ERIM

FORMERLY WILLOW RUN LABORATORIES, THE UNIVERSITY OF MICHIGAN

7 June 1976

MEMORANDUM TO: NRAO File

FROM: J. S. Zelenka JSZ

SUBJECT: Circular Recording Formats

A radio telescope can obtain a sampled version of the mutual coherence function associated with radio sources in the sky. If a sufficient number of mutual coherence samples are obtained, a sky map can be generated by Fourier transforming the resulting sampled data. A straightforward way to obtain these radio sky maps is to store the individual interferometric signals on an optical recording media. A coherent optical processor can then be used to perform the required two-dimensional transform.

For an interferometer spacing of B_N in the northerly direction and of B_W in the westerly direction, the ideal elliptical trajectory in the u-v plane are

$$u = -B_W \cos h - B_N' \sin h$$

and

$$v = \sin\delta\,(B_W \sin h + B_N' \cos h) + v_o$$

where

$B'_N = B_N \sin \ell,$

$v_o = V_N \cos \ell \cos \ell,$

δ is the declination angle associated with the center of the object field, h is the hour angle of the object, and ℓ is the latitude of the radio telescope.

The above equation can be rewritten as

$$u = -B_o \sin (h + \alpha)$$

and

$$v = -B_o \sin \delta \cos (h + \alpha) + v_o$$

where

$$B_o = \sqrt{B_W^2 + (B_N \sin \ell)^2}$$

and

$$\alpha = \tan^{-1} \frac{B_W}{B_N \sin \ell}$$

Hence the required u - v trajectory is an ellipse with major axis

$$a = B_o$$

and minor axis

$$b = B_o \sin \delta .$$

The center of the ellipse is displaced a distance v_o along the positive v-axis.

The eccentricity for the elliptical trajector is

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \cos \delta$$

Note that the eccentricity is only dependent upon the declination angle of the source. Similarly the minor-to-major axis ratio is

$$\frac{b}{a} = \sin \delta$$

which also just depends upon the declination angle of the source.

Foreshortening the u-axis by a factor of sin δ will cause the elliptical trajectories in the u-v plane to become circular. To remove the resulting distortion in the object field estimate (image) obtained by Fourier transforming the data, one must reduce the scale of the image in the u-direction of a factor of sin δ. To change the aspect ratio of an image optically would require the use of some cylindrical optics. However, these cylindrical optics probably would not be required since the optically processed image will be scanned with an array of photodetectors which could be programmed to remove the image distortion resulting from using circular trajectories in the u-v plane.

If the u-axis is foreshortened so that circular trajectories are produced, the circular scan rate would be a constant if interferometer samples are collected at uniform time intervals. This fact also simplifies the requirements imposed on the optical recorder.

In conclusion the optical recorder has to only record along circular paths at a constant angular scan rate. The radius of these scans is a function of the array latitude and element spacings associated with the interferometer. The recording radius could be varied electronically or optically, depending on which approach is easier to implement. The required displacement of each circular path along the v-axis could be done with a precision mechanical drive (e.g., a Moore table). <u>If we can write a perfect circle in the u-v plane with either a laser scanner or an electron gun, we got it made</u>.

JZ/pw
cc: C. Aleksoff
    I. Cindrich
    J. Fienup
    E. Leith
    P. McInnes