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CROSS POLARIZATION IN A CASSEGRAIN ANTENNA WITH A "TRICONE" FEED SYSTEM

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In this paper expressions are derived for the cross-polarization in a Cassegrain antenna in which the line passing through the primary focus, the vertex of the hyperbola and the secondary focus is tilted with respect to the paraboloid axis. Such a situation is useful when a Cassegrain system is to be equipped with multiple feeds (Levy and Katow, 1968). The analysis follows the analysis of a symmetrical Cassegrain carried out by Potter (1961).

The coordinate system used for the polarization analysis is shown in Figure 1. The field at the point F' is assumed to be given by

$$E(F') = E_o(\xi_H', \gamma)\ \tilde{a}_{x'} \tag{1}$$

The projection of this vector onto the line defined by the intersection of the Z' = 0 plane and the ξ' = constant plane is given by

$$E_\gamma\ (\xi_H', \gamma{=}0) = E_o(\xi_H', \gamma)\ \cos\ (\xi_H)\ \tilde{a}_\gamma \tag{2}$$

The projection of this vector onto the plane normal to the line F'P is given by

$$E_\gamma(\xi_H', \gamma) = E_o(\xi_H', \gamma)\ \cos\ (\xi_H')\ \cos\ (\gamma)\ \tilde{a}_\gamma \tag{3}$$

Similarly $$E_{\xi_H'}(\xi_H', \gamma) = E_o(\xi_H', \gamma)\ \sin\ (\xi_H')\ \tilde{a}_{\xi_H'} \tag{4}$$

It is shown by Potter (1961) that the relative intensities of the ξ' and ψ' components of the field are preserved in reflection from the hyperboloid. This can also be easily seen from consideration of the normal and tangential field components at the hyperboloid surface. Thus

$$E_{\psi'} = E_o(\xi', \psi')\ \cos\ (\xi')\ \cos\ (\gamma)\ \tilde{a}_{\psi'} \tag{5}$$

$$E_{\xi'} = E_o(\xi',\psi')\ \sin\ \xi'(-\tilde{a}_{\xi'}) = E_o(\xi',\psi')\ \sin\ (\xi')\ \tilde{a}_{\xi'} \tag{6}$$

where (from Potter (1961))

$$E_o(\xi',\psi') = \frac{\sin\gamma}{\sin\psi'}\ E_o(\xi',\gamma)$$

Since $\tan \frac{\psi'}{2} = M \tan (\frac{\gamma}{2})$. For $\gamma_{max} < 10°$, $\sin \gamma \approx 2 \tan \frac{\gamma}{2}$

$$\therefore \frac{\sin \gamma}{\sin \psi'} = \frac{2 \tan \frac{\gamma}{2}}{\sin \psi'} = \frac{2 \tan (\frac{\psi}{2})}{M \sin \psi} = \frac{\mathrm{Sec}^2 \frac{\psi}{2}}{M}$$

$$\therefore E_o(\xi',\psi') = \frac{\mathrm{Sec}^2 (\frac{\psi'}{2})}{M} E_o(\gamma,\xi') \qquad (7)$$

The minus signs in (5) and (6) arise from the change of coordinates and the zero tangential field restriction.

Since the paraboloid will transform rays from the point F into rays normal to the X-Y plane, the polarization in the aperture plane will be given by the X and Y components of (5) and (6).

From Figure 1 it can be seen that

$$E_x = (E_{\psi'} \cdot \tilde{a}_{\psi'} \cos \xi' - E_{\xi'} \cdot \tilde{a}_{\xi'} \sin \xi') \tilde{a}_x \qquad (8)$$

$$E_y = (E_{\psi'} \cdot \tilde{a}_{\psi'} \sin \xi' + E_{\xi'} \cdot \tilde{a}_{\xi'} \cos \xi') \tilde{a}_y \qquad (9)$$

Combining (5), (6), (8), (9)

$$E_x = (- E_o(\xi',\psi') [\cos \xi' \cos \gamma \cos \xi' + \sin \xi' \sin \xi'] \tilde{a}_x \qquad (10)$$

$$E_y = (- E_o(\xi',\psi') [\cos \xi' \cos \gamma \sin \xi' - \sin \xi' \cos \xi']) \tilde{a}_y \qquad (11)$$

Eq. (10) and (11) give the co- and cross-polarized components in the aperture of the parabola. Before (10) and (11) can be evaluated it is necessary to express ξ' and γ in terms of the aperture angles ξ and ψ and the tilt angle ϕ. From Figure 1

$$y' = y \qquad (12)$$

$$x' = x \cos \phi - z \sin \phi \qquad (13)$$

$$z' = x \sin \phi + z \cos \phi \qquad (14)$$

Expressing (12), (13), (14) in spherical coordinates gives

$$\sin\psi' \sin\xi' = \sin\psi \sin\xi$$

$$\sin\psi' \cos\xi' = \sin\psi \cos\xi \cos\phi - \cos\psi \sin\phi$$

$$\cos\psi' = \sin\psi \cos\xi \sin\phi + \cos\psi \cos\phi$$

so that

$$\psi' = \cos^{-1} (\sin\psi \cos\xi \sin\phi + \cos\psi \cos\phi) \tag{15}$$

$$\xi' = \cos^{-1} \left(\frac{\sin\psi \cos\xi \sin\phi - \cos\psi \sin\phi}{\sin\psi'}\right) \tag{16}$$

$$\gamma = 2 \tan^{-1} \left(\frac{\tan(\psi'/2)}{M}\right) \tag{17}$$

If it is wished to express (10) and (11) as functions of the aperture rectangular coordinates x, y, the relationships

$$x = \rho \sin\psi \cos\xi$$

$$y = \rho \sin\psi \sin\xi$$

$$r^2 = 4\ az$$

can be used to obtain

$$\xi = \tan^{-1} \left(\frac{y}{x}\right) \tag{18}$$

$$\psi = \tan^{-1} \left[\frac{4a \sqrt{x^2 + y^2}}{4a^2 - (x^2 + y^2)}\right] \tag{19}$$

For a y' polarized field at F', the expressions equivalent to (10) and (11) are

$$E_x = (-E_o(\xi', \psi')\,[\sin\xi' \cos\gamma \cos\xi' - \cos\xi' \sin\xi']) \,\tilde{a}_x \qquad (20)$$

$$E_y = (-E_o(\xi', \psi')\,[\sin\xi' \cos\gamma \sin\xi' + \cos\xi' \cos\xi']) \,\tilde{a}_y \qquad (21)$$

REFERENCES

Levy, G. S. and Katow, S. M. 1968, "Multi-feed Cone System for the Advanced Antenna System", JPL Space Programs Summary, 37-45, Vol. III, pp. 48-51.

Potter, P. D. 1961, "The Aperture Efficiency of Large Paraboloidal Antennas As a Function of Their Feed System Radiation Characteristics", JPL Tech. Report 32-149.

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parabola focal length = a

xz plane and x'z' plane are coplanar.

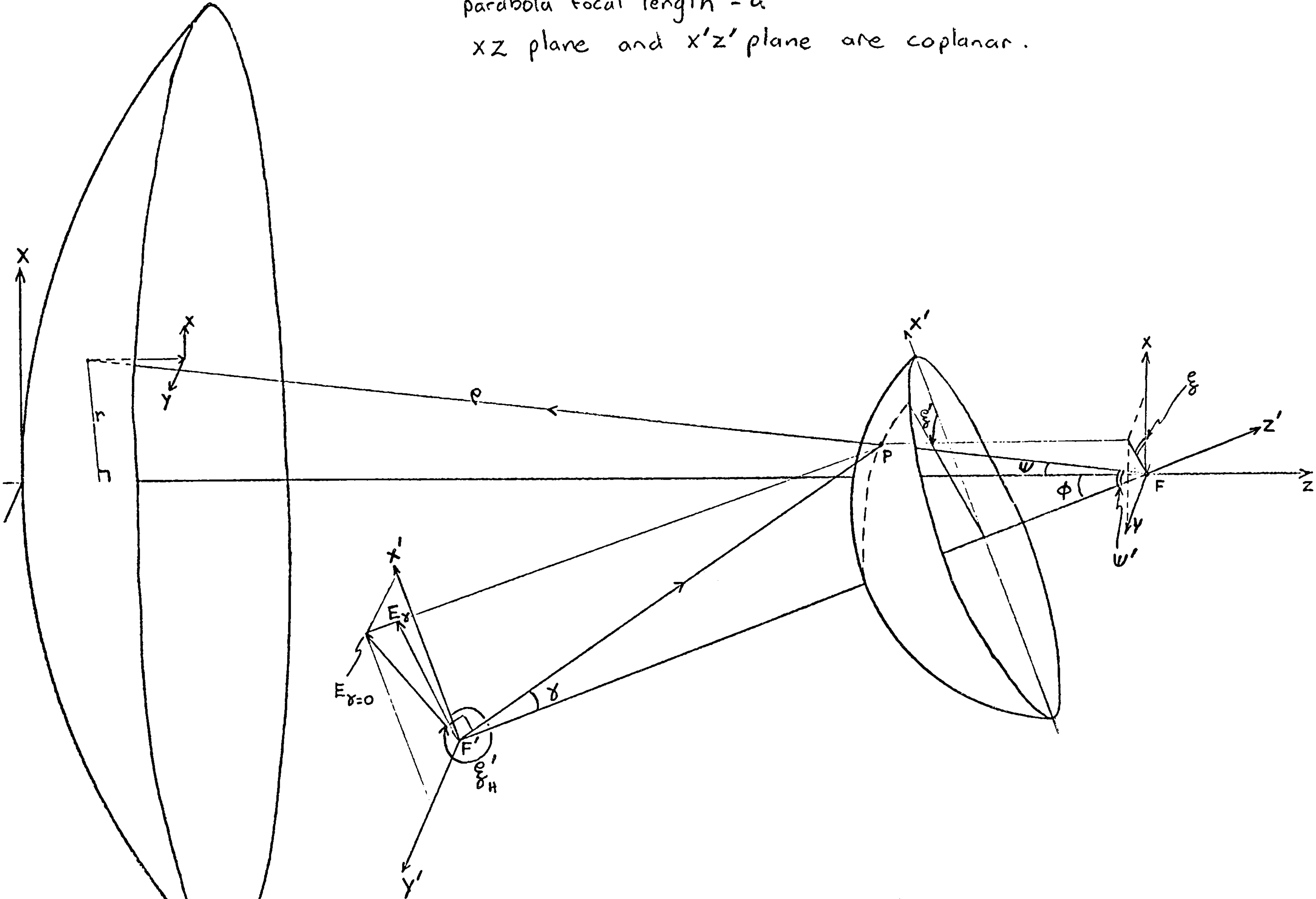

Figure 1 Cassegrain coordinates.