NATIONAL RADIO ASTRONOMY OBSERVATORY
Green Bank, West Virginia

VLA ELECTRONICS MEMORANDUM NO. 5
November 29, 1967

# COMPARISON OF MODULATION SYSTEMS FOR TRANSMISSION THROUGH DISPERSIVE CABLES

S. Weinreb

## Introduction

The VLA system requires the transmission of 50 MHz bandwidth signals through up to 21 km of coaxial cable. The amplitude variation over this band must be under ± 0.5 dB and the phase nonlinearity must be less than ± 10°. These requirements are made difficult because of the frequency variation of attenuation and time delay in coaxial cables. These dispersive effects can be minimized by the choice of modulation system and carrier frequency.

Three types of modulation systems are evaluated. These are single-sideband (SSB), double-sideband (DSB), and double-sideband quadrature (DSB-Q) modulation. Besides the dispersive effects evaluated here, the following considerations apply to the choice of modulation system:

1) SSB and DSB-Q require a transmission band equal to the modulation bandwidth (50 MHz) plus a guard band ($\sim$ 25 MHz). DSB requires twice this bandwidth.

2) SSB modulators and demodulators are more complex than those required for the DSB and DSB-Q systems. In particular, frequencies close to DC are not easily modulated. A modulation frequency range of 8 to 50 MHz is feasible.

3) The delay lines connected to the output of the IF transmission system require a SSB signal in the 50 to 100 MHz band. The interface is more convenient with a SSB transmission system.

4) The DSB-Q system allows two DSB signals to be put in one frequency band by modulating on carriers which are in quadrature. The crosstalk problem will be investigated in this memorandum.

A DSB-Q will be analyzed first; the other systems can then be analyzed by applying special conditions to this system.

## Analysis

Consider the system shown in Figure 1. The input of the dispersive transmission line is given by:

$$\cos \omega_0 t + \frac{p}{2} \cos (\omega_0 + \omega_1)t + \frac{p}{2} \cos (\omega_0 - \omega_1)t$$

$$+ \sin \omega_0 t + \frac{q}{2} \sin [(\omega_0 + \omega_1)t + \Theta] + \frac{q}{2} \sin [(\omega_0 - \omega_1)t - \Theta]$$

where $\omega_0$ and $\omega_1$ are the carrier and modulation frequencies and p, q, and $\Theta$ the amplitudes and relative phase of the in-phase and quadrature modulation signals. (Note that the carrier of the quadrature signal must be 90° out of phase with the in-phase carrier but the phase, $\Theta$, of the modulation signals can be arbitrary.)

The transmission line is a linear system with properties completely described by the amplitude and phase response at the three input frequencies:

| | $\omega_0$ | $\omega_0 + \omega_1$ | $\omega_0 - \omega_1$ |
|---|---|---|---|
| Amplitude ..... | 1 | a | b |
| Phase ........ | $\varphi$ | $\varphi + \alpha$ | $\varphi - \beta$ |

The carrier is extracted at the output of the transmission line and after suitable phase adjustment is used to product-detect the in-phase transmission line output signal. The result is:

$$g(t) = a\,p \cos (\omega_1 t + \alpha) + b\,p \cos (\omega_1 t + \beta)$$

$$+ a\,q \sin (\omega_1 t + \alpha + \Theta) - b\,q \sin (\omega_1 t + \beta + \Theta) \qquad (1)$$

This result is illustrated in the vector diagram of Figure 2. The first two terms are the desired signals arising from upper and lower sidebands of the modulated carrier; for an ideal transmission line they will be identical with $\alpha = \beta = \omega_1 \tau$. The last two terms represent cross-talk; for an ideal transmission line they will be equal and opposite.

The parameters we wish to evaluate are the crosstalk, the amplitude variation with frequency, and the dispersion. The crosstalk is defined as the ratio of the amplitudes of the desired and undesired signals. In the case of a real transmission line the amplitude parameters (a and b) have a higher order effect than the phase parameters ($\alpha$ and $\beta$) and the crosstalk is given by:

$$R = 20 \log \frac{a + b}{a - b} \tag{2}$$

Some numerical results are given in the next section.

The amplitude variation and dispersion effects can be evaluated by solving for the amplitude and phase of the sum of the first two terms of Equation (1). That is, we wish to solve the following equation for c and $\gamma$ (we will assume p = 1):

$$c \cos (\omega_1 t + \gamma) = a \cos (\omega_1 t + \alpha) + b \cos (\omega_1 t + \beta) \tag{3}$$

The dispersion, $\epsilon$, will be defined as the departure of $\gamma$ from a linear function of frequency ($\omega_1$),

$$\epsilon = \gamma - \omega_1 \tau \tag{4}$$

where

$$\tau = \frac{d\gamma}{d\omega_1} \qquad \omega_1 = 0 \tag{5}$$

The solution of (3) for c and $\gamma$ gives:

$$c = \sqrt{a^2 + b^2 + 2\, ab \cos (\alpha - \beta)} \tag{6}$$

$$\sim (a + b) \left[ 1 - \frac{ab}{(a + b)^2} \quad \frac{(\alpha - \beta)^2}{2} \right] \tag{7}$$

and

$$\gamma = \tan^{-1} \frac{(a \sin \alpha + b \sin \beta)}{(a \cos \alpha + b \cos \beta)} \tag{8}$$

$$\sim \frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2} \quad \frac{(a - b)}{(a + b)} \tag{9}$$

The approximations in (7) and (9) assume $\alpha - \beta << 1$; they are valid for dispersion effects much less than a radian. The approximation in going from (8) to (9) is illustrated by the construction in Figure 3 and can also be proved with trigometric identities.

It is of interest to evaluate the above quantities for a lossy transmission line. At UHF frequencies the transfer function of the line is given by,

$$H(\omega) = e^{-j\omega\tau_0 - (1+j)k\omega^{1/2}} \qquad (10)$$

where $\tau_0 = 1\sqrt{\epsilon\mu}$ and k is a loss constant. The amplitude function is thus,

$$|H(\omega)| = e^{-k\omega^{1/2}} \qquad (11)$$

and the phase function is,

$$\varphi(\omega) = \omega\tau_0 + k\omega^{1/2} \qquad (12)$$

The amplitudes (a, b, and c) and phases ($\alpha$, $\beta$, and $\gamma$) can be expressed in terms of a normalized frequency, $x \equiv \omega_1/\omega_0$, and the line loss in nepers at the carrier frequency, $\rho \equiv k\omega_0^{1/2}$. These results are given in Table I. The dispersion is then given by,

$$\epsilon = \frac{\rho x^2}{8} \tanh \frac{\rho x}{2} + \frac{\rho x^3}{16} \qquad (13)$$

The quantities corresponding to c and $\gamma$ for a SSB system can easily be seen to be a and $\alpha$ for an upper sideband and b and $\beta$ for the lower sideband system. The dispersion in a SSB system (upper sideband) is then,

$$\epsilon_{SSB} = -\frac{\rho x^2}{8} + \frac{\rho x^3}{16} \qquad (14)$$

Thus for small values of $\rho x/2$ ($\tanh \rho x/2 \sim \rho x/2$) the DSB system has a factor of $\rho x/2$ less dispersion while for large values of $\rho x/2$ ($\tanh \rho x/2 \sim 1$) the dispersions are equal.

The amplitude variations which result in the DSB and SSB systems are plotted in Figure 4. The plot shows on a logarithmic scale the amplitude variation as the modulation frequency is varied or, alternatively, the amplitude variation at a given $\omega_1$ as the cable attenuation is increased.

Numerical results for five different examples of cable diameter, cable length, and carrier frequency are presented in Table II.

## Conclusions

The most surprising result is the very small value of phase nonlinearity with either modulation system. This is true even when the cable attenuation is very high and the amplitude variation with frequency is large. For example, cable No. 1 in Table II has 547 dB attenuation at 1 GHz, 13.6 dB variation in attenuation over a 50 MHz band starting at 1 GHz, and yet the phase nonlinearity is $\sim$ 1° for either SSB or DSB modulation. This result arises because the additional phase shift introduced by the cable loss (6.6°/dB) is very nearly a linear function of frequency for small ratios of modulation to carrier frequencies. In the above example the 13.6 dB attenuation variation causes a 90° phase shift as the modulation frequency is increased from 0 to 50 MHz. However, this phase shift is a linear function of frequency to within 1° and has the effect of increasing the cable time delay by 5 ns.

The general point is that cable attenuation variation will be more troublesome than phase nonlinearity. The phase shift and loss are the same functions of frequency (i.e., $\alpha\ f^{1/2}$). However, a linear attenuation variation is harmful; a linear phase shift is not.

The results show that there will be crosstalk between the in-phase and quadrature signals in the DSB-Q modulation technique if the sidebands do not have equal amplitude. One dB amplitude difference gives 24 dB crosstalk; 3 dB gives 16 dB. These figures are tolerable but not comfortable.

The comparison of DSB and SSB modulation systems indicates that the DSB system is more tolerant to cable attenuation variation and phase linearity. However, the difference becomes small for the very large cable attenuations which are necessary in the array. This is shown clearly in Examples No. 1 and No. 4 in Table II. This difference is not worth the factor of 2 greater bandwidth requirement.

The DSB modulation system becomes extremely good for short ($\sim$ 2 km) lengths of cable between modulator and demodulator. Note that a cascade of sections, each consisting of DSB modulator-cable-DSB demodulator, achieves much better performance than one long section. Thus, 10 sections of the type of Example No. 2 would give 0.8 dB attenuation variation and 0.15° phase nonlinearity over a 21 km path with no equalization. This is approximately a factor of 10 better than the single 21 km cable of Example No. 1. However, alarge number of fairly complex demodulator-modulator repeaters would be required. In the case of SSB transmission the insertion of demodulator-modulator units in the transmission line does not improve the performance.

The results can be used to predict the accuracy required to equalize the transmission lines. For example, if the cable No. 4 in Table II is used, the attenuation variation across a 50 MHz band is 56 dB with SSB modulation (50 dB with DSB); thus equalizers with 2 percent attenuation accuracy are required to meet a 1 dB specification. This high accuracy is undesirable for the following reasons:

1) The equalizer manufacture and adjustment become difficult.

2) The cable loss varies 2 percent for a 8 °C temperature change; thus the equalizers would have to be readjusted for seasonal changes.

3) It is desirable to be able to move antennas without changing equalizers. A move may require differences of up to 1 km in cable length to the first equalizer. This length of cable produces a 5 percent attenuation variation change.

The cable described as Example No. 1 in Table II has 13.6 dB amplitude variation with SSB transmission. This cable would require $\sim$ 7 percent equalizers which is more reasonable.

## TABLE I — PARAMETERS OF LOSSY TRANSMISSION LINE

$\rho \equiv k\omega_0^{1/2} = \dfrac{\text{attenuation at } \omega_0 \text{ in dB}}{8.68}$ $\qquad$ $x \equiv \dfrac{\omega_1}{\omega_0}$

| | Exact Value | Series Expansion |
|---|---|---|
| a $\equiv$ amplitude transmission at $\omega_0 + \omega_1$ = amplitude of SSB demodulated signal. | $\dfrac{e^{-\rho(1+x)^{1/2}}}{e^{-\rho}}$ | $e^{-\frac{\rho x}{2}}$ |
| b $\equiv$ amplitude transmission at $\omega_0 - \omega_1$ | $\dfrac{e^{-\rho(1-x)^{1/2}}}{e^{-\rho}}$ | $e^{+\frac{\rho x}{2}}$ |
| $\alpha \equiv$ phase shift at $\omega_0 + \omega_1$ minus phase shift at $\omega_0$. | $\omega_1\tau_0 + \rho\,[(1+x)^{1/2} - 1]$ | $\omega_1\tau_0 + \dfrac{\rho x}{2} - \dfrac{\rho x^2}{8} + \dfrac{\rho x^3}{16} - \ldots$<br>$x \ll 1$ |
| $\beta \equiv$ phase shift at $\omega_0$ minus phase shift at $\omega_0 - \omega_1$. | $\omega_1\tau_0 + \rho\,[(1-x)^{1/2} - 1]$ | $\omega_1\tau_0 + \dfrac{\rho x}{2} + \dfrac{\rho x^2}{8} + \dfrac{\rho x^3}{16} + \ldots$ |
| c $\equiv$ amplitude of demodulated signal. | $\dfrac{e^{-\rho(1+x)^{1/2}} + e^{-\rho(1-x)^{1/2}}}{2e^{-\rho}}$ | $\cosh \dfrac{\rho x}{2}$ |
| $\gamma \equiv$ phase of demodulated signal. | (Too Complex) | $\omega_1\left(\tau_0 + \dfrac{\rho}{2\omega_0}\right) + \dfrac{\rho x^2}{8} \tanh \dfrac{\rho x}{2} + \dfrac{\rho x^3}{16}$ |

TABLE II — TRANSMISSION CHARACTERISTICS OF UNEQUALIZED CABLES WITH DSB AND SSB MODULATION

| Parameter | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cable Diameter | in. | 1 5/8 | 1 5/8 | 7/8 | 7/8 | 1 5/8 |
| Length | km | 21 | 2.1 | 1.0 | 21 | 5.1 |
| Carrier Frequency | $\frac{\omega_0}{2\pi}$ | $10^9$ | $10^9$ | $10^9$ | $.33 \times 10^9$ | $.15 \times 10^9$ |
| Attenuation at Carrier Frequency | $\rho$ dB | 547 | 54.7 | 36.0 | 756 | 47.0 |
| Rejection of Quadrature Signal | dB | 0.7 | 16 | 20 | 0 | 3 |
| Amplitude Variation across 50 MHz Band SSB Modulation | dB | 13.6 | 1.4 | 0.8 | 56 | 7.8 |
| Amplitude Variation across 50 MHz Band DSB Modulation | dB | 8.0 | .08 | .04 | 50 | 3 |
| Phase Nonlinearity SSB Modulation | | 1.12° | .12° | .07° | 13.7° | 4.3° |
| Phase Nonlinearity DSB Modulation | | 1.03° | .015° | .01° | 13.7° | 3.1° |
| Increase in Time Delay Due to Loss | $\frac{\rho}{2\omega_0}$ | 5 ns | .5 ns | .33 ns | 20.7 ns | 2.9 ns |

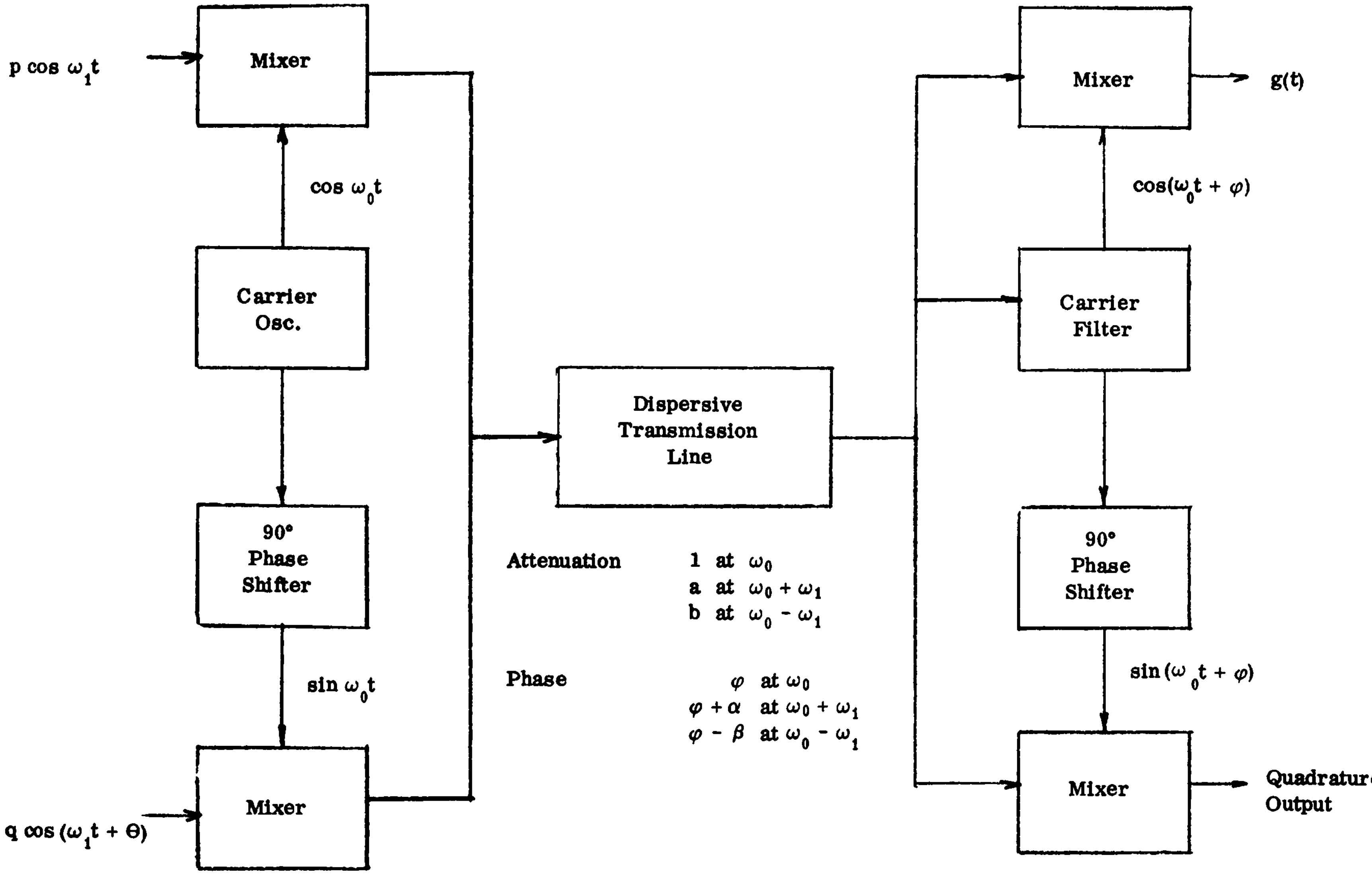



Figure 1 — Block diagram of doubled-sideband quadrature modulation transmission system. Two signals, p and q, are transmitted thru a common frequency band by modulation on carriers which are in quadrature.

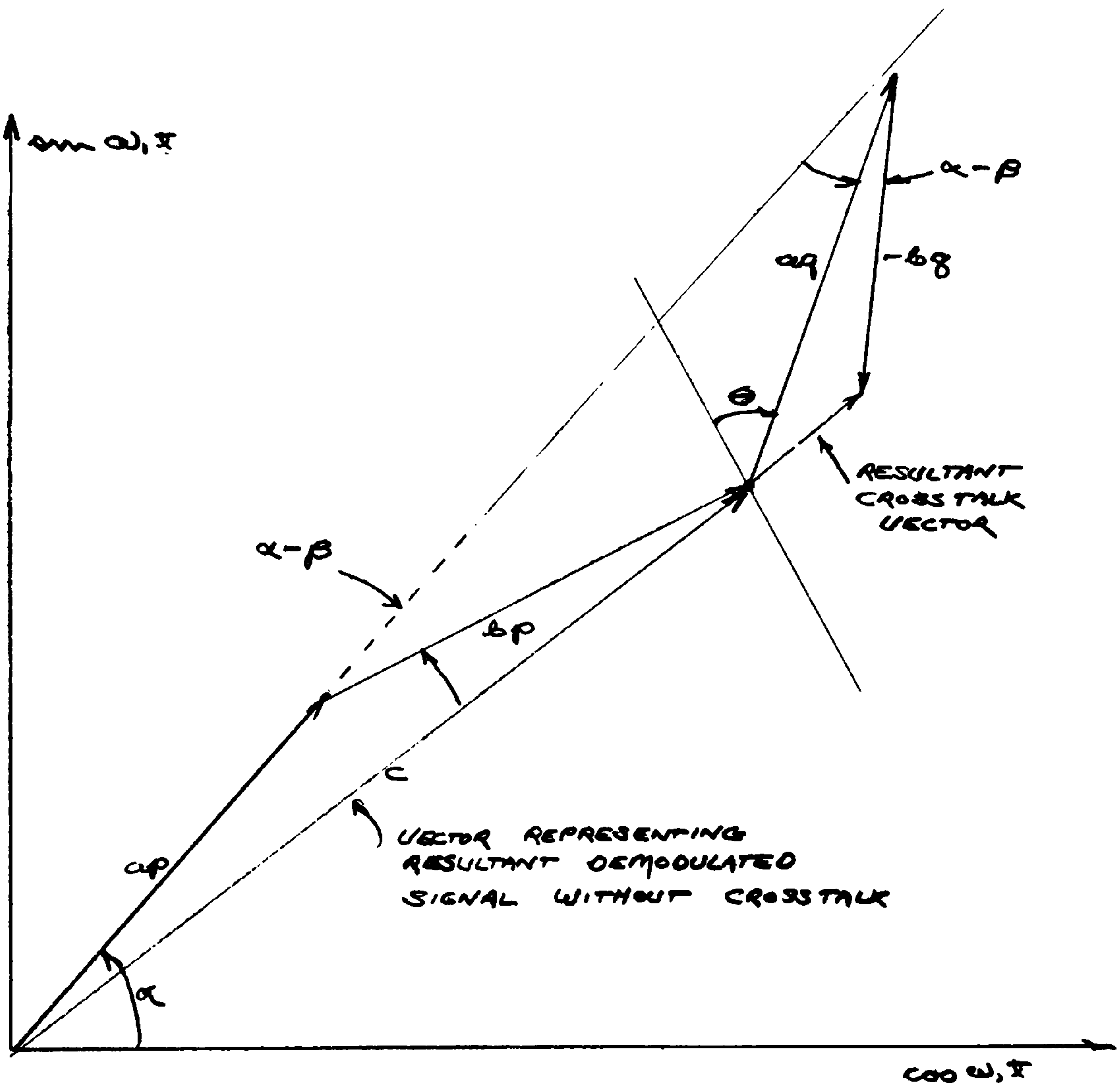
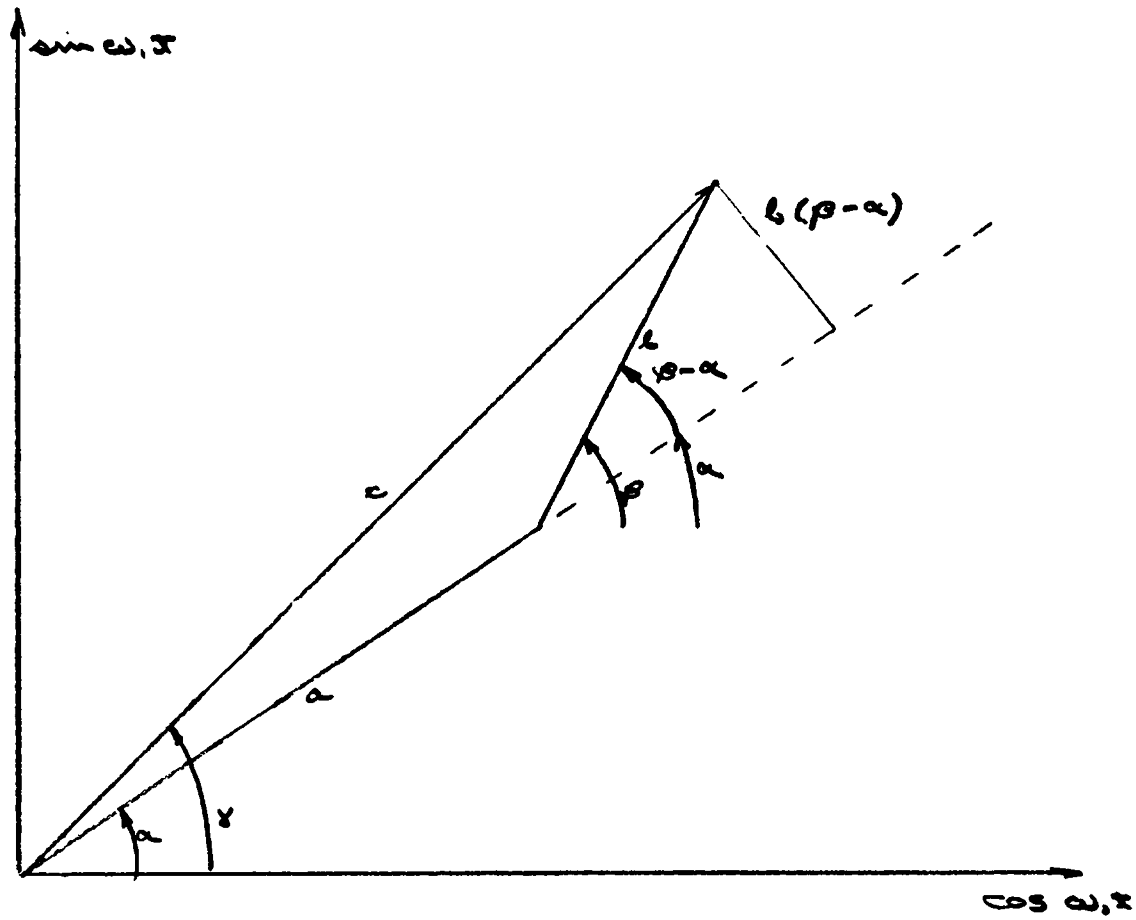



Figure 2 — Vector diagram illustrating the output of a double-sideband quadrature modulation system. The demodulated voltage is the sum of vectors ap, bp, aq, and -bq. If the transmission line was lossless, then a = b and $\alpha = \beta$.




$$\gamma = \alpha + \frac{b(\beta-\alpha)}{a+b} \qquad |\alpha-\beta| << 1$$

$$= \frac{\alpha+\beta}{2} + \frac{(a-b)}{a+b}\frac{(\alpha-\beta)}{2}$$

Figure 3 — Vector diagram used to make approximation used in Equation (9).

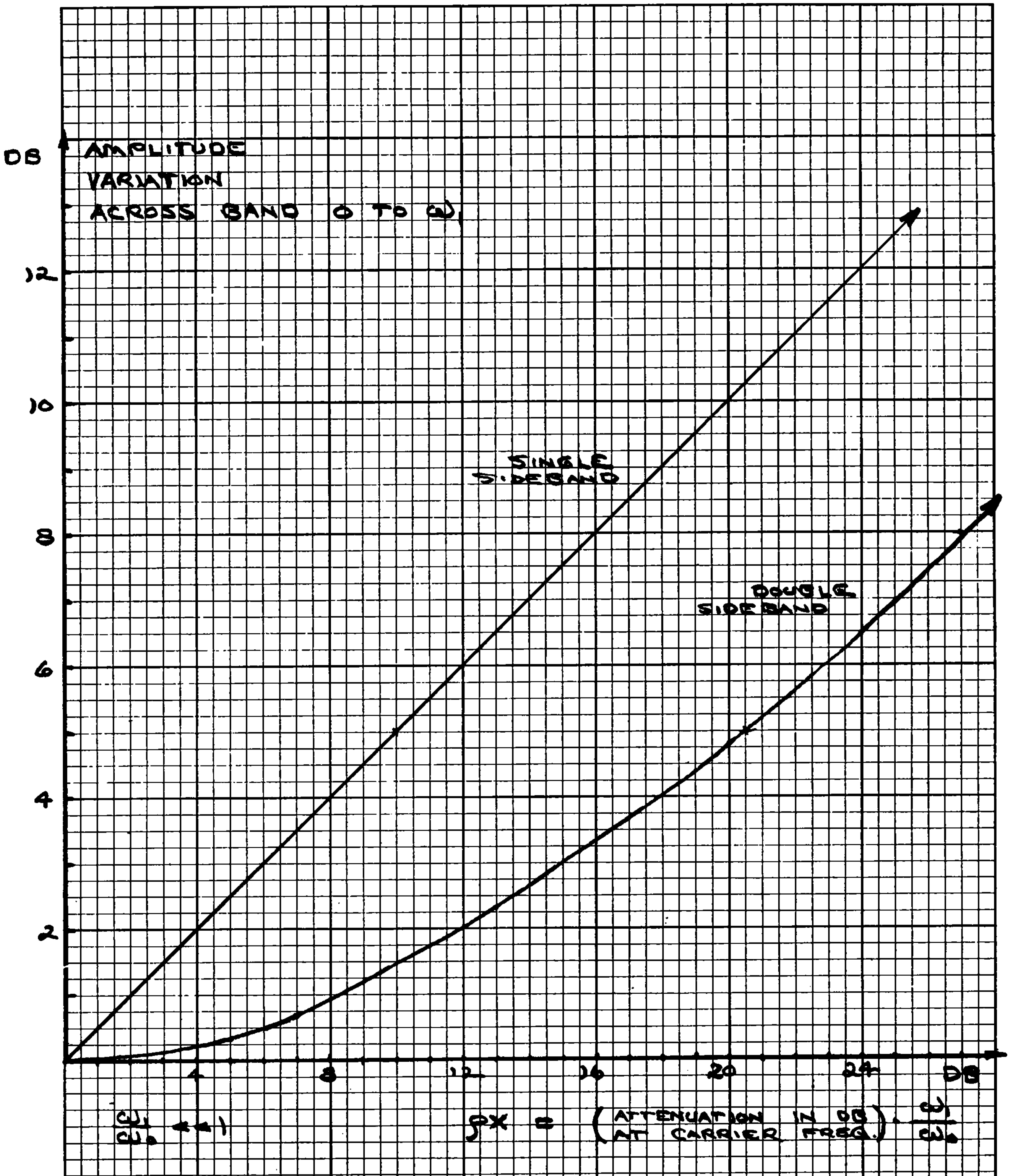



Figure 4 — Amplitude variation of signal thru lossy transmission line for DSB and SSB modulation.