NATIONAL RADIO ASTRONOMY OBSERVATORY

VLA ELECTRONICS MEMO #116

SOME TOLERANCES RELATING TO SPURIOUS RESPONSES

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This report contains some approximate estimates of the levels of spurious responses resulting from unwanted frequencies in the local oscillator system and from cross coupling between the IF signal channels. Tolerances on the levels of such unwanted signals and coupling are thereby obtained.

1. The Local Oscillator System

To provide flexibility in operating frequencies the oscillator system of the VLA operates by combining selected harmonics of standard frequencies of 5 and 500 MHz. The wanted frequencies are selected by filters or phase locked oscillators and unwanted frequencies are rejected only to some finite extent. One must therefore expect that the local oscillator signals may be accompanied by low-level unwanted sidebands.

To understand the effects of such sidebands consider the way in which they combine in a mixer both with the signal being converted and with the oscillator. The first of these combinations produces a small component of the signal converted by the wrong oscillator frequency, which generally has the effect of slightly decorrelating the signal or effectively increasing the system noise. This effect is unimportant so long as the unwanted oscillator frequencies are at least 20 dB below the wanted oscillator.

More serious problems arise from the interaction of the sidebands and the wanted oscillator frequency. If a sideband occurs which is separated from the oscillator by a frequency difference which falls within the signal passband at the mixer, a low level CW signal will be introduced into the IF. For a balanced mixer the conversion efficiency for the unwanted sideband, which enters the mixer at the oscillator port, is less than that for the signal by an isolation factor which depends upon the particular mixer used but is typically more than 20 dB. This effect however is generally not sufficient to eliminate the unwanted signal. Since the master oscillators are synchronized at all antennas such CW signals can combine in the correlators to produce spurious responses. The output of each correlator is phase detected with a waveform which is derived from the phase-switch control waveforms for the two corresponding antennas. It is therefore necessary to distinguish three cases; (a) unwanted signals introduced ahead of or at the mixer driven by the phase switched oscillator, (b) signals introduced after this mixer but before the mixer at which the fringe frequency compensation is introduced, and (c) signals introduced after the fringe compensation mixer. Spurious signals can cause problems in all three cases and these will be discussed in detail below. In case (c) the unwanted signals could only result from pick-up in the final IF since the fringe compensation occurs in the final mixer.

Case (a) Phase-Switched Spurious Signals

Unwanted signals which suffer the same phase reversals as the signals received at the antennas can produce serious spurious responses. In passing through the final mixer they receive a rate of change of phase equal to the fringe frequency compensation and at any correlator they combine to produce a sinusoidal output almost equal to the expected fringe frequency[1]. Superimposed on this are the 180° phase reversals of the phase switching sequence. After phase detection the resulting output tends to average to zero if the fringe frequency is high but produces a spurious output if the fringe frequency is about one cycle or less during the averaging period.

For any antenna pair the fringe frequency is equal to

$$f_f = \omega_o D \cos \delta \cos d \sin (H-h)$$

where ω_o is the angular rotation frequency of the earth (7.27×10^{-5} rad sec^{-1}), D is the antenna spacing in wavelengths, H and δ are the hour angle and declination of the center of the field being observed and h and d are the hour angle and declination of the projection of the baseline nearest the north pole. The fringe frequency thus becomes low when H-h approaches zero, i.e., when the source is moving parallel to the fringes, and also when the observing declination is very high. Consider the first condition which is likely to be the more important one. During a twelve hour mapping observation the fringe frequency for

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[1] The compensation for the fringe frequency which is introduced is equal to the rate of change of phase over the path lengths to the antennas for a local oscillator frequency (see VLA E.M. #109) which is a combination of the oscillator frequencies used in the various superheterdyne conversions from the input band to the final IF band. The output of the correlator is not exactly equal to the corresponding fringe frequency because a further small effect is introduced by the rate of change of the settings of the delay system.

each antenna pair will go through zero once. Consider one antenna pair and suppose that for a time interval τ_1 centered on H=h the field center remains within the same half fringe. This is a measure of the period during which the sign of the spurious output, after phase detection, remains unchanged. If the output from the antenna pair is averaged over this period τ_1 the ratio of the spurious response to the rms noise is

$$5\text{x}10^7 \frac{P_u}{P_n} \sqrt{\tau_1/B} \qquad (2)$$

where P_u is the power in the unwanted component in an IF channel, P_n is the noise power in a 50 MHz IF bandwidth, and B is the IF bandwidth at the correlator input in Hz. Outside of the period τ_1 the spurious response tends to average to zero, so over a 12-hour ($4.32\text{x}10^4$ sec) observing period its averaged value relative to the rms system noise becomes[2]

$$5\text{x}10^7 \frac{P_u}{P_n} \sqrt{\frac{\tau_1^2}{B \text{ x } 4.32\text{x}10^4}} \qquad (3)$$

τ_1 being measured in seconds. In the resulting map the spurious outputs will combine with essentially random phases since the varying settings of the delay system delay them by different amounts. The fluctuations resulting from the system noise combine in a similar manner, so if in (3) we replace τ_1 by $\bar{\tau}_1$ a value which is an average for the 351 antenna pairs, it provides an estimate of the ratio of the rms fluctuations resulting from spurious responses to those from the system noise in the derived map. If the spurious responses are not to degrade the sensitivity by more than 10%,

$$\frac{P_u}{P_n} \leqslant 4.2\text{x}10^{-7} B^{\frac{1}{2}}/\bar{\tau}_1 \qquad (4)$$

To estimate τ_1 for any given antenna pair one can take twice the time interval required for the fringe phase to change by $\pi/2$, starting at H-h = 0.

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[2] In this approximate treatment details such as the use of separate sine and cosine correlators have been omitted.

Then

$$\tau_1 = \omega_0^{-1} \sqrt{\frac{2}{D \cos \delta \cos d}} \qquad (5)$$

Maximum values of τ_1, for which the spurious responses are greatest, occur with the smallest values of D, i.e. at 20 cm wavelength and with the most compact configuration. Under these circumstances the antenna positions extend for about 0.5 km or 2500 wavelengths from the center of the wye, and a value of 1000 wavelengths can be taken as representative of the mean antenna separation. Inserting this in eqn. (5) with $\cos \delta = \cos d = 1$ one obtains $\tau_1 = 615$ sec. We shall therefore use 10 min. as a worst-case estimate of $\overline{\tau}_1$. Then for B = 50 MHz, the full bandwidth of an IF channel, eqn. (4) indicates $P_u/P_n < 4.9\text{x}10^{-6}$ or P_u must be 53 dB below P_n. For B = 0.5 MHz, the minimum bandwidth in the continuum mode, the tolerable limit on P_u is reduced by a further 10 dB to 63 dB below P_n. If one considers observations using a line receiving system in which the channel bandwidths may be as low as 1 kHz, eqn. (4) indicates P_u should be 77 dB below P_n. Achieving this level is somewhat less essential than the 63 dB required for the continuum system since with a multichannel output the effects of spurious responses in one channel would be easily detected, and the adjustable frequency of the final local oscillator could be changed slightly to place the wanted line in a different part of the IF passband. For observations at high declinations $\overline{\tau}_1$ is increased, but only by a factor of two at $\delta = 75°$. In the limit at $\delta = 90°$ the fringes are stopped altogether and $\overline{\tau}_1$ is equal to the full 12 hour mapping period. The tolerable levels of P_u are then decreased by 18 dB. This is again a less essential condition since observations very close to the pole are likely to be rarely made. Thus P_u at least 63 dB below P_n can be taken as a definite requirement with a further 20 dB as a desirable goal.

The tolerable level of a spurious signal relative to the local oscillator signal which it accompanies depends of course on the point in the system at which the frequency conversion occurs. In the 1.4 GHz upconverter or the 14 and 20 GHz cooled mixer P_n, the noise level in a 50 MHz bandwidth, is of the order of kTB or 10^{-13}W. The local oscillator power, $\sim 10^{-2}$W, is 110 dB greater. With 20 dB isolation in the mixer a spurious signal level 63 dB below P_n is produced by an oscillator sideband 153 dB below the level of the wanted oscillator frequency. At later points in the system the tolerable level is greater by a factor equal to the system gain from the input of the 5 GHz parametric amplifier.

Case (b) Spurious Signals Which Are Not Phase Switched

Spurious signals introduced into the IF which do not undergo the 180° phase changes of the phase switched oscillator, but which do undergo the fringe frequency compensation, produce a fringe frequency component at the correlator output. When the fringe frequency and the phase switching frequency are equal, or are related in an odd-harmonic manner, a spurious response can occur. The phase switching waveform is different for different antenna pairs, and is not a simple squarewave, but it is periodic with a maximum period of 9.6s. A value of 0.104 Hz will therefore be used for the phase switching frequency, f_s, in the following calculations.

For those antenna pairs for which the maximum fringe frequency (which occurs when $H-h = \pi/2$) exceeds f_s, the fringe rate will be equal to f_s twice during a 12-hour mapping period. What is the longest time for which the fringe frequency waveform can remain in phase with the switching waveform? This maximum in-phase duration occurs when the antenna spacing and δ are such that the fringe frequency is equal to f_s at $H-h = \pi/2$, since then the rate of change of fringe frequency goes through zero. For a given declination, configuration and frequency there may or may not be an antenna pair for which the above condition occurs exactly, but it is likely to occur sometimes and provides a realistic worst case. If τ_2 is the in-phase duration we take $\tau_2/2$ to be the time taken for the phase difference of the two waveforms to change by $\pi/2$ from the condition at $H-h = 90°$. Since $f_f = f_s$ at $H-h = 90°$ we have, from (1)

$$f_s = D \cos \delta \cos d$$

and putting $H-h = \frac{\pi}{2} + \omega_0 t$

$$\int_0^{\tau_2/2} 2\pi\, f_s\, dt - \int_0^{\tau_2/2} 2\pi\, f_s \sin (\omega_0 t + \pi/2) dt = \pi/2$$

from which

$$\tau_2 \simeq 2(1.5/f_s\, \omega_0^2)^{1/3} = 46 \text{ min.}$$

The mean value of the in-phase duration over the 351 antenna pairs will be considerably less than 46 min. At 20 cm wavelength with the most compact configuration the minimum duration occurs for the longest spacing which is $4.2\mathrm{x}10^3$ wavelengths east-west (except for the shortest spacings for which the fringe frequency is always less than f_s). An estimate of τ_2 is then obtained from

$$2\pi \int_0^{\tau_2/2} f_s \, dt - 2\pi \int_0^{\tau_2/2} (f_s - \frac{df_f}{dt} t) dt = - \frac{\pi}{2}$$

which using eqn. (1) becomes approximately

$$\int_0^{\tau_2/2} D \cos d \cos \delta \, \omega_0^2 \cos (H_0 - h) \, t \, dt = \frac{1}{4}$$

where H_0 is the hour angle at which $f_f = f_s$. This yields

$$\tau_2 = \frac{1}{\omega_0} \sqrt{\frac{2}{D \cos d \cos \delta \cos (H_0 - h)}} = 5.2 \text{ min.}$$

taking $\delta = 0$. Thus a figure of 10 min. must be a fairly good estimate of the mean of τ_2, and this is the same as the value used for $\overline{\tau}_1$, in case (a) above.

Apart from some minor effects, the way in which the spurious outputs from the various pairs combine is the same as described for case (a). (In the present case the fringe waveform is sinusoidal rather than the quasi-squarewave of the switching waveform; on the other hand $f_f = f_s$ twice in twelve hours whereas f_f goes to zero only once, and thus the differences between the two cases tend to cancel out.) So the tolerance figures on the level of a spurious component derived for case (a), which were -63 dB relative to P_n with a goal of a further -20 dB, can be taken to apply in this case also.

Case (c) Spurious Signals Entering After the Fringe Compensation Mixer

Spurious signals which do not suffer phase changes from either the phase switching or the fringe frequency compensation combine at the correlators to produce outputs which vary slowly with the changing delay settings. Since the delays exactly compensate for the changing path lengths from a source to the antennas, the frequency of the output variations is the same as would be observed using the same physical antenna spacings but receiving at a frequency equal to that of the spurious signal in the IF. Spurious responses can be injected into the output map when this output frequency is equal to, or oddharmonically related to, the phase switching frequency. The minimum antenna spacing at which an output frequency equal to f_s (0.104 Hz) can occur is found from eqn. (1) to be 1430 wavelengths or 8.6 km at the shortest wavelength in the IF passband (6m). Deleterious effects can therefore arise with the largest configuration, and to a small extent with the second largest but not with the two most compact ones. The frequencies of the output variations that would be observed with the largest configuration and a spurious IF signal of 33 MHz are the same as the fringe frequencies with the most compact configuration and an observing frequency of 1400 MHz. Thus the overall effect of the spurious responses in the present case will be, at the worst, about the same as that for case (b) discussed above, and the same maximum tolerable levels can again be applied.

(d) The Single Sideband Mixer

An application of the discussion under case (b) is provided by the single sideband mixer in the fringe-frequency compensation system. For each IF channel a low frequency, f_l, of about 260 kHz is generated digitally in a manner that allows the computer to impose upon it a controlled initial phase and rate of change of phase. This waveform is used in the final conversion of the IF signals, and the phases of the latter are thereby adjusted in such a way as to remove the effects of the motion of the fringe patterns across the sky. Before being used in the final mixer, however, the frequency f_l is added to a higher frequency f_h in the 1-2 GHz range. Only the frequency $f_h + f_l$ is wanted and the frequencies f_h & $f_h - f_l$ which accompany it must be suppressed by filtering, phasing, or locked oscillator techniques. These latter frequencies could beat with $f_l + f_h$ in the final mixer to produce spurious signals at f_l & $2f_l$ which at the correlators would produce outputs at the fringe frequency f_f and at $2f_f$. From the above discussion of case (b) the maximum tolerable levels of f_l and $2f_l$ at the correlator inputs are 63 dB below P_n, the IF noise power in a 50 MHz bandwidth. The frequencies f_l & $2f_l$ are however not much greater than 0.5 MHz and the gain in this low edge of the IF band is reduced by a high pass filter to about 30 dB below the mid-band gain. Thus at the output of the final mixer the f_l & $2f_l$ frequencies need only be 33 dB below P_n, which at that point is about -13 dBm. The unwanted frequencies $f_h - f_l$ & f_h therefore must not exceed -26 dBm (allowing 20 dB mixer isolation) which is 36 dB below the wanted oscillator frequency f_1+f_h. This level of rejection is barely achievable using phasing techniques, and it is therefore proposed to add f_1 to f_h in two steps, first adding f_1 to a frequency f_2 between 60 & 70 MHz which lies outside the IF passband and at which a crystal filter can be inserted. The required suppression of the unwanted frequencies, including the 20 dB additional goal, can then be readily achieved. In the second step, in which (f_1+f_2) is added to a 1-2 GHz frequency, unwanted outputs occur at approximately f_2 & $2f_2$ below the wanted frequency. Components at frequency f_2 which are thereby generated in the final IF mixers could again produce fringe-frequency outputs at the correlators. If the IF response at f_2 is again -30 dB relative to the band center it follows as before that a minimum of 36 dB suppression is required for the unwanted outputs of the second frequency conversion step. This time a conventional coaxial filter suffices.

2. Cross Coupling of the IF Signals

Spurious responses similar to those described above can also result from cross coupling of the IF waveforms. Such coupling is only likely to occur in the IF modem and transmission system or the later stages. First consider coupling that occurs before the final mixer where the fringe frequency compensation is introduced. Suppose that a fraction α of the power in one channel can be coupled into a second channel in such a way that after conversion of both IF waveforms to the 0-50 MHz baseband the coupled component remains correlated with the signal in the first channel. When these two channels are fed into a correlator an output at the expected fringe frequency will appear. If the settings of the delays in the two channels differ by more than the reciprocal of the IF bandwidth then this response will be substantially attenuated, but assume for the moment that the delay difference is small. After an averaging period τ_2 the amplitude of this fringe frequency component relative to the rms noise is approximately equal to

$$\sqrt{\alpha\ B\ \tau_2} \qquad (6)$$

Note that in contrast to the spurious CW signals the unwanted effects here are greatest with the maximum bandwidth. The way in which the spurious responses can enter into a map is similar to that discussed for case (b) above except that the overall effect will be substantially reduced because the delay settings which occur when the fringe and phase switching frequencies coincide will generally not be equal. To obtain a criterion of the maximum tolerable coupling consider the resulting error in any visibility measurement. The worst case occurs when the delay settings happen to be equal and the duration of the measurement coincides with the period τ_2 during which the fringe frequency and phase switching waveforms remain in phase. Then if the error is not to exceed the rms noise[3] we have from (6)

$$\alpha < (B\,\tau_2)^{-1}$$

For $B = 5x10^7$ Hz and τ_2=10 min. this shows α must not exceed -105 dB.

Coupling between IF channels after the final mixer results in an output from the correlator which varies slowly as the delay settings for the two channels vary. This effect is similar to that discussed under case (c) above and here the frequency of the variation corresponds to that of the fringe pattern that would be observed with the antennas receiving signals at the IF center frequency of 25 MHz. The minimum spacing at which a fringe frequency equal to the 0.104 Hz phase switching frequency can occur is 1430 wavelengths at 25 MHz or 17.2 km. At the largest configuration frequencies equal to the phase switch frequency could occur for about half of the antenna pairs, so the effect here is somewhat less than that discussed above for coupling ahead of the final mixer, but probably not less by an order of magnitude. A figure of -100 dB can be taken as an estimate of the maximum tolerable cross coupling.

The above discussion applies to cross coupling between IF channels of different antennas. Cross coupling between two IF channels of the same antenna is less serious since the inputs to any correlator always come from different antennas. The most serious effect of coupling for the same antenna appears to be the introduction of an instrumental component in polarization measurements. If such instrumental polarization is not to exceed 0.1% the coupling should not exceed -60 dB.

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[3] In the case of the spurious signals discussed in section 1 where the criterion was based upon the overall map the effect on a single visibility measurement is about the same as under the present criterion and can be found by substituting eqn. (4) in (2).

3. Concluding Remarks

For spurious CW signals introduced into the IF three cases have been distinguished, (a) in which both phase-switching and fringe-compensation phase shifts occur, (b) in which only the latter occur and (c) in which neither occur. The most serious effects occur in case (a) for observations near the pole, otherwise the overall level of spurious responses in the final map is about the same in each case under the conditions which maximize these undesirable effects. In case (a), and to a lesser extent in case (b), the responses are maximized when the antenna spacings in wavelengths are smallest, but in case (c) they are maximized when the antenna spacings in meters are greatest. In cases (b) and (c) the responses could be greatly reduced by increasing the frequency of the phase switching which is introduced in the early stages. The maximum fringe frequency at 1.5 cm wavelength is 176 Hz, so with a switching frequency a few times greater than this, responses in case (b) would be limited to the effects of odd-harmonics of the fringe frequency coinciding with the switching frequency. The amplitudes of these harmonic responses have an inverse dependence upon the harmonic number and their durations are decreased because the rate of change of frequency of the harmonics is correspondingly greater. Another possibility would be to increase the switching frequency by a factory of 10 to decrease the responses in case (c) and then move the fringe compensation mixing to an earlier part of the receiving system. This would require increased suppression of the unwanted frequencies in 1(d). The present arrangement is convenient for computer control since the phase switching is slow enough to allow the computer to generate the required waveforms and perform the phase detection. One of the above modifications could however be contemplated if achieving the tolerances described here proves difficult. The slow switching rate is, of course, quite satisfactory for achieving the principal objective of the phase switching which is to eliminate the effects of DC offsets in the correlator outputs that can result from non-linearity in the IF amplifiers or DC offsets at the samplers.

Maximum tolerable levels of spurious CW signals in the IF are -63 dB relative to the IF noise level in a 50 MHz bandwidth, and a further -20 dB is desirable for observations with a line system or for case (a) when observing at very high declinations. For cross coupling in the IF the maximum tolerable coupling is about -105 dB for channels from different antennas and -60 dB for channels from the same antenna.

ADDENDUM TO VLA ELECTRONICS MEMO #116

September 6, 1973

A. R. Thompson

SOME TOLERANCES RELATING TO SPURIOUS RESPONSES

This addendum gives some statistical data on the separation of the antenna pairs in the VLA and thus provides a more definite basis for some of the estimates used in VLA E.M. #116. The data are presented in Figures 1 to 4. Figures 1 & 2 are histograms of the distribution of spacings of the 351 antenna pairs for the largest and the most compact configurations (A & D) respectively. Figures 3 & 4 are similar histograms of the spacing multiplied by cos d for each pair and this data is particularly useful in determining the range of fringe frequencies encountered as can be seen from the expression for f_f on p.2. For configuration A the mean spacing is 16.76 km and the mean of the spacing x cos d is 14.11 km. For configuration D the equivalent values are 399.9 m and 338.4 m respectively. The data are based on antenna positions in memorandum by J. H. Lancaster dated 8/30/73 and the computations were performed by Steve Burgan.

The above data are used to check estimates in VLA E.M. #116 as follows. On p.4 a figure of 1000 wavelengths was estimated for the mean value of D at 20 cm wavelength with the most compact configuration. The new data indicate a value of 2000 wavelengths. However to determine a representative value of $\overline{\tau}_1$ it is more appropriate to use in eqn (5) the mean value of D cos d which is 1690 wavelengths. This gives $\overline{\tau}_1 = 7.9$ minutes and using this value instead of 10 minutes the tolerable level of P_n is increased by 1 dB.

In the discussion of case b on p.5 one may inquire how many of the antenna pairs have fringe frequencies which do not exceed f_s with the most compact configuration and a wavelength of 20 cm. For $\delta=0°$ this is the number of antenna pairs for which D cos d does not exceed 1430 wavelengths, and this turns out to be 44% of the total number. This effect was ignored in considering case b; including it would increase the tolerable level of P_n by a factor $(1-0.44)^{-1/2}$ or 1.25 dB.

On p.9 the fraction of antenna pairs for which D cos d exceeds 1430 wavelengths at 25 MHz with the largest configuration is estimated to be 50%. The new data indicates a value of 37%. Spurious responses resulting from coupling in the final IF channels can thus produce correlator output frequencies equal to f_s for 37% of the pairs at the largest configuration.

One can conclude that with the new data the tolerable levels in the memo are confirmed within about 1 dB.

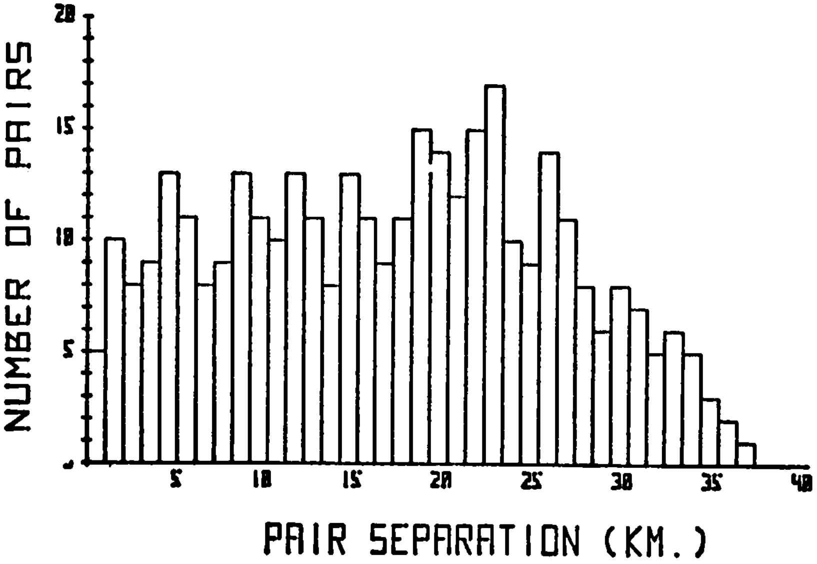

Fig. 1 Histogram of Spacings for Largest Configuration (A)

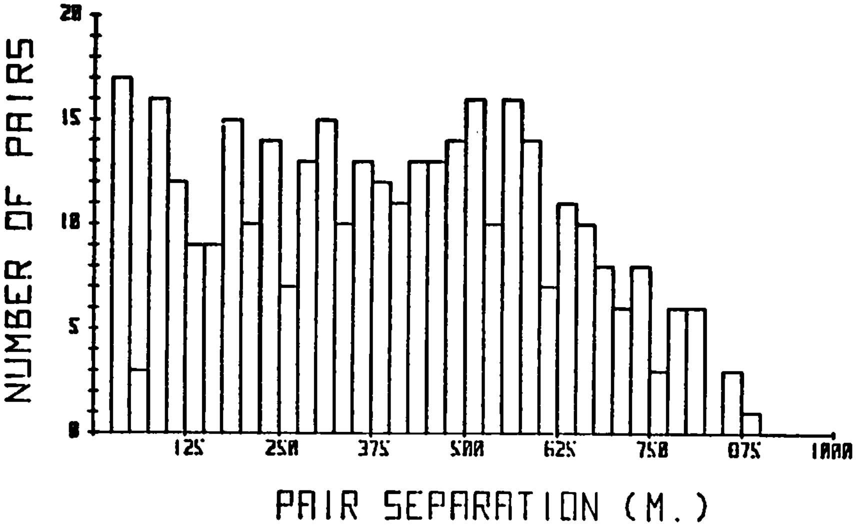

Fig. 2 Histogram of Spacings for Most Compact Configuration (D)

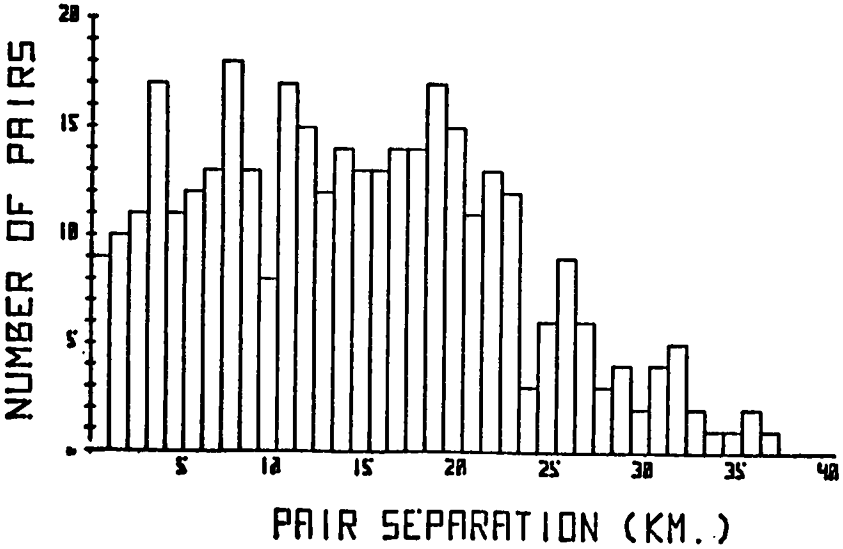

Fig. 3 Histogram of Spacing x Cos d for Configuration A

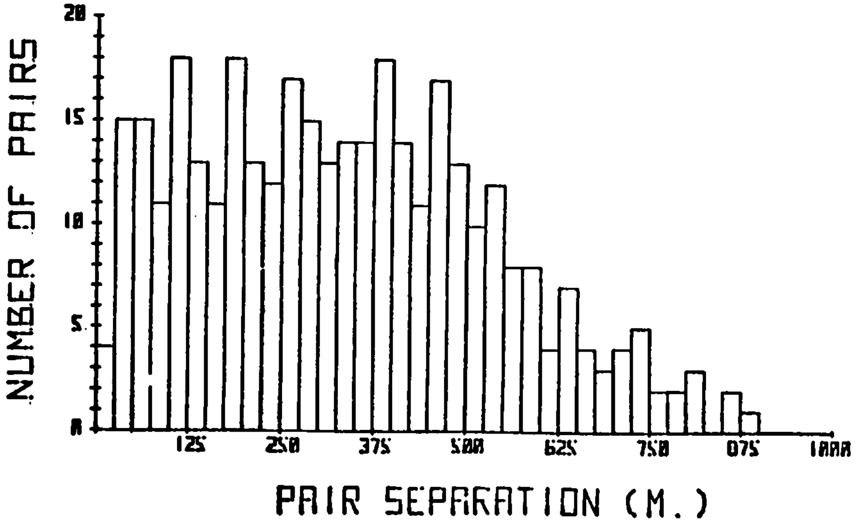

Fig. 4 Histogram of Spacing x Cos d for Configuration D