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# A SYNCHRONOUS-PULSE CALIBRATION SYSTEM FOR THE VLA

A. R. Thompson

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A discussion is given of the feasibility and uses of a calibration scheme using synchronized trains of fast pulses injected at the antennas. The spectrum of such a pulse train consists of discrete lines which are harmonics of the repetition frequencies. It is thus possible to match the spectrum to the frequency channels of the spectral processor, and thereby investigate the phase and amplitude characteristics of the receiving passband from the pulse injection point to the samplers. Such a calibration system would be useful both as a diagnostic tool for examining the response of such critical items as the waveguide, and as a means of rapidly calibrating the relative responses of the frequency channels during spectral line observations.

In studying this problem a series of schemes of increasing sensitivity and complexity were analyzed. Although not all are useful, they are described because they show the development of ideas that lead to a feasible system.

## 1.0 BASIC IDEAS

An infinite train of narrow pulses with repetition frequency $f_p$ has a power spectrum that consists of discrete lines at harmonics of $f_p$. Such a pulse train could be injected into the front-end or IF amplifiers at an antenna to provide a calibration signal. If the pulses at each antenna were synchronized the resulting signals would be correlated, and could be processed by the correlators to provide calibration of the response of the instrument in both amplitude and phase. Such a scheme is potentially more useful than one in which incoherent noise is injected at the antennas, since such noise produces no response from the correlators and aids only the amplitude calibration.

Synchronization of the pulses can be most practically achieved by using the local oscillator signals at the antennas. The pulse calibration scheme would not, therefore, measure the absolute phase stability of the instrument, and observations of calibration sources would remain the fundamental calibration procedure. The pulse scheme would be most useful for calibrating the relative responses of the individual channels in the spectral line mode of operation, and should be able to provide more rapid and convenient monitoring during a long run than repeated observations of calibration sources. The latter method is slow: with a source as strong as 10 Jy it would take about 4 hours to achieve a signal-to-noise ratio of 100 when observing with the narrowest channel bandwidth of 381 Hz. It is envisaged, therefore, that when observing with narrow channels the pulse scheme would be used to monitor any changes in the instrumental response, and calibration on a radio source would be required at relatively infrequent intervals. It might also prove possible to perform the calibration observation in the continuum mode, which would require much less observing time. The result could then be related to the mean of the channel responses determined by the pulse technique, to obtain an absolute phase calibration for each channel.

An infinite train of rectangular pulses can be most conveniently represented as a function of time by using Bracewell's (1965) notation:

$$f_p \mathrm{III}(f_p t) * \sqcap\!\!\sqcap\left(\frac{t}{\Delta t}\right) , \tag{1}$$

where $\mathrm{III}(f_p t)$ is a sequence of delta functions at intervals $f_p^{-1}$ in time, $\sqcap\!\!\sqcap\left(\frac{t}{\Delta t}\right)$ is a rectangular function of width $\Delta t$, and the asterisk denotes convolution. The power spectrum of the pulse train is the squared modules of the Fourier transform of (1), which is

$$\Delta t^2 \mathrm{III}\left(\frac{f}{f_p}\right) \mathrm{sinc}^2(\Delta t f) . \tag{2}$$

Here we have a series of lines at frequencies that are harmonics of $f_p$, the amplitudes of which decrease with frequency following the sinc-squared function $\sin^2(\pi\Delta tf)/(\pi\Delta tf)^2$. The envelope falls by 1 dB at $\Delta tf=0{\cdot}26$; for example at $f=5{\cdot}2$ GHz for $\Delta t=50$ ps, which is achievable with a tunnel diode pulse generator (Rogers, 1975). It should therefore be possible to generate pulses useful at L- and C-Bands and the 1-2 GHz intermediate frequencies at the antennas.

The spectral processor for the VLA uses 10 different sampling bandwidths varying by factors of two from 50 MHz down to 97 kHz. The numbers of frequency channels vary from 16 to 256. The bandwidths and various other data are given in Table 1[1]. To obtain a calibration line in each channel $f_p$ must be made equal to the chosen channel bandwidth, and the final local oscillator must be appropriately adjusted to center the channels on harmonics of $f_p$. Since all of the bandwidth values in Table 1 can be expressed as 100 MHz/$2^n$, n being an integer, all of the required pulse frequencies can be generated by binary division from a 100 MHz clock.

As will become apparent, a major problem in designing a pulse calibration system is in obtaining sufficient signal strength with narrow IF bandwidths and large numbers of spectral channels. It would be convenient to be able to obtain a calibration line in each channel for all modes of operation, but this is not essential. The closest required spacing of the calibration lines actually depends upon the fineness of the frequency structure in the receiving passbands. The finest structure results from reflections within the waveguide transmission system. For example, a ripple of amplitude 0·1 dB p-p could have a period as short as 16 kHz, assuming a reflection coefficient of 0·1 and waveguide attenuation of 1·4 dB per km.

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[1]Some details that are not important for this feasibility study have been omitted from the table. For example, the correlator has the capacity to generate 512 channels for the narrowest bandwidths although it is not planned to implement more than 256, and in some cases oversampling is used.

Thus a minimum goal should be to measure calibration lines down to, say, the 12 kHz channel spacing. Other design goals are to achieve a signal-to-noise ratio of 100 for each calibration line and to do this within an integration time of 10 minutes.

## 2.0 SENSITIVITY LIMITATIONS OF A SIMPLE PULSE-CALIBRATION SCHEME

Consider a system in which synchronized pulse generators are located at the antennas and used to inject pulse trains into the receiver inputs. The pulse amplitudes must be small enough that the signals remain within the linear operating range of the electronics until they have traversed all components that affect the frequency response. It can be assumed that the a.l.c. circuits do not respond in the time scale of the sharp pulses. Let $B_1$ be the widest receiver bandwidth where overloading is likely to occur. In this bandwidth there will be $B_1/f_p$ calibration lines, each of peak amplitude a. During each pulse the amplitudes of the lines add in phase, so the peak pulse power is $(aB_1/f_p)^2$. (Note that the peak power is proportional to $B_1{}^2$ but the mean power is proportional to $B_1$ because the pulse duration is proportional to $B_1{}^{-1}$.) To avoid overloading, the peak power must not exceed some fraction, α, of the system noise level. Thus the maximum pulse level is given by

$$a^2 B_1{}^2 f_p{}^{-2} = \alpha k T B_1 \ , \tag{3}$$

whence

$$a = \frac{f_p \sqrt{\alpha k T}}{\sqrt{B_1}} \ , \tag{4}$$

where k is Boltzmann's constant and T is the system noise temperature. In calculating the signal-to-noise ratio for the calibration lines it will be assumed that the correlator acts like a filter receiver with identical bandwidths. The pulse repetition frequency, $f_p$, is chosen to equal the spectral channel bandwidth, so the signal-to-noise power ratio in each channel is

$$\frac{a^2}{kTf_p} = \frac{\alpha f_p}{B_1} . \tag{5}$$

After integration for time $\tau$ the output signal-to-noise ratio is

$$S_1 = \frac{\alpha f_p^{3/2} \tau^{1/2}}{B_1} . \tag{6}$$

With $S_1$=100, $\tau$=600 secs, $\alpha$=0·1 and $B_1$ equal to the parametric amplifier bandwidth of 500 MHz one obtains $f_p$=7·5 MHz. This is the lowest value of $f_p$ for which the required sensitivity can be obtained, but it is too large for even the widest channel bandwidths. The situation can be improved a little by injecting the signals into the 50 MHz-wide IF bands centered in the 1-2 GHz range. Thus $B_1$ becomes 50 MHz and equation (6) gives $f_p$=1·6 MHz. This is still far from meeting the performance goals.

## 3.0 AN IMPROVED SCHEME USING A NOISE-FREE REFERENCE

One way of obtaining better sensitivity would be to build a special receiver, located in the Control Building, that would provide a reference against which the signals from each of the antennas would be correlated. The reference receiver would be designed to handle high pulse levels to improve the signal-to-noise ratio. As an incidental advantage, the reference passband could be stable and well calibrated, and thus be easily separated from the required band-pass measurements. Further consideration shows that it is not necessary to build the complete reference receiver, but only to simulate its sampled output and supply it to the multipliers. In this way the reference channel can be entirely noise free.

To calculate the output from the reference receiver, infinitely narrow input pulses and a rectangular passband can be used. The output as a function of frequency is simply a band-pass filtered series of calibration lines:

$$\mathrm{III}\left(\frac{f}{f_p}\right)\{\sqcap\left(\frac{f}{B_{IF}}\right)*[\delta(f-f_o)+\delta(f+f_o)]/2\} \ . \qquad (7)$$

Here $B_{IF}$ and $f_o$ are the bandwidth and center frequency of the final IF passband. The output pulse train, as a function of time, is the Fourier transform of (7):

$$B_{IF}f_p\mathrm{III}(f_pt)*[\mathrm{sinc}(B_{IF}t)\cos 2\pi f_ot] \ . \qquad (8)$$

Expressions (7) and (8) are illustrated in Figure 1. Expression (8) represents a series of sinc-function pulses multiplied by $\cos 2\pi f_o t$, which repeat at frequency $f_p$. To obtain the required signal at the multipliers, expression (8) must be digitized at the appropriate sampling frequency which is $2B_{IF}$. The IF filters of the spectral processor are designed for low-pass or band-pass sampling, and this results in the following relationship:

$$f_o = (p + 1/2)B_{IF} \ , \qquad (9)$$

where p is an integer that is zero for low-pass filtering. Then if the timing of the sampling pulses is chosen so that pulses coincide with the sinc function maxima, as in Figure 1, it will be found that, except at these maxima, the sampling times all coincide with zeros of the sinc or cosine functions of expression (8). So the required reference is simply a sequence of logic ones coincident with the signal pulses at the antennas, the intervening samples being zero[2].

In the spectral processor, correlations between the reference signal and the signals from the antennas are made for a series of time offsets from $-(N-1)/2B_{IF}$ to $N/2B_{IF}$ where N is the number of channels ($N=B_{IF}/f_p$). The resulting 2N integrated products are Fourier transformed to give N complex visibility values. Consider

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[2]This result conveniently disposes of the question of where the reference levels should be set for the three-level sampling of the waveform represented by (8).

the multiplications of a pulse train from one antenna with the reference described above. Of the 2N multiplications that occur at each sample time, on average only one involves a non-zero reference value. The reference acts rather like a gate, selecting certain signal samples for storage in certain integrators. The operation is such that the waveform from an antenna is effectively divided into lengths of $\frac{1}{f_p}$ each centered on a pulse occurrence time, and the 2N successive samples in each length are stored in the integrators for successive delay offsets. Thus the integrators accumulate an interval of the signal waveform averaged in a superposed-epoch manner. The final output is the Fourier transform of this averaged signal, and it provides directly the required frequency response in amplitude and phase.

How much do we gain in sensitivity, compared with the scheme of section 2.0, by using the noise-free reference? In time $\tau$ the zero-offset multiplier receives $2B_{IF}\tau/2N$ samples of the pulse amplitude $aB_{IF}/f_p$. These add up to a signal voltage $aB_{IF}{}^2\tau/Nf_p$. In the same time interval each multiplier also receives $B_{IF}\tau/N$ samples of noise of rms amplitude $\sqrt{kTB_{IF}}$, which combine to produce an rms voltage $B_{IF}\sqrt{kT\tau/N}$. The 2N integrator outputs are then Fourier transformed to produce N measurements of amplitude and phase. Now let s be the signal amplitude in each channel in the frequency domain and apply Rayleigh's theorem (see, for example, Bracewell loc. cit.) which states that the infinite integral of the squared modules of a function is the same as the corresponding quantity for its Fourier transform. Thus

$$Ns^2 = a^2B_{IF}{}^4\tau^2/N^2f_p{}^2 \ . \tag{10}$$

Application of the same theorem to the noise component gives an rms amplitude $B_{IF}\sqrt{2kT\tau/N}$ for each channel. The output signal-to-noise ratio is

$$S_2 = \frac{f_p\sqrt{\alpha\tau}}{\sqrt{2B_1}} \ , \tag{11}$$

where equation (4) has been used to substitute for a.

If the calibration pulses are inserted into the 50 MHz-wide IF band we have $B_1$=50 MHz, and, as before, we put $\alpha$=0·1, $\tau$=600 secs, and $S_2$=100. These values give $f_p$=129 kHz, a result which is an order of magnitude better than for the previous scheme, but still far from satisfactory. Note that $S_2$ is equal to the signal-to-noise voltage ratio in a bandwidth $2/\tau$ centered on one of the calibration lines. Thus the noise in the receiving channel beats with the line frequencies in the reference, and only those components with low enough frequency to pass through the integrators appear in the output.

There is no obvious way of further improving the signal-to-noise ratio with uniformly spaced pulses. The basic problem is, of course, that so little power is carried by the pulses. As $f_p$ is reduced to give more calibration lines one would like to increase the input power to maintain the measurement accuracy for each line, but instead the reduction of $f_p$ decreases the input power.

## 4.0 THE USE OF PSEUDO-RANDOM PULSE TRAINS

By.using pseudo-random pulse sequences instead of uniformly spaced pulses, it is possible to increase the mean input power without increasing the peak level, and also retain the required spacing for the calibration lines. The generation of pseudo-random pulse sequences using shift registers with feedback is a well-known technique (see, for example, Krishnaiyer and Donovan, 1973). A register with m stages can be made to generate sequences of length $(2^m-1)$ bits without repeating. These are known as maximally long sequences, and shorter ones can be generated by varying the feedback arrangement. The feedback involves linear combinations of the outputs of two or more stages, commonly using exclusive-or gates to perform modulo-two summation.

The autocorrelation function of an infinite train of pulses that repeats every M clock cycles is unity for time offsets of zero and multiples of M clock cycles. At other offsets that are integral numbers of clock cycles the autocorrelation function has an expectation of 1/2. This is because the pulses occur with approximately random probability and the number of logical ones in M clock cycles should be closely equal to M/2. If M is sufficiently large, the autocorrelation can be represented by

$$[(f_c/M)\,III(f_c t/M) + f_c III(f_c t)]/2 \,, \qquad (12)$$

where $f_c$ is the clock frequency. The power spectrum is the Fourier transform of (12) which is

$$[III(\frac{Mf}{f_c}) + III(\frac{f}{f_c})]/2 \;. \qquad (13)$$

The spectrum thus consists of a series of lines at harmonics of the sequence repetition frequency and a series at harmonics of the clock frequency, as shown in Figure 2, each series containing half the total power in the pulses. To put one calibration line in each spectral channel the sequence repetition frequency should be made equal to the channel spacing, so

$$f_c/M = B_{IF}/N \;. \qquad (14)$$

It seems logical to make the clock frequency equal to the sampling frequency for the final IF bandwidth, and this case will now be considered. Thus

$$f_c = 2B_{IF} \;. \qquad (15)$$

From (14) and (15)

$$M = 2N \,, \qquad (16)$$

that is, the number of clock pulses in each sequence should be equal to twice the number of spectral channels in the IF bandwidth. The lines at harmonics of the clock frequency are spaced at twice the IF bandwidth and have amplitudes $\sqrt{M}$ times those of the wanted calibration lines. One would try to avoid having one of the clock harmonics fall within the IF passband, but if it does occur it should be possible to correct for it if M is not too large.[3]

Now suppose that a pulse calibration system is implemented with the occurrence of the fast pulses controlled by a pseudo-random generator with clock and repetition frequencies as just described. An identical generator is used to produce a noise-free reference with which to correlate the signals from the antennas. To calculate the sensitivity with this scheme, first note that the peak power in the pulses must remain $\alpha kTB_1$ as in section 2.0. The mean power in bandwidth $B_1$ is the peak power multiplied by the pulse duration, $1/B_1$, and by the mean repetition frequency, $f_c/2=B_{IF}$, and is

$$\alpha kTB_{IF} \tag{17}$$

In bandwidth $B_{IF}$ the mean power is

$$\alpha kTB_{IF}^2/B_1 \ , \tag{18}$$

There are N calibration lines of amplitude a in bandwidth $B_{IF}$ and equating their mean power to half the pulse power one obtains

$$Na^2 = \alpha kTB_{IF}^2/2B_1$$

$$a = \frac{B_{IF}\sqrt{\alpha kT}}{\sqrt{2NB_1}} \tag{19}$$

The line amplitude is greater by a factor $\sqrt{N/2}$ than for the case of uniformly spaced pulses given in equation (4). This is easily explained since there are N=M/2 times as many pulses in the

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[3]The effect of the absence of clock harmonics can be seen as follows. The clock harmonics must combine in phase when the pulses occur, and thus they represent a series of pulses at frequency $f_c$. Since half the pulses are missing in the pseudo-random train the other lines in the spectrum must represent pulses of opposite signs that cancel or reinforce the clock-harmonic pulses.

pseudo-random case but half the power goes into the clock-harmonic lines. The sensitivity may therefore be expected to be $\sqrt{N/2}$ greater than for the uniform pulse case, but setting aside this anticipated result we follow the procedure of the preceding section.

To estimate the pulse amplitude after filtering through the final IF amplifier note that the pulse duration becomes $1/B_{IF}$ and the mean recurrence frequence is $f_c/2=B_{IF}$. Thus the peak pulse power is approximately equal to the mean power and the pulse amplitude is the square root of expression (18),

$$\frac{B_{IF}\sqrt{\alpha kT}}{\sqrt{B_1}} \;. \tag{20}$$

In time $\tau$ the zero-offset integrator receives $B_{IF}\tau$ pulses of amplitude given by (20) and hence accumulates a voltage

$$V_s = \frac{B_{IF}^{\,2}\tau\sqrt{\alpha kT}}{\sqrt{B_1}} \;. \tag{21}$$

In the same interval each of the other integrators receives, on average, half as many pulses and hence accumulates half the voltage given by (21).[4] The accumulated voltages are proportional to the autocorrelation values in Figure 2(a). In the Fourier transformation the component of amplitude $V_s/2$ in each of the 2N integrators gives rise to spectral lines at intervals of $2B_{IF}$ which correspond to the clock-frequency harmonics. The remaining component $V_s/2$ in the zero offset integrator gives rise to the wanted calibration lines. In addition, in time $\tau$ each of the integrators receives $B_{IF}\tau/2$ random noise samples of rms amplitude $\sqrt{kTB_{IF}}$. By application of Rayleigh's theorem, as in section 3.0, the final signal-to-noise ratio for the calibration lines at harmonics of the recurrence frequency is found

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[4]In the absence of a clock harmonic in the IF passband the pulses are of half amplitude and both positive and negative occur. In the integrators they tend to cancel, except for the zero offset case for which all negative pulses coincide with zeros of the reference. The result in equation (22) remains unchanged.

to be

$$S_3 = \frac{B_{IF}\sqrt{\alpha\tau}}{2\sqrt{NB_1}} \quad , \tag{22}$$

which is $\sqrt{N/2}$ greater than $S_2$. Then for $\alpha$=0·1, $\tau$=600 secs, $B_1$=50 MHz and $S_3$=100 one obtains $B_{IF}/\sqrt{N}$=183 kHz. For the narrower channels N=256, so that the above result becomes $B_{IF}/N$=11·4 kHz. This just nicely matches the minimum goal of calibration lines down to the 12 kHz channel spacing. A further improvement can be obtained if the narrower filters in the Antenna IF Filters Module (F7)[5] are used. Filters are provided to reduce the 50 MHz IF bandwidth at the antennas to 25 or 12·5 MHz and a value of 12·5 MHz for $B_1$ allows a factor of two decrease in the line spacing to 5·7 kHz. Table 2 shows practical parameters and performance for all ten IF bandwidths derived from equation (22). Note that for the 50 to 6·25 MHz bandwidths the signals can be injected into the full front-end bandwidth if desired. One can conclude that, with regard to sensitivity, this scheme meets the minimum specified goals.

## 5.0 A PSEUDO-RANDOM SCHEME WITH HIGH CLOCK FREQUENCY

It is useful to inquire whether better sensitivity could be obtained by the use of genuine random noise, if it were feasible to generate fully correlated noise at the antennas. Suppose that random noise of power level $\alpha$ relative to the system noise is introduced at an antenna, and that noise fully correlated with the introduced noise is used as a reference. The signal-to-noise level for a channel bandwidth $B_{IF}/N$, after integration for time $\tau$ is approximately

$$\sqrt{\alpha B_{IF}\tau/N} \ . \tag{23}$$

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[5]Not in the prototype electronics but will be retrofitted in 1979.

For comparison, equation (22) can be written as

$$S_3 = \tfrac{1}{2}\sqrt{\alpha B_{IF}\tau/N}\ \sqrt{B_{IF}/B_1}\ . \qquad (24)$$

For the narrow bandwidths the factor $\sqrt{B_{IF}/B_1}$ can be as small as 1/71·5, so the pseudo-random pulses, implemented as in Section 4.0, leave room for considerable improvement. As will be apparent from what follows, the factor $\sqrt{B_{IF}/B_1}$ comes from the choice of the clock frequency, and it can be eliminated.

Suppose that the clock frequency is made equal to the sampling frequency corresponding to the input bandwidth $B_1$. The pulse duration in that bandwidth is $1/B_1$ and the mean pulse frequency is $f_c/2=B_1$. Thus the mean power is closely equal to the peak power, and is about as high as one could hope for without overloading. As shown in the previous section, half of the mean power is in the lines at harmonics of the sequence repetition frequency, and within the IF bandwidth the power in these lines is

$$\tfrac{1}{2}\alpha kTB_1 \times B_{IF}/B_1 = \tfrac{1}{2}\alpha kTB_{IF}\ . \qquad (25)$$

The power in the lines thus remains a constant fraction of the noise power in any IF bandwidth, and comparison with expression (18) shows that the factor $B_{IF}/B_1$ has disappeared. A much better performance can be expected with the high clock frequency, but is there any danger of overloading as the bandwidth is decreased to $B_{IF}$?

In a pseudo-random pulse train, as in a uniformly spaced train, the pulse amplitude is proportional to the bandwidth, as explained in Section 2.0. When the pulses pass from bandwidth $B_1$ to the final IF stages their amplitudes drop by a factor $B_{IF}/B_1$ and their durations are increased by the inverse of this factor. The overall signal at any instant therefore contains contributions from something like $f_c/B_{IF}$ clock cycles.

The way in which these pulse contributions combine is given by expression (8), if $f_p$ is replaced by $f_c$ which is greater than $B_{IF}$ or $f_o$. The pulses do not simply combine additively in amplitude since the phase of their r.f. component varies. Indeed, the occurrence of any signal from the combination of pulses depends upon the randomness of the pulse occurrences, since a uniform train with recurrence $f_c$ would generally produce no lines within the final IF passband. Additive combination of the pulse powers perhaps more closely describes the situation, and is consistent with expression (25) for the mean power. One would expect the peak power to occur when, for an isolated interval of order $1/B_{IF}$, input pulses occur on all clock cycles, resulting in an input power of twice the mean level. The possibility of overloading therefore seems unlikely, but should be checked for in any practical situation. If necessary the pulse power can be reduced below the value for $\alpha=0{\cdot}1$ and still produce better performance than with the clock frequency limitation of Section 4.0.

In calculating the sensitivity with the high clock frequency it is useful to include the possibility of injecting the pulses into the 500 MHz bandwidth of the front-end, but with $f_c$ limited to 100 MHz to avoid special logic circuitry. If $f_c/2$ is used for the mean pulse frequency in expression (25), the mean power in the calibration lines within the IF bandwidth becomes

$$\alpha k T f_c B_{IF}/4B_1 \ . \tag{26}$$

Within any spectral channel of bandwidth $B_{IF}/N$ the ratio of the power in the calibration lines to that in the noise is

$$\alpha f_c/4B_1 \ . \tag{27}$$

If the IF signals from two antennas, both equipped with identical pulse generators, are correlated, the final signal-to-noise ratio for each channel is (27) multiplied by the usual bandwidth-time constant factor:

$$S_4 = \frac{\alpha f_c \sqrt{\tau B_{IF}}}{4B_1\sqrt{N}} . \tag{28}$$

If a noise-free reference waveform can be generated and correlated with the signal from each antenna, the sensitivity is the voltage signal-to-noise ratio, i.e., the square root of (27), multiplied by the bandwidth-time constant factor:

$$S_5 = \frac{\sqrt{\alpha f_c \tau B_{IF}}}{2\sqrt{B_1 N}} . \tag{29}$$

To compare the sensitivity with that for the scheme of Section 3.0 consider the case where the pulses are injected into the IF amplifiers at the antennas so $B_1$=50 MHz=$f_c/2$. Then $S_3$ is less than $S_5$ by the factor $\sqrt{B_{IF}/2B_1}$. In Tables 3 and 4 equations (28) and (29) are used to estimate practical performance parameters for all of the ten IF bandwidths.

How can a noise-free reference be generated for use with the present scheme? What is required is a one-bit or two-bit sequence at the sampling frequency, 2 $B_{IF}$, highly correlated with the input pulse train. The simplest possibility is a series of logic ones at the sequence repetition frequency since this has a spectrum of lines at the same frequencies as the calibration lines, and is correlated with the input for any single channel. The action of the correlators is identical to that described in Section 3.0, and each of the 2 N integrators accumulates the signal at a specific time interval in the sequence. The series of integrator voltages represents an averaged version of the pseudo-random sequence, and Fourier transformation of the integrator voltages gives the spectrum of the pseudo-random sequence as modified by transmission through the IF chain. Each integrator receives $\tau B_{IF}/N$ signal samples in an integration time $\tau$, so the signal-to-noise voltage ratio in each integrator is the square root of this factor multiplied by the square root of (27), a result which is identical to (29). After Fourier transformation the ratio of the rms over the spectral channels of the signal and noise levels is equal to the ratio of the same quantities over the 2 N integrators. Hence the final sensitivity is that given by (29), and the simple reference signal appears to be effective. Unfortunately it has one drawback, which is that the spectrum of the pseudo-random sequence must be known so that it can be separated from the wanted frequency response in the final result. The line amplitudes in the spectrum are almost equal, but the phases vary randomly over $2\pi$. The amplitudes and phases can be computed and stored, but the amount of data is not small. The spectrum repeats at intervals of 100 MHz (the proposed clock frequency) and for a channel spacing of 381 Hz there are $2{\cdot}6 \times 10^5$ lines within each interval. Further study may show that it is more attractive to generate a reference by suitably filtering the pseudo-random sequence so that the reference has a spectrum identical to that part of the input that falls within the IF bandwidth.

## 6.0 TIMING TOLERANCES

The remaining critical area with regard to feasibility concerns the synchronization of the pulse trains. The basic timing waveform that would be used at each antenna is a 100 MHz signal from the local oscillator. Phase noise in the local oscillator is sufficiently small that decorrelation is negligible, certainly for signals up to 10 GHz. Obtaining the required pulse frequencies requires division from 100 MHz, and hence some other timing standard must be used to avoid ambiguities. Timing signals at the 19·2 Hz rate used for the waveguide transmission are available at all antennas, and a pulse derived from them could be used to reset all of the dividers and pseudo-random generators as nearly simultaneously as possible. The accuracy of the 19·2 Hz timing signals is estimated to be about 50 ns or a few cycles at 100 MHz. There are also fixed timing differences for the local oscillator signals resulting from the different waveguide travel times for the reference signals. These are measurable using the round-trip phase monitoring facility for the local oscillator, and can thus be allowed for.

The timing in the signal paths from the antennas to the correlators must be considered, and when using the pulse calibration system the digital compensating delays would be set to equalize the signal travel times. The delay parameters are calibrated approximately to the smallest delay increment of 625 ps, so timing errors here should be insignificant. If the delays are correctly calibrated, observations of a calibration source should show no linear phase variation across the band. Such an observation can therefore be used to check the delay calibration, and for this purpose the use of the widest bandwidth and a small number of channels should suffice.

It appears that the only serious timing error is the estimated 50 ns in the 19·2 Hz waveforms, and the extent to which this would vary randomly for different resets of the timing logic at a given antenna is not known. A difference of 50 ns between an antenna and the reference would produce a phase shift of $18^{o}$ per MHz, linear with frequency, in the measured phases of the calibration lines. This would amount to 2·5 rotations across the 50 MHz bandwidth but would be only a few degrees across the narrowest ones. One could, of course, simply ignore any linear component of phase and use the calibration scheme to measure only bumps or ripples in the responses, which would still be quite a useful function. Suppose however, the pulse generators are sufficiently reliable that, when allowed to run after once being started, the timing errors remain constant for a period of, say, a day. The calibration signals will then provide measurement of any changes that occur during such a period. It is also very probable that more accurate timing than presently exists at the antennas can be developed from the existing local oscillator signals.

## 7.0 MISCELLANEOUS POINTS

Thus far, in discussing pseudo-random sequences, it has been assumed the sequence repetition frequency is equal to the spectral channel bandwidth, so that there is one line in each channel. Another possibility is to make the sequences very long, so as to put a large number of lines into any channel. As an example of the possible sequence length a 32-stage shift register clocked at 100 MHz repeats its output every 42·9 seconds. The signal power going into any channel bandwidth remains the same as discussed above so the sensitivity calculations still apply. A minor advantage is that it would not be necessary to adjust the final local oscillator to center a single line within each channel whenever the calibration system is used. The scheme described in Section 5.0, with the simple reference consisting of pulses at the sequence repetition frequency could not be used. This is because the responses to the numerous lines within any spectral channel would combine with random phases.

The non-linear response of three-level sampling must not be forgotten. J. Granlund has devised a correction algorithm that can be used for Gaussian signals. When the pseudo-random, high-clock-rate scheme is used with narrow IF bandwidths the effective averaging over many input clock cycles should result in sufficiently Gaussian characteristics for the signals at the samplers. For wide IF bandwidths it could be necessary to derive a different correction, but it may suffice to reduce the signal level as allowed by the higher sensitivity for wide bandwidths.

In designing the fast pulse generator the best approach seems to be to derive a 100 MHz squarewave from the local oscillator and use this to drive a tunnel diode with no intervening logic to introduce timing errors. The tunnel diode pulses would then be gated by an rf switch controlled by the pseudo-random generator.

The switch would be opened on high-level output pulses from the generator, and the gating signal would be timed to pass either only positive-going or only the negative-going pulses. This essentially follows the design by Rogers (loc.cit.).

## 8.0 CONCLUSIONS

1. A synchronous pulse calibration scheme would enhance the accuracy and efficiency of the VLA for spectral line observations and be a valuable diagnostic aid for the electronics. Calibration observations of standard sources would still be required to determine absolute phases, but it should suffice to make these in the continuum or wide bandwidth modes and thus avoid long integration times.

2. Construction of a pulse calibration system is feasible. The most serious limit on the accuracy results from the estimated 50 ns timing uncertainties at the antennas. The precise nature of these errors and the extent to which they can be reduced is likely to remain uncertain until some tests are made with a prototype calibration system.

3. Considerations of sensitivity clearly indicate the use of pseudo-random pulse trains with a clock frequency as high as twice the input bandwidth. A clock frequency of 100 MHz, for which the logic is highly reliable, is a good choice at the present time. The number of clock cycles within a sequence can be controlled by resetting the pseudo-random generator from a clock-pulse counter.

4. Three ways of implementing the system have been suggested:

(a) Correlate signals from each antenna with one another and then separate the responses for individual antennas by subsequent computation. (The VLA software is designed to handle calibrations on a per-antenna rather than a per-baseline basis.)

(b) Correlate each antenna signal with a noise-free reference which takes the form of logic ones at the sequence repetition frequency. Correct for the spectrum of the input signal.

(c) Correlate each antenna signal with a reference for which the spectrum is the same as that part of the input signal which falls within the receiving passband.

Schemes (b) and (c) have the following advantages over (a): higher sensitivity (compare Tables 3 and 4); results obtained directly on a per-antenna basis which avoids further computing and the possibility of bad data from one antenna contaminating all the rest; the possibility of making measurements on only one or two antennas for electronics testing. Scheme (b) is the simplest in terms of electronics. It requires computation and storage of about $5 \times 10^5$ complex numbers to cover input spectra for all ten bandwidths. The storage is comparable to that for data from the full array used in the continuum mode with four IF channels for one hour. Scheme (c) may require generation of the reference by analog means, using a simplified receiver. Applying a running mean to the output of a pseudo-random generator would not suffice to provide a reference since the result would contain no information on what part of the input spectrum is being used. For schemes (a) and (c) it may also be necessary to apply a small correction for the amplitudes in cases where the number of clock cycles in a sequence is not large. Scheme (b) seems good but some further thought should be given to scheme (c) before making a final choice.

5. The strong clock-harmonic lines in the pseudo-random trains present a problem, and may restrict the usable frequency range in their vicinity. Attention must be given to adequate suppression or correction in any practical design. The level of the clock harmonics can be as much as 54 dB above the wanted calibration lines, as can be seen from column 3 of Table 4.

6. The feasibility of radiating the calibration signal into the feed horn in some manner that allows calibration of standing waves on the antenna has not been considered.

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TABLE 1: PARAMETERS OF VLA SPECTRAL PROCESSOR

| IF Bandwidth $B_{IF}$ | Number of Channels[1] N | Channel Bandwidth | Final Intermediate Frequency[2] $f_o$ | Minimum Integrating Time |
|---|---|---|---|---|
| 50 MHz | 16 | 3·125 MHz | 25 MHz (Low-pass) | 312·5 ms |
| 25 " | 32 | 781 kHz | 12·5 " " | 625 ms |
| 12·5 " | 64 | 195 " | 6·25 " " | 1·25 s |
| 6·25 " | 128 | 48·8 " | 3·125 " " | 2·5 s |
| 3·125 " | 256 | 12·2 " | 1·56 " " | 5 s |
| 1·56 " | 256 | 6·10 " | 2·34 " (Band-pass) | 10 s |
| 781 kHz | 256 | 3·05 " | 1·17 " " | 10 s |
| 391 " | 256 | 1·53 " | 586 kHz " | 10 s |
| 195 " | 256 | 763 Hz | 293 " " | 10 s |
| 97·6 " | 256 | 381 " | 244 " " | 10 s |

[1]Single band mode which gives maxim number of channels.

[2]provisional data.

TABLE 2: PERFORMANCE PARAMETERS FOR PSEUDO-RANDOM PULSES WITH $f_c = 2\ B_{IF}$, CALCULATED FROM EQUATION (22) WITH $\alpha = 0{\cdot}1$

| IF Bandwidth $B_{IF}$ | Number of Channels N | Input Bandwidth $B_1$ | Output Signal-to-Noise Ratio $S_3$ | Integration Time $\tau$ |
|---|---|---|---|---|
| 50 MHz | 16 | 500 MHz | 100 | 1·3 s |
| | | 50 " | 100 | ·13 s |
| 25 " | 32 | 500 " | 100 | 10·2 s |
| | | 50 " | 100 | 1·02 s |
| 12·5 " | 64 | 500 " | 100 | 1·4 m |
| | | 50 " | 100 | 8·2 s |
| 6·25 " | 128 | 500 " | 100 | 11 m |
| | | 50 " | 100 | 1·1 m |
| 3·125 " [1] | 256 | 50 " | 100 | 8·7 m |
| 1·56 " | 256 | 50 " | 100 | 35 m |
| 781 kHz | 256 | 25 " | 100 | 1·16 h |
| 391 " | 256 | 12·5 " | 100 | 2·3 h |
| 195 " | 256 | 12·5 " | 50 | 2·3 h |
| 97·6 " | 256 | 12·5 " | 50 | 9·3 h |

[1] 12 kHz channel bandwidth; performance within minimum specified goals down to and including this bandwidth.

TABLE 3: PERFORMANCE PARAMETERS FOR PSEUDO-RANDOM PULSES WITH $f_c$ = 100 MHz, CORRELATION OF SIGNALS FROM TWO ANTENNAS, CALCULATED FROM EQUATION (28) WITH $\alpha = 0{\cdot}1$

| IF Bandwidth $B_{IF}$ | Number of Channels N | Input Bandwidth $B_1$ | Output Signal-to-Noise Ratio $S_4$ | Integration Time $\tau$ |
|---|---|---|---|---|
| 50 MHz | 16 | 500 MHz | 100 | 2·1 m |
| | | 50 " | 100 | 1·3 s |
| 25 " | 32 | 500 " | 100 | 8·5 m |
| | | 50 " | 100 | 5·1 s |
| 12·5 " | 64 | 500 " | 100 | 34 m |
| | | 50 " | 100 | 20·5 s |
| 6·25 " | 128 | 50 " | 100 | 1·4 m |
| 3·125 " | 256 | 50 " | 100 | 5·5 m |
| 1·56 " | 256 | 50 " | 100 | 10·9 m |
| 781 kHz | 256 | 25 " | 100 | 5·5 m |
| 391 " | 256 | 25 " | 100 | 10·9 m |
| 195 " | 256 | 12·5 " | 100 | 5·5 m |
| 97·6 " | 256 | 12·5 " | 100 | 10·9 m |

TABLE 4: PERFORMANCE PARAMETERS FOR PSEUDO-RANDOM PULSES WITH $f_c$ = 100 MHz, CORRELATION WITH NOISE-FREE REFERENCE, CALCULATED FROM EQUATION (29) WITH $\alpha = 0{\cdot}1$

| IF Bandwidth $B_{IF}$ | Number of Channels $N$ | Number of Clock Cycles per Sequence[1] $M = \frac{Nf_c}{B_{IF}}$ | Input Bandwidth $B_1$ | Output Signal-to-Noise Ratio $S_5$ | Integration Time $\tau$ |
|---|---|---|---|---|---|
| 50 MHz | 16 | 32 | 500 MHz | 100 | 0·64 s |
| | | | 50 " | 100 | 0·064 s |
| 25 " | 32 | 128 | 500 " | 100 | 2·6 s |
| | | | 50 " | 100 | 0·26 s |
| 12·5 " | 64 | 512 | 500 " | 100 | 10·2 s |
| | | | 50 " | 100 | 1·0 s |
| 6·25 " | 128 | 2048 | 500 " | 100 | 41 s |
| | | | 50 " | 100 | 4·1 s |
| 3·125 " | 256 | 8192 | 500 " | 100 | 2·7 m |
| | | | 50 " | 100 | 16·4 s |
| 1·56 " | 256 | 16384 | 500 " | 100 | 5·5 m |
| | | | 50 " | 100 | 33 s |
| 781 kHz | 256 | 32768 | 500 " | 100 | 11 m |
| | | | 50 " | 100 | 1·1 m |
| 391 " | 256 | 65536 | 50 " | 100 | 2·2 m |
| 195 " | 256 | 131072 | 50 " | 100 | 4·4 m |
| 97·6 " | 256 | 262144 | 50 " | 100 | 8·7 m |
| | | | 12·5 " | 100 | 2·2 m |

[1]Minimum number, i.e., one calibration line per channel. M is also equal to the ratio of the power level in a clock-harmonic line to that in a sequence-harmonic line.

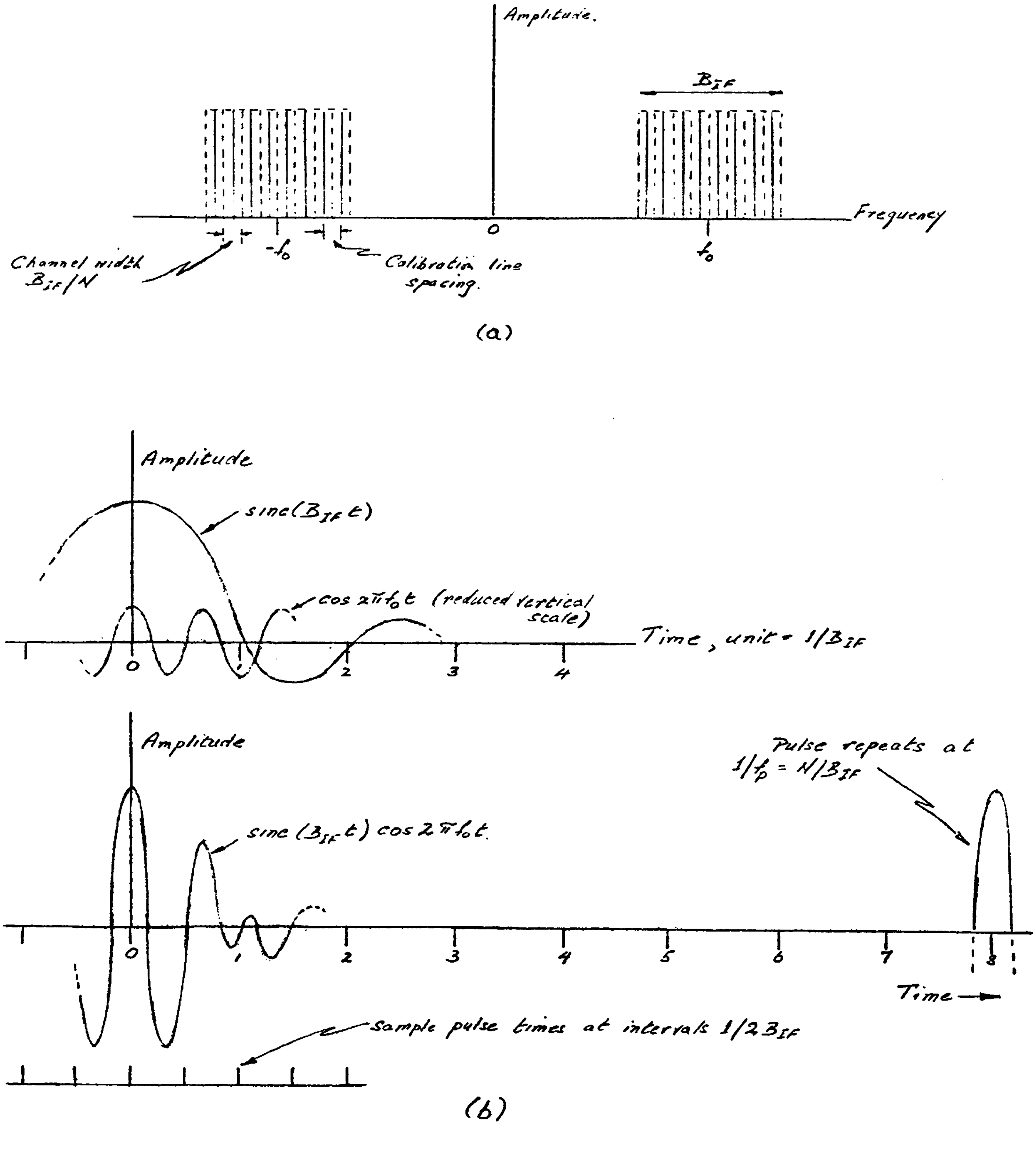


Figure 1: (a) The spectrum of band-pass filtered calibration lines represented by expression (7).
(b) Filtered pulse waveforms represented by expression (8).
In these examples N = 8 and $f_o = 3\ B_{IF}/2$.

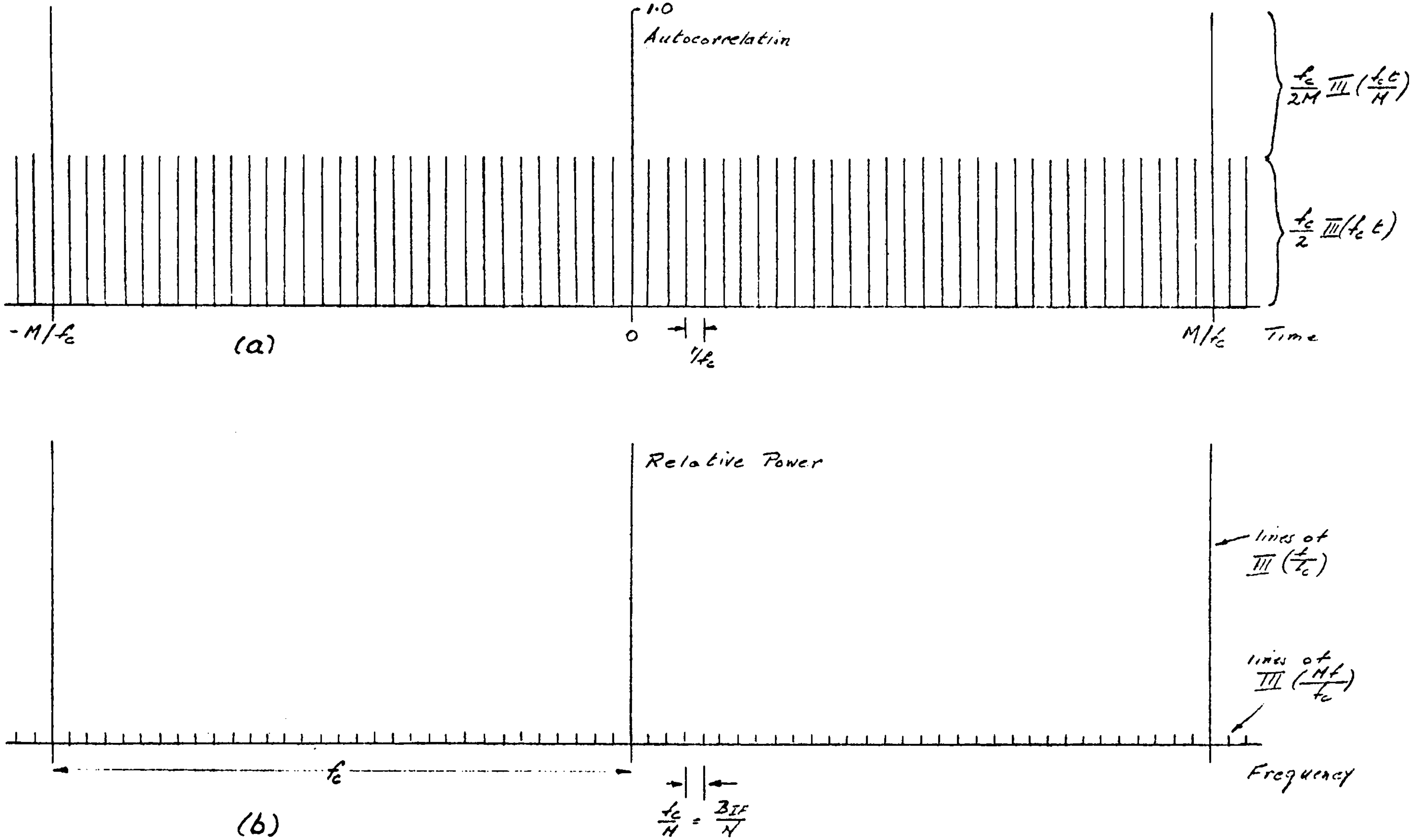


Figure 2: (a) The autocorrelation function of a pseudo-random pulse train represented by expression (12).
(b) The corresponding power spectrum represented by expression (13).
In these examples M = 32.