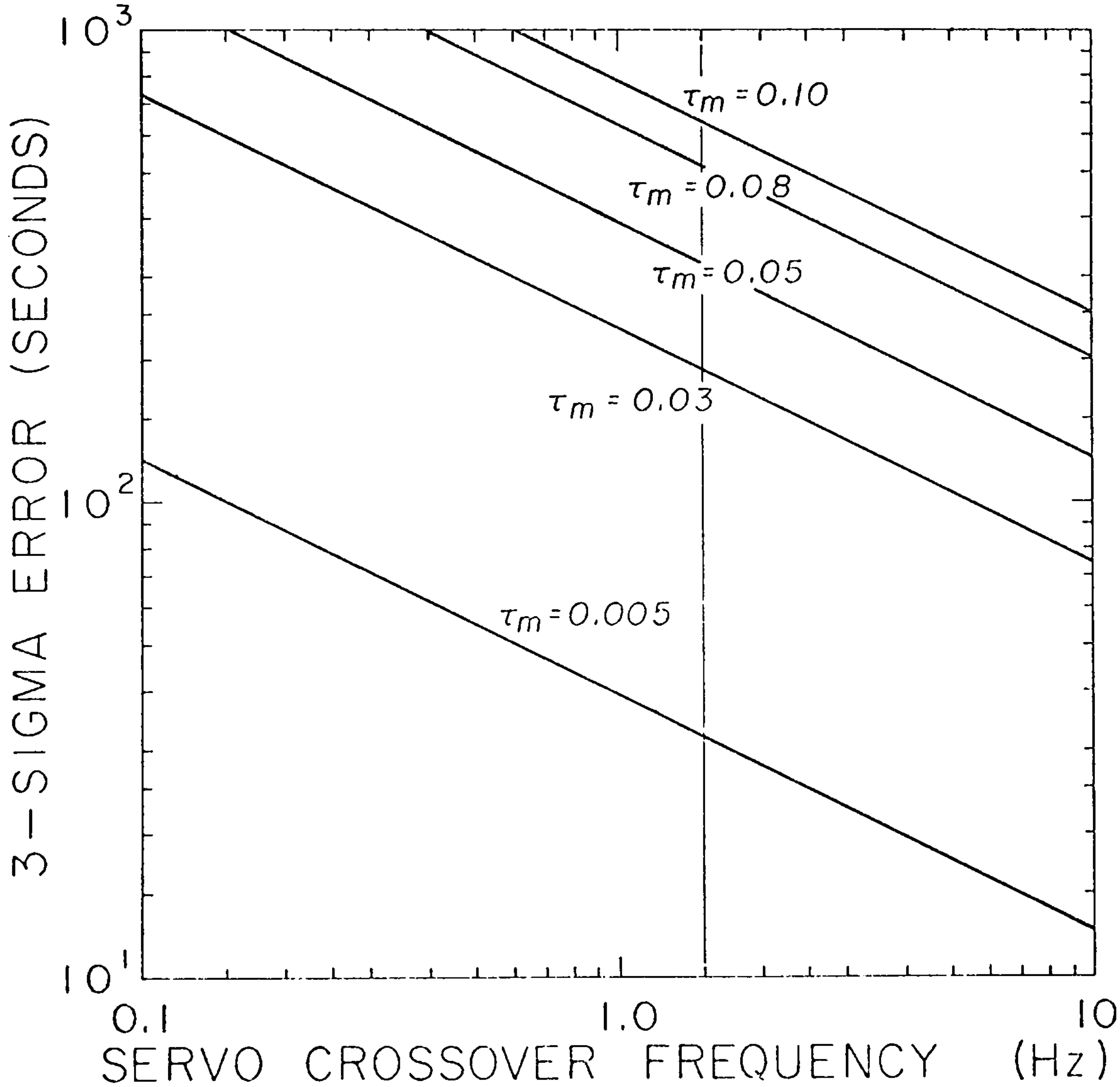

Fig. 4.3 Error for different values of servo motor time constant.

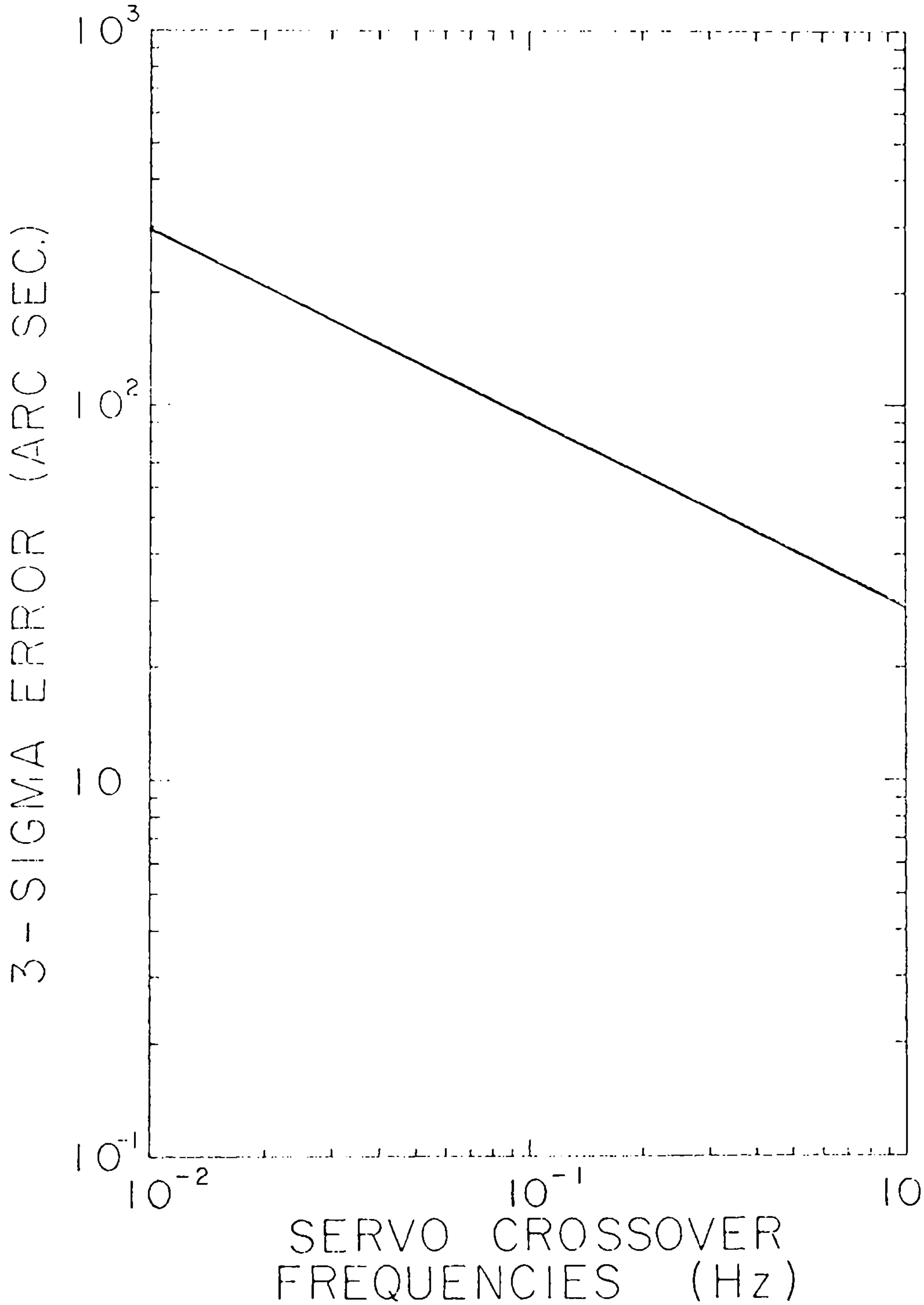

Fig. 4.4 3-sigma error for model of Fig. 1.7 for Omega between 0.144 and 0.720 Hz.

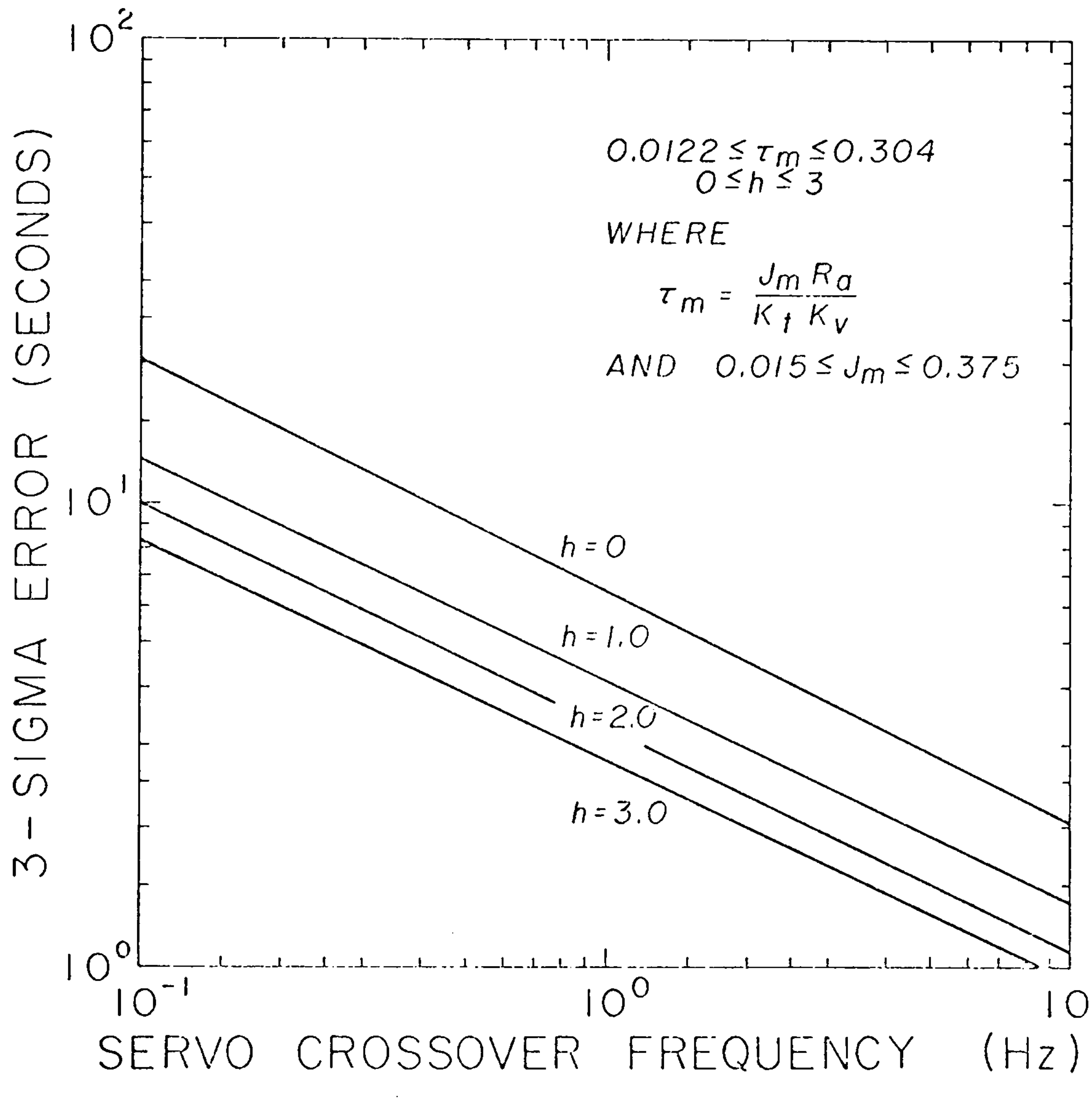

Fig. 4.5 3-sigma error for model with tachometer gain and motor time constant varied. Note the motor time constant had negligible effect on the error.

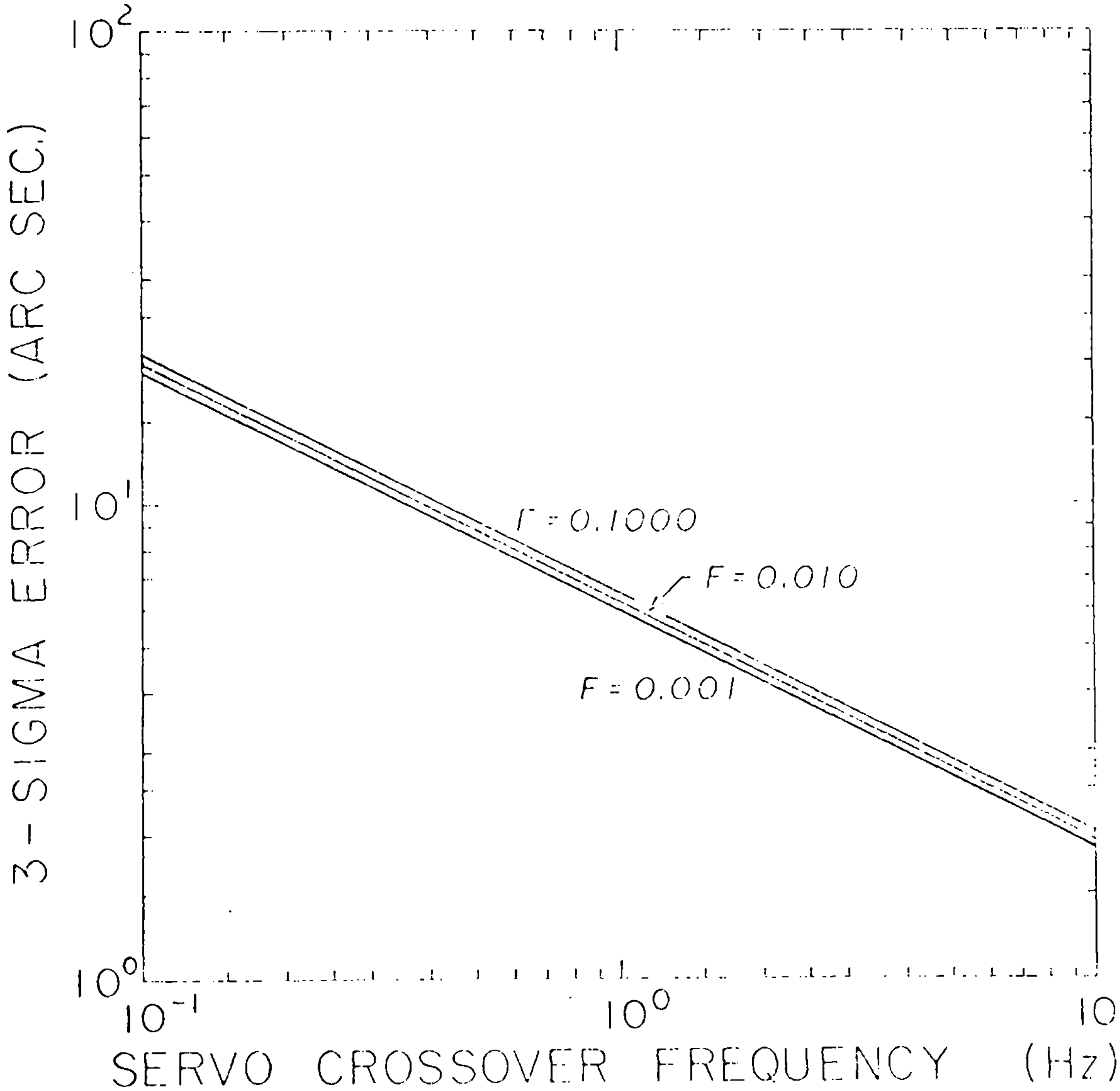

Fig. 4.6 3-sigma error for model with antenna damping factor varied.

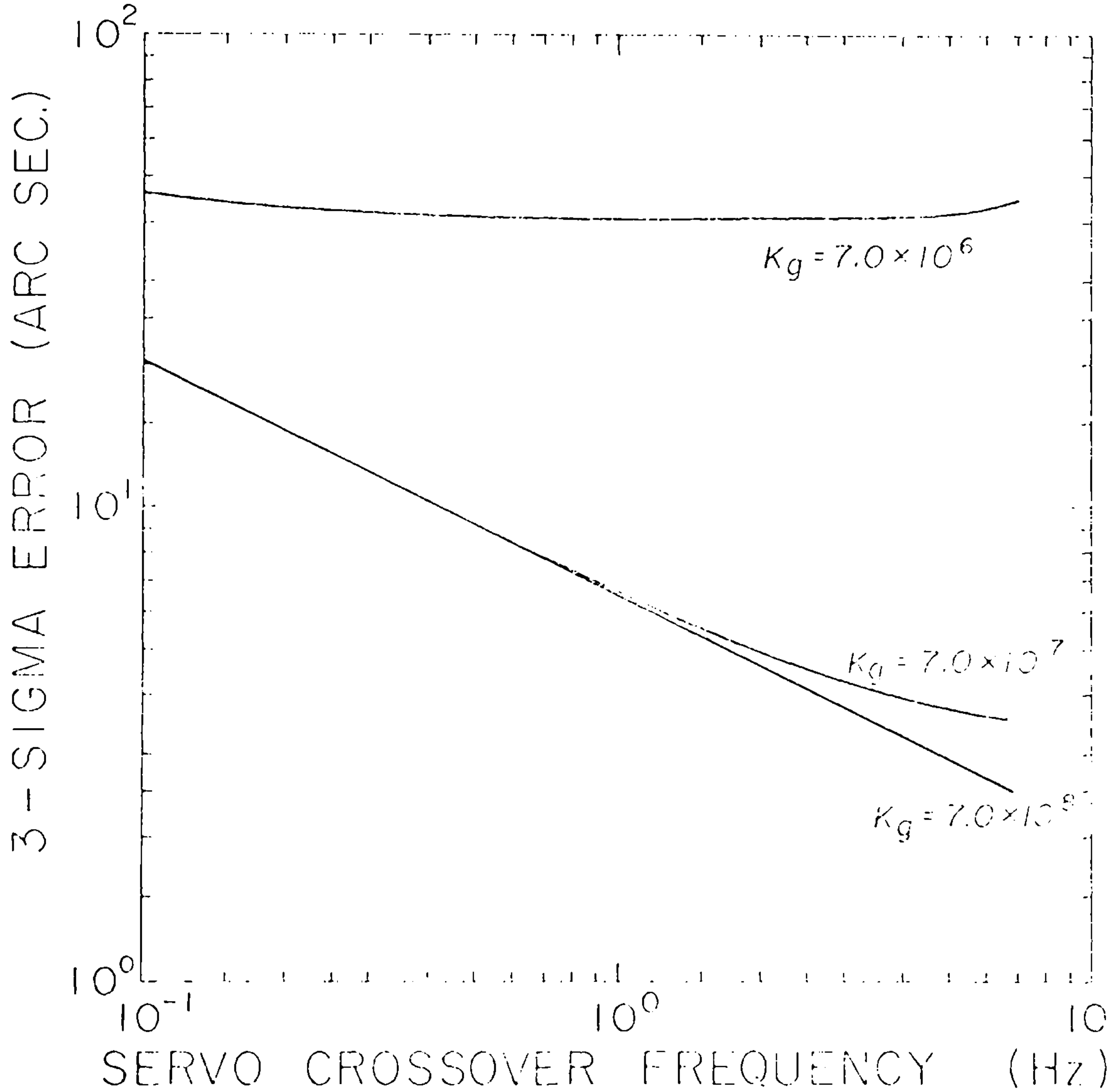

Fig. 4.7 3-sigma error for model with gear train spring constant varied.

SECTION V

SPECIFICATIONS FOR THE SERVO ASSEMBLY OF A 25 METER DIAMETER VERY LARGE ARRAY (VLA) RADIO TELESCOPE ANTENNA SYSTEM

1.0 SCOPE

1.1 CONTENT

This specification defines the requirements for the design and fabrication of the servo assembly (control system) of a steerable 25-meter diameter radio telescope antenna . Although this specification is for a single VLA prototype antenna control system, the design and other considerations are to be oriented towards large scale production.

1.2 PURPOSE

The purpose of the servo assembly is to control the elevation and azimuth positions and rates of the antenna while observing celestial sources.

2.0 APPLICABLE

TBA

3.0 FUNCTIONAL DESCRIPTION

3.1 GENERAL

The servo assembly consists of two separate control systems which indepencently control the elevation and azimuth of the antenna. The are type 2 (zero velocity error) electro-mechanical control systems. They are identical except for compensation and rotation requirements. Compensation takes into account the differences in azimuth and elevation parameters such as locked rotor resonant frequency.

The major sections of the servos are:

1) Drive section
2) Control sections
3) Encoder and decoder section

3.2 MODES OF OPERATION

The control system shall have the following modes and sub-modes of operation:

1) Non-operating modes:

   Emergency off

   Power on-off

   Pre-standby

   Azimuth interlocks

   +270 degree limit and override

   -270 degree limit and override

   drive interlock

   stow interlock

   Elevation interlocks

   +5 degree limit and override

   +125 degree limit and override

   drive interlock

   stow interlock

   Standby

2) Operating modes:

   Local rate

   Local position

   Remote position

3.2.1 NON-OPERATING MODES

In the non-operating modes, the antenna brakes are applied power is removed from the drive system.

3.2.1.1 Emergency Off

This mode shall completely remove power from the antenna control system and set the brakes. It may be initiated throughout the antenna structure and remotely by computer command.

3.2.1.2 Power On-Off

The power on-off mode shall allow the antenna operator at the computer control location or the local control unit to apply power to the complete control system.

3.2.1.3 Pre-Standby

This mode shall indicate the presence of an antenna interlock. On completion of power start-up, this mode shall be activated if an interlock condition exists. If an interlock condition should occur during an operating mode the system shall drop back into pre-standby. Upon clearing the interlock condition the system shall enter standby automatically. Provision shall be made to insure that the system remains in pre-standby until start-up transients have passed.

3.2.1.4 Azimuth Interlocks

A minimum of four interlock conditions shall be provided for azimuth; +270 degree limit and override, -270 degree limit and override, drive interlock (overspeed, overload, etc.) and stow interlock.

The limits shall be activated by multiple antenna limit switches connected in a fail safe arrangement. The limit override shall allow the operator to remove the antenna from the limits.

3.2.1.6 Elevation Interlocks

A minimum of four interlock conditions shall be provided for elevation: +5 degrees limit and override, +125 degrees limit and override, drive interlock, and stow interlock. Operation of these interlocks shall be identical to azimuth.

3.2.2 OPERATING MODES

Three operating modes shall be furnished: local rate, local position, and central computer or remote position.

3.2.2.1 Local Rate

This is a continous mode of operation. The local operator shall be able to command continuous velocity of the antenna in both azimuth and elevation with the local control unit for testing purposes. The velocity shall be smoothly variable from 0 to 20 degrees per minute in either direction in azimuth and elevation and shall not depend on the central computer or encoding system for operation. Simultaneous operation of azimuth and elevation modes is to be avialable.

3.2.2.2 Local Position

This is a sampled data mode of operation. The azimuth and elevation axis position shall be controllable by digital inputs from a local control unit for testing purposes.

3.2.2.3 Central Computer Position

The azimuth and elevation axis position shall be controllable by digital inputs from a central computer. An encoder output and various overlap and monitor bits for each axis are to be made available for transmission to a central computer. Bit levels shall be 0 volt and +3 volts.

The input to the drive system is to be a position error signal.

3.2.3 POWER AND MODE SWITCHING AND STATUS INDICATORS

Power and mode switching will be activated either from the local control unit or from the central computer.

Operating mode switching shall be designed so that it is unnessary to pass thru intermediate operating modes. Means shall also be provided to eliminate transient affects when switching modes.

3.2.3.1 Limit Switches

Redundancy shall be provided for each antenna limit and shall operate as described in paragraph 3.2.1.4.

3.2.4 CONTROL LOOP

The control system shall contain a rate loop which operates as a velocity servo in the manual mode. The digital position loop shall be closed about the rate loop. All control loop electronics shall be solid state. Careful attention shall be given to gearing arrangements to insure that gear box imperfections such as windup and backlash do not excessively degrade performance.

Other loops as required to meet the specified performance shall be provided.

3.2.4.1 Rate Loop

The rate loop for each shall contain dual D.C. forced cooled drive motor, D.C. tachometers, gear trains for the tachometer, power amplifiers, rate loop amplifiers, and a D.C. motor field power supply. Torque bias and motor current limit circuitry shall be provided.

3.2.4.1.1 Power Amplifier

The power amplifier used shall be high performance, solid state units. They shall have sufficient bandwidth to introduce negligible effect on system relative stability in accordance with the bandwidth specified in paragraph 3.4.2.1.

3.2.4.1.2 Drive System Prime Power

The subcontractor will be furnished commercially available 440±

44 volts, 3-phase, 4 wire, 60±1 Hz prime power for the drive equipment.

All motors used in the equipment shall be of a type which do not produce radio-frequency interference, or shall be equipped with interference filters. All contacts which open and close during normal tracking mode operation shall be equipped with interference suppression devices. No gaseous-discharge devices, except noise sources for test and neon pilot lamps, shall be employed. Means shall be employed to reduce static-electricity and the consequent R.F. noise generated in any rotating machinery. All items of equipment suppled under this specification shall be properly grounded and shielded to preclude any interference to or by other equipment installed at the station.

3.2.4.1.3 Motor Protection

Each D.C. drive motor shall be provided with thermal overload protection as a means for measuring motor overheating. Overheating shall be indicated by signal to the central computer.

3.2.4.1.4 Field Supply

The field exciter shall be turned on when the power amplifiers are on.

3.2.4.2 Position Loop

The digital position loop shall be closed about the rate loop for each axis.

3.2.4.2.1 Servo Loop Performance

Rate Loop Performance

Rate loop and position crossover frequencies shall be established to achieve loop shall have maxium M_p of 1.4.

3.2.5 CONTROL ELECTRONICS POWER

The servo system electronics shall operate from a standard public utility prime power source with the following nominal characteristics:

a) Frequency - 60 ± 1 Hz

b) Voltage - 220 ± 22 volts, 1 phase, 3 wire

3.2.6 ENCODING SYSTEM

The azimuth and elevation position angles shall be measured using a 17 bit encoder directly coupled to the azimuth and elevation shafts. The encoders must be accessible for servicing in any antenna position.

3.2.7 MECHANICAL

3.2.7.1 Drive Equipment - Both Axes

A one-speed range electric drive system shall be provided for each axis. The drive system shall be supplied as pairs, and torque biasing shall be provided so that paired gear trains on each axis oppose each other to minimize backlash. The drive shall be operable at reduced performance with only on to the two drive motors on an axis in winds of 20 mph gusting to 30 mph. Antenna velocities of 20°/min about each axis and accelerations of 15°/min^2 about each axis, in the environment specified shall be provided. Wind induced torques shall not be based on the use of spoilers and/or sails.

3.2.7.1.1 D.C. Motors

The D.C. drive motors shall be forced cooled. The motors selected shall have a nominal rated load speed not to exceed 1750 rpm. The drive motors shall be able to withstand the following current conditions:

100% rated - continuous

150% rated - 2 minutes out of every 20 minutes after remaining 18 minutes at rated load

200% rated - instantaneous, 0.5 seconds or less repeated not oftener than once every minute after remaining at rated load for 30 minutes.

The nominal drive rating of each axis shall be 6 hp.

3.2.7.1.2 Gearing

The overall gear ration from drive motor shaft output to antenna position shall be 31,000:1 on both axis. The overall mechanical drive stiffness shall be equal or greater that 10^9 ft-lb/rad. The rated torque for each axis shall be 283,000 ft-lb.

3.2.7.1.3 Brakes

The azimuth and elevation drives shall each be provided with spring-set mechanical brakes which operate on a "fail-safe" basis. The brakes shall have manual and automatic reset and release. There shall be one brake for each drive motor. Each brake shall have the capacity to stall the drive motor to which it is connected at rated torque.

3.2.7.2 Lubrication

Lubrication shall be provided where dictated by sound design practice and shall be adequate to meet the preformance and environmental requirements specified herein.

3.2.7.3 Bearings and Gears

All main axis bearings and power train gearing shall be conservatively designed and selected to ensure desired life expectancy. Running friction for the drive system shall be held to minimum practical levels.

3.2.7.4 Cable Loop

All cables shall be looped about each axis.

3.2.7.5 Stow Locks

Stow locking devices shall be installed to provided locking at elevation zenith and azimuth reference positions.

3.2.8 PERFORMANCE

The nominal design errors are to be as follows:

	Three Sigma Axis Error In Seconds of Arc	
	Az Axis	El Axis
Servo Noise	3.6	3.6
Breakout Friction	1.3	1.1
Resolution Limit Cy	5.0	5.0
Servo Drift	2.0	2.0
Blacklash	0	0
Wind Gusts	5.1	5.1
RSS Axis Error	8.8	8.3
RSS of Both Axes	11.73	

These values assume that the individual errors are random with normal disturbances. Thus, the sum of the indivivual variances (σ^2) equal the total variance. The term RSS is defined here as the square root of the sum of the three sigma values squared. The wind gust errors are based on the wind inducing an antenna torque spectrum,

$$\phi(s) = \frac{\beta^2}{\pi} \quad \frac{\omega_c}{-s^2+\omega_g^2}$$

where the worst case values are

ω_g = 0.36 rad/sec

β = 26,400

4.0 ENVIRONMENT

TBA

5.0 LIFE EXPECTANCY

TBA

6.0 RADIO FREQUENCY INTERFERENCE

The amplifiers and D.C. drive motors shall be capable of meeting the requirements of MIL-16910 (ships) with the option of paragraph 3.5.1.4 (1,000 to 10,000 mHz) required.

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