NATIONAL RADIO ASTRONOMY OBSERVATORY
SOCORRO, NEW MEXICO
VERY LARGE ARRAY PROGRAM

VLA ELECTRONICS MEMORANDUM NO. 190

MECHANICAL MEASUREMENT OF WAVEGUIDE ALIGNMENT USING "MOUSE"

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March 1976

(Revised by J. W. Archer December 1979)

1.0 PRINCIPLE OF STRAIGHTNESS MEASUREMENT

In the curved tube, shown in Figure 1, the bending radius

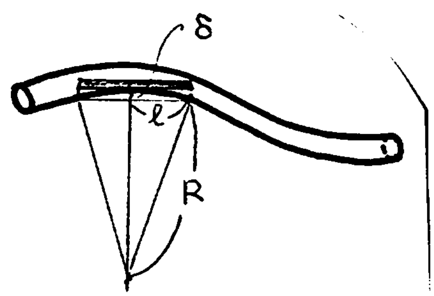


Figure 1

along the longitudinal axis can be related to the value δ as follows.

$$R^2 = \ell^2 + (R-\delta)^2 \qquad (1\text{-}1)$$

or when $\delta^2 \ll \ell^2$

$$R \sim \frac{\ell^2}{2\delta}$$

To obtain the value δ along the waveguide axis in the horizontal and vertical planes, a special instrument called a "mouse" has been used. The mouse has six dimensional-electrical sensors, as shown in Figure 2. "δ", for a fixed value of ℓ, can be obtained as follows.

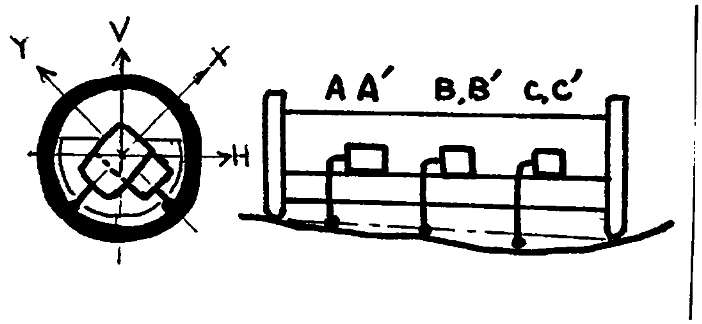


Figure 2

$$\delta_x = \delta_B - \tfrac{1}{2}(\delta_A + \delta_C) \tag{1-2}$$

$$\delta_y = \delta_B' - \tfrac{1}{2}(\delta_A' + \delta_C') \tag{1-3}$$

From δ_x, δ_y, deviations δ_V, δ_H in the vertical and horizontal planes can be obtained as follows.

$$\delta_v = \frac{\sqrt{2}}{2}(\delta_x + \delta_y) \tag{1-4}$$

$$\delta_H = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(\delta_x - \delta_y) \tag{1-5}$$

2.0 DEFINITION OF STRAIGHTNESS (rms curvature)

Loss increase caused by random axis curvature is almost in inverse proportion to the square of the radius as follows.

$$\Delta\alpha \propto \iint \frac{1}{R(X)} \frac{1}{R(X+\ell)} e^{-j\Delta\beta\cdot\ell}\, dx\, d\ell \qquad (2\text{-}1)$$

So as a parameter that has "close relationship to loss increase", curvature $\frac{1}{R}$ should be defined as follows.

$$\frac{1}{R^2} = \frac{1}{L} \int_0^L \frac{1}{R(X)^2}\, dx \qquad (2\text{-}2)$$

and it follows that

$$\frac{1}{R^2} = \left(\frac{2}{\ell^2}\right)^2 \frac{1}{L} \cdot \int_0^L \delta_v(x)^2\, dx \qquad (2\text{-}3)$$

R may be calculated using the following expressions, applied to discrete data points sampled at different positions along the waveguide:

$$\frac{1}{R_v^2} = \left(\frac{2}{\ell^2}\right)^2 \sum_i^N \delta_{vi}^2/N \qquad \frac{1}{R_H^2} = \left(\frac{2}{\ell^2}\right)^2 \sum_i^N \delta_{Hi}^2\ N \qquad (2\text{-}4)$$

$$R_v = \frac{\ell^2}{2} \Big/ \sqrt{\sum_2^N \delta_{vi}^2/N} \qquad R_H = \frac{\ell^2}{2} \Big/ \sqrt{\sum_2^N \delta_{vi}^2\ N}. \qquad (2\text{-}5)$$

$$R = 1/\sqrt{(1/R_v)^2 + (1/R_H)^2}$$

The value R is usually called "straightness of waveguide" or "rms curvature of waveguide". R_V and R_H are straightness in the vertical plane and horizontal plane respectively.

3.0 CONNECTION IMPERFECTION

A mouse can also obtain information about the imperfections of waveguide section joints. When the mouse passes an offset, as in Figure 3-A, δ_V changes as in Figure 3-B.

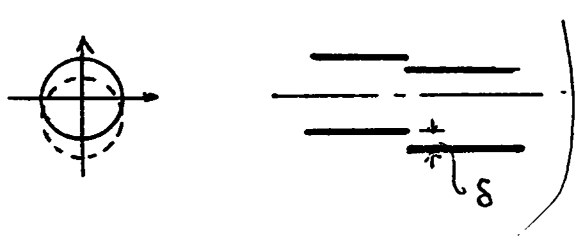


Figure 3-A

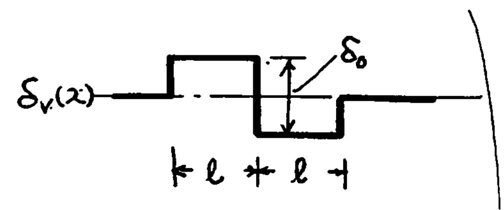


Figure 3-B

and when the mouse passes a tilted joint, shown in Figure 4-A, the signal δ_H changes as shown in Figure 4-B.

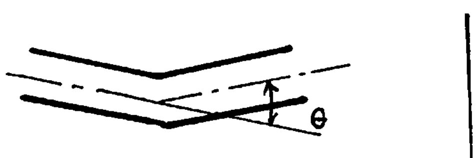


Figure 4-A

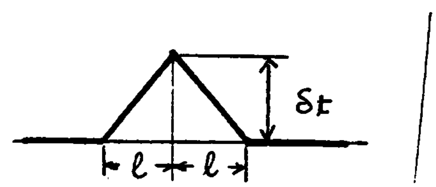


Figure 4-B

From this data, δ_o, δ_t, we know the magnitude of the offset and tilt.

When a joint has both an offset and a tilt, as shown in Figure 5-A, $\delta(x)$ becomes as shown in Figure 5-B. In this case, δ_o, δ_t can be obtained from $\delta(x)$ as in Figure 5-B.

Figure 5-A

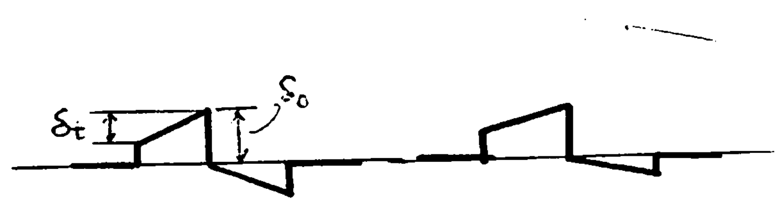


Figure 5-B

4.0 ANALYSIS OF MOUSE DATA

4.1 Calibration

The calibration of the mouse is usually done by having ±10 mil deviation on a center sensor (sensor B_x or B_y) after setting x direction (or y) sensors on a perfectly straight line or flat platform. In such case we get deviations on the output recorder as shown in Figure 6.

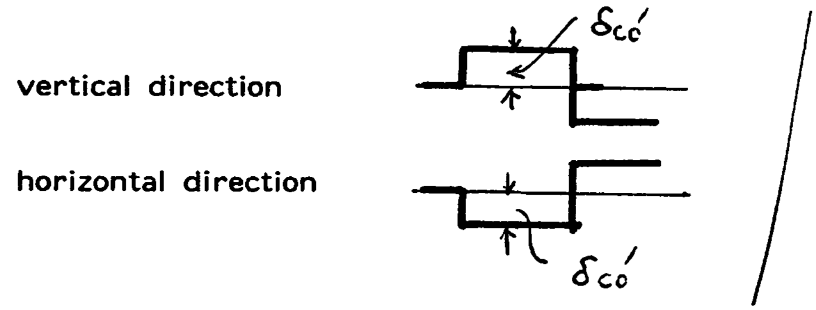


Figure 6

In this case the actual deviations in horizontal and vertical direction are

$$\delta_{vc} = \frac{\sqrt{2}}{2} (\delta_x + \delta_y)$$

$$= \pm \frac{\sqrt{2}}{2} \cdot 10^{-2''} = \pm .18 \text{ mm} \qquad (4\text{-}1)$$

$$\delta_{HC} = \frac{\sqrt{2}}{2} (\delta_x - \delta_y)$$

$$= \mp \frac{\sqrt{2}}{2} \cdot 10^{-2''} = \mp .18 \text{ mm} \qquad (4\text{-}2)$$

So deviation δ'_{co} on the recorder corresponds to δ_{vc}, δ_{hc} (= 0.18 mm). Usually the mouse sensors have good linearity in mechanical deviation and output electrical signal response in the usual range. So we can relate the deviation on the recorder chart to the actual deviation on the mouse.

4.2 How To Get Offset Volume

From the recorder output of the mouse, actual offset magnitude δ_{HO}, δ_{VO} is as follows.

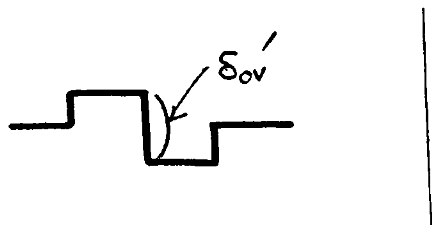


Figure 7: Offset pattern on data.

$$\delta_{OV} = \frac{0.18}{\delta_{CO}} \cdot \delta'_{OV} \text{ (mm)} \tag{4-3}$$

$$\delta_{OH} = \frac{0.18}{\delta_{CO}} \cdot \delta'_{OH} \text{ (mm)} \tag{4-4}$$

In recent cases:

$$\delta_{CO} = 12 \text{ mm} \tag{4-5}$$

and the correspondence shown in Table I holds true.

4.3 How To Get Tilt Angle

Tilt angle θ can be defined in terms of the maximum deviation δ'_{vt} as follows.

TABLE I

DATA CORRECTION OF MOUSE TEST

(10^{-2}" deviation on sensor B or B' corresponds to 12 mm deviation output recorder chart.)

< OFFSET >

δ(mm) on chart	actual
1	0.015 mm
2	0.030 mm
3	0.045 mm
4	0.060 mm
5	0.075 mm

< TILTS > $\theta_{hc} = \frac{2\delta_{hc}}{\ell}$

δ(mm) on chart	actual tilt
1 (mm)	0.12 mrad
2	0.24 mrad
3	0.36 mrad
4	0.48 mrad
5	0.60 mrad
6	0.72 mrad
7	0.84 mrad
8	0.96 mrad
9	1.08 mrad
10	1.2 mrad

< RADIUS > $R_c = \frac{\ell^2}{2\delta_{hc}}$

δ(mm) on chart	
12	173.6 m
10	208.3 m
5	416.6 m
4	521. m
3	605. m
2	1.041 m

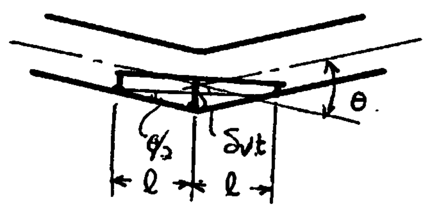


Figure 8

$$\theta_{vt} = 2 \cdot \frac{\theta_{vt}}{\ell}, \qquad \theta_{Ht} = 2 \cdot \frac{\delta_{Ht}}{\ell} \tag{4-6}$$

This volume can be shown by deviations on the chart.

$$\theta_{vt} = \frac{2}{\ell} \cdot \frac{0.18}{\delta_{CO}} \delta'_{vt} \text{ (mm)} \tag{4-7}$$

$$\theta_{Ht} = \frac{2}{\ell} \cdot \frac{0.18}{\delta_{CO}} \delta'_{Ht} \text{ (mm)} \tag{4-8}$$

In recent cases the correspondence shown in Table I holds true.

4.4 <u>How To Get Curvature</u>

From the chart recorder output of the mouse, we can obtain the actual deviation from straightness as follows.

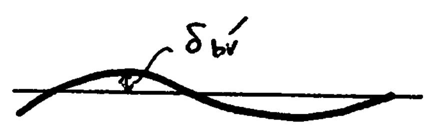


Figure 9

$$\delta_b = \frac{0.18}{\delta_{CO}} \cdot \delta_{bv}' \tag{4-9}$$

The bending radius is given by

$$R = \frac{\ell^2}{2\delta_p} \tag{4-10}$$

In recent case correspondence shown in Table I holds true.

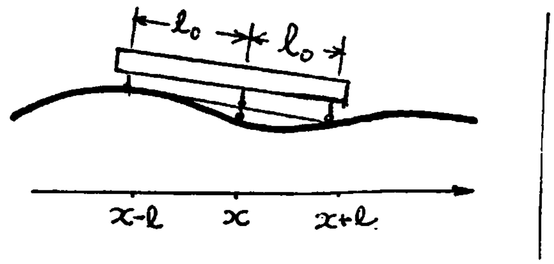
5.0 METHOD TO OBTAIN PROFILE CURVE OF WAVEGUIDE AXIS FROM MOUSE DATA

5.1 Principle

[definition]

profile along waveguide axis: $f(x)$

distance between two adjacent sensors: ℓ



usually $\ell \doteqdot \ell_o$

Figure 10

[information picked up by mouse]
deviation from straightness $\delta(x)$:

$$\delta(x) \doteqdot \frac{f(x+\ell) + f(x-\ell)}{2} - f(x) \tag{5-1}$$

When the change of $f(x)$ is gentle enough, $f(x+\ell)$ is given by

$$f(x+\ell) = f(x) + \ell f'(x) + \frac{\ell^2}{2} f''(x) + \frac{\ell^3}{6} f'''(x) +$$

$$f(x-\ell) = f(x) - \ell f'(x) + \frac{\ell^2}{2} f''(x) - \frac{\ell^3}{6} f'''(x) \tag{5-2}$$

and $\delta(x)$ can be written simply as follows

$$\delta(x) \doteqdot \frac{\ell^2}{2} f''(x). \tag{5-3}$$

[profile]

To obtain profile curve of waveguide axis f(x), integration must be done twice.

$$f(x) = \frac{2}{\ell^2} \iint \delta(x)\ dx \qquad (5\text{-}4)$$

5.2 Simulation of Mouse Data to Obtain Profile Curve

A. CASE I - SINUSOIDAL CHANGE

[simulation formula]

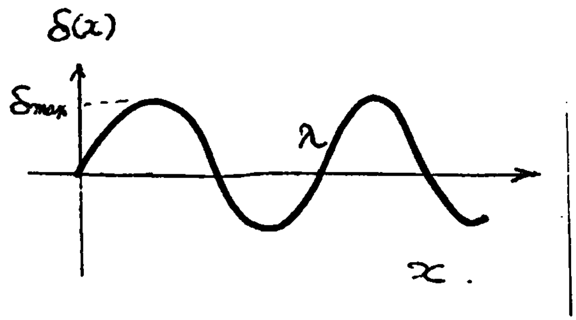


Figure 11

$$\underline{\delta(x) = \delta_{max} \sin\left(2\pi . \frac{x}{\lambda_o}\right)} \qquad (5\text{-}5)$$

$$f(x) = \iint f''(x)\ dx.$$

$$= \frac{2}{\ell^2} \iint \delta(x)\ dx$$

$$= -\frac{2}{\ell^2} \left(\frac{\lambda_o}{2\pi}\right)^2 \lambda_{max} \sin\left(2\pi \frac{x}{\lambda_o}\right) + DX + E \qquad (5\text{-}6)$$

when we have the following boundary conditions

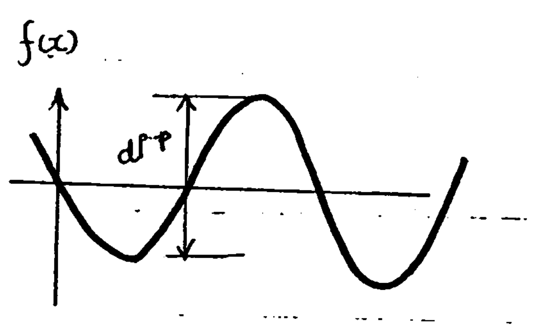


Figure 12

$$f(0) = 0$$
$$f'(\lambda/4) = 0 \qquad (5\text{-}7)$$

we get the following

$$D = E = 0. \qquad (5\text{-}8)$$

and f(x) becomes

$$f(x) = -\frac{2}{\ell^2}\left(\frac{\lambda_o}{2\pi}\right)^2 \lambda_{max} \sin\left(2\pi \frac{x}{\lambda_o}\right) \qquad (5\text{-}9)$$

From this we obtain peak-to-peak deviation from straightness

$$d^{P-P} = f_{max} - f_{min}$$

$$= - \frac{4}{\ell^2} \left(\frac{\lambda_o}{2\pi}\right)^2 \cdot \delta_{max}$$

$$= \frac{1}{\pi^2} \left(\frac{\lambda_o}{\ell}\right)^2 \cdot \delta_{max}$$

$$\div \frac{1}{10} \left(\frac{\lambda_o}{\ell}\right)^2 \cdot \delta_{max} \qquad (5\text{-}10)$$

[examples]

1.

ℓ = 25 cm (in our case)

ℓ_o = 2.5 m

δ_{max} = 0.15 mm

(10 mm deviation from center line on charts corresponding minimum bending curvature of 210 m)

$$d^{P-P} \div \frac{1}{10} \cdot 4 \times 10^2 \times 0.15 = 6 \text{ mm}$$

2.

ℓ = 25 cm

λ_o = 2.5 m

δ_{max} = 0.15 mm

$$d^{P-P} \div 1.5 \text{ mm}$$

B. CASE II - SAWTOOTH CHANGE
[simulation formula]

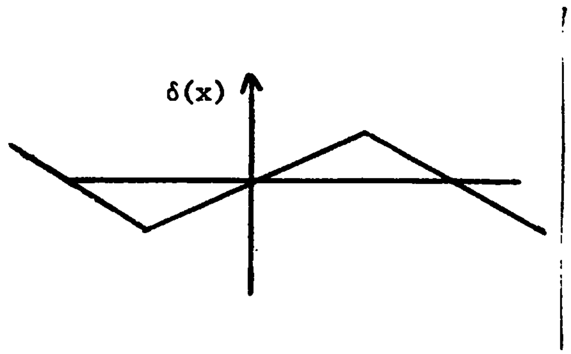


Figure 13

$$\underline{\delta(x) = CX \qquad (-\lambda \le x \le \lambda)}$$

$$= - CX + 2C\lambda \quad (\lambda \le x \le 3\lambda) \tag{5-11}$$

$$f_1(x) = \iint f''(x)\, dx$$

$$= \frac{2}{\ell^2} \iint \delta(x)\, dx$$

$$= \frac{2}{\ell^2} \cdot [\frac{C}{6} x^3 + Ex + f] \quad (-\lambda \le x \le \lambda) \tag{5-12}$$

$$f_2(x) = \frac{2}{\ell^2} \cdot [\frac{C}{6} (x - 2\lambda)^3 - E(x - 2\lambda) + F] \quad (\lambda \le x \le 3\lambda) \tag{5-13}$$

when we place the following boundary conditions

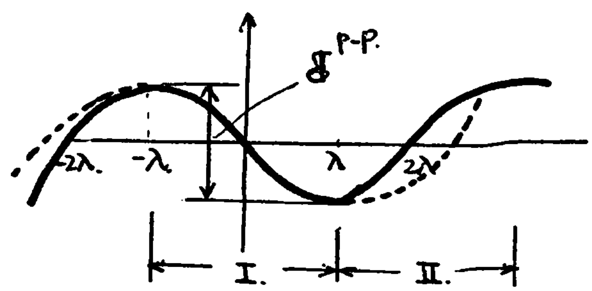


Figure 14

$$f_1(0) = 0$$

$$f_1(X) = f_2(\lambda) \tag{5-14}$$

$$f_1'(\lambda) = f_2'(\lambda)$$

we obtain the following

$$F = F' = 0$$

$$E = -\frac{C}{2}\lambda^2 \tag{5-15}$$

f(x) becomes

$$f_1(x) = \frac{2}{\ell^2}\left(\frac{C}{6}x^3 - \frac{C}{2}\lambda^2 x\right) = \frac{C}{3\ell^2} \times (x^2 - 3\lambda^2) \tag{5-16}$$

$$f_2(x) = \frac{2}{\ell^2} \cdot \frac{C}{6}(x - 2\lambda)\,[(x - 2\lambda)^2 - 3\lambda^2) \tag{5-17}$$

From these formulas we can obtain peak-to-peak deviation from straight line as follows

$$d^{P-P} = f_{max} - f_{min}$$

$$= 2 \mid f(\lambda) \mid$$

$$= 2 \frac{C}{3\ell^2} \lambda \cdot 2\lambda^2$$

$$= \frac{4}{3} \cdot \frac{C}{\ell^2} \lambda^3 = \frac{4}{3} (\frac{\lambda}{\ell})^2 \cdot \delta_{max} = \frac{1}{12} (\frac{\lambda_o}{\ell})^2 \delta_{max} \qquad (5\text{-}18)$$

$$(\delta_{max} = C\lambda) \qquad (\lambda_o = 4\lambda;\ \text{period})$$

[examples]

1.

ℓ = 25 cm (in our case)

λ_o = 5.0 m

δ_{max} = 0.15 mm

$$d^{P-P} = \frac{1}{12} \times 4 \times 10^2 \times 0.15 = 5 \text{ mm}$$

2.

ℓ = 25 cm

λ_o = 25 m

δ_{max} = 0.15 m

$$d^{P-P} = 1.25 \text{ mm}$$

The relationship between minimum deviation of mouse data and maximum deviation on profile curve is shown in Figure 15 in

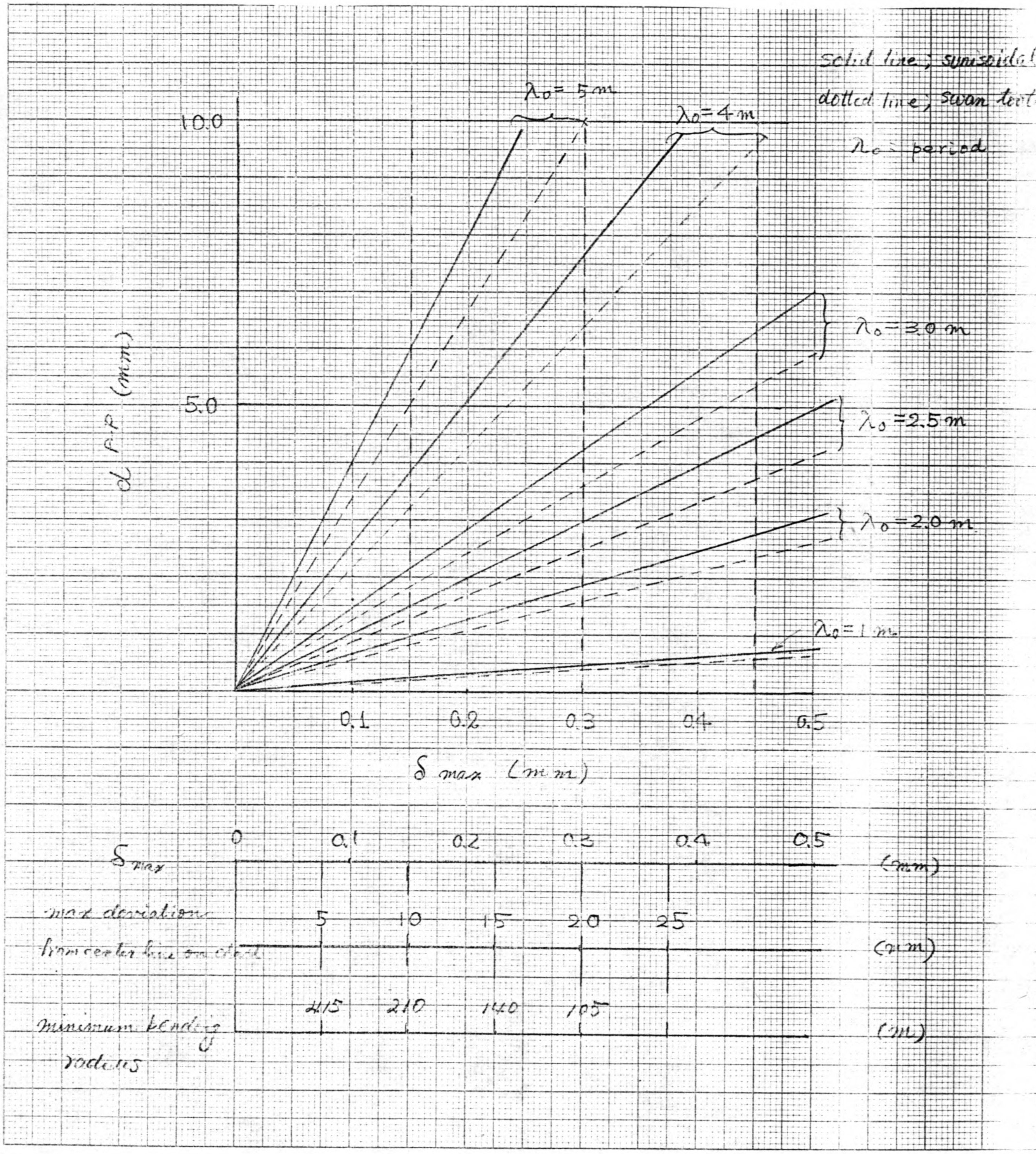


Figure 15: Relationship between maximum deviation of mouse data and maximum deviation on profile curve.

the case that mouse data can be simulated by sinusoidal curves and sawtooth curve.

5.3 Profile Curve By Numerical Integration

From the definition of integration, the following formula can be derived

$$\int_{x_o}^{x_1} \delta(x)\ dt = \lim_{N\to\infty} \frac{(x_1\ x_o)}{N} \sum_{n=1}^{N} \delta\ (t_o) + \frac{n(x_1 - n_o)}{N} + C \qquad (5\text{-}19)$$

C (constant)

When $\delta(x)$ changes gradually enough and can be sampled at appropriate intervals, integration can be simulated as follows

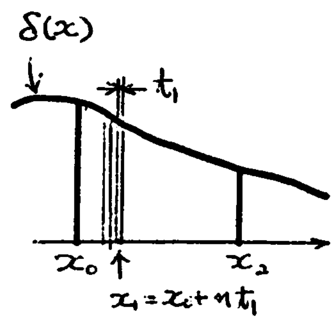


Figure 16

$$\int_{x_o}^{x_1} \delta(x)dt \doteqdot t_1 \sum_{n=1}^{N} \delta(n_o + nt_1) + \delta(x_o) \qquad (5\text{-}20)$$

$$t_1 = \frac{x_2 - x_o}{N_o}\ , \qquad N = \frac{x_1 - x_o}{x_2 - x_o} \cdot N_o$$

From this formula, second order integration of δ(x) can be derived

$$\int_{x_o}^{x_2} \int_{x_o}^{x_1} \delta(x)\, dx\, dx, \div \int_{x_o}^{x_1} t_1 \sum_{n=1}^{N} \delta(x_o + nt_1)\, dx + C(x_2 - x_o) + D$$

$$= \sum_{N=1}^{M} t_1^2 \sum^{N} \delta(x_2 - x_o) + D. \qquad (5\text{-}21)$$

Profile curve of waveguide axis f(x) can be derived from mouse data δ(x) as follows

$$f(x_2) = \frac{2}{\ell^2} \int_{x_o}^{x_2} \int_{x_o}^{x_1} \delta(x)\, dx,\, dx,$$

$$= \frac{2}{\ell^2} [\sum_{N=1}^{N_o} t_1^2 \sum_{n=1}^{N} \delta(x_1 + nt_1) + C(x - x_o) + D]$$

$$= \frac{2}{\ell^2} \cdot \kappa_1^2 [\sum_{N=1}^{N_o} \sum_{n=1}^{N} \delta(x_1 + mt_1) + C'(x - x_o) + D'] \qquad (5\text{-}22)$$

where

$$t_1 = \frac{x_2 - x_o}{N}$$

Profile Curve of 60 mm dia. Waveguide Axis


SANBORN Recording Permapaper
+20
+10
0
-20
-30
7
6
5