NATIONAL RADIO ASTRONOMY OBSERVATORY

Charlottesville, Virginia

June 10, 1976

MEMORANDUM

TO: Addressee

FROM: R. Burns

SUBJECT: Optical Processor Design Information

The enclosed information will be given to ERIM to form the basic specifications around which ERIM will design the optical processor. It is very important that these specifications reflect the correct astronomical requirements. The gedanken acceptance tests on pages 4 and 5 are especially important.

Please read this and give me your suggested changes, if any, in writing. The document must be finalized by the end of this month.

INFORMATION FOR ERIM

INPUT DATA

Description of Input (u,v) Plane

The input data lies on a plane, referred to as the (uv) plane. The (uv) plane is real; it serves to define the positions of the measured data. The data values themselves are complex numbers. The input data lies on ellipses, these ellipses are sometimes referred to as baselines or (uv) tracks. Normally measurements will be made on not more than 351 such tracks. The function sampled (complex visibility function) is known to be Hermitian. By invoking this property, the 351 tracks may, if desired, be conjugated to form up to 702 tracks. Or, the 351 tracks themselves may be processes by taking into consideration the conjugate nature of the data, (i.e., the Fourier Transform is real).

On occasion, twice this number of tracks will be used. This case will occur, when it is desired to take the difference of two maps. Since the optical processor will presumably have limited accuracy in the output plane, it is sometimes desirable to do this differencing in the input plane. When such subtraction occurs, the uv tracks corresponding to the two maps will either lie on top of one another (in which case the differencing may be done digitally) or will form pairs of (uv) tracks, the tracks forming each pair being very close to one another.

For purposes of this design, we shall restrict consideration to those portions of the ellipses within a circle of radius R; the portions of the largest ellipses which extended beyond this circle will be suppressed. For every track b_k, there are sample points in time along the track designated t_1, t_2, t_3, etc. The data (each point is complex) may be written on the (uv) plane in two orders (bt: b_1t_1, b_2t_1, -- b_nt_1, b_1t_2 -- b_2t_2, b_nt_2, b_1t_3 -- $b_nt_3$or (t,b): b_1t_1,b_1t_2 -- b_1t_n, b_2t_1, b_2t_2 -- b_2t_n, b_3t_1 -- etc. Of these two orderings, (bt) is preferred. Both are preferred to a raster scan of (u,v).

Coordinate Input - (bt)

For (bt) order the optical processor will be given 27 complex position vectors for each value to t_k. The 351 coordinates forming b_1t_1, b_2t_1, -- $t_{351}t_1$ will then be obtained in the processor by differencing these 27 positions in groups of two at a time. For time t_2 another set of 27 complex positions are given the processor. The number of samples is the same for each track and is approximately 4000. At the conclusion of drawing the approximately (351 x 4000) points, if a map subtraction were desired the same process would be done again on the same piece of film.

Coordinate Input (tb)

For (tb) order, the processor will be given the parameters for each ellipse, one ellipse at a time. The exact coordinate equation of the ellipse will then be internally generated based on these parameters. The nth ellipse is described by the relation $u = a_n \cos (t-h_n)$, $v = b_n \sin (t-h_n) + V_n$ where a_n, b_n, h_n and V_n are constants. The input parameters a_n, b_n, h_n and V_n will be given as a 16 bit word each. Data will be time synchronized with the ellipse generation. The arc length between successive data points along the ellipse varies. In general the points lie close together in the center of the plane and are widely separated at the edge of the plane.

Test Data to be Supplied

NRAO will supply ERIM test data giving the (uv) coordinates for what may be assumed to be typical (u,v) plane coverage.* The information will be supplied on 9 track magnetic tape (1600 bpi) either bcd or binary, as desired. Only data giving the (uv) coordinates of the approximately 351 x 4000 sample points (16 bit numbers) shall be given, not the values of the sampled data at each point. NRAO refers to the data itself as the Visibility Funciton, V(u,v), and the Fourier transform as the Brightness Distribution, B(x,y). All given (uv) coordinates will lie within a circle of radius

$$R = \sqrt{u_{max}^2 + v_{max}^2} = 10^4$$

OUTPUT DATA

The data output shall be digital data, 16 bit words. Because of the computer industry byte standardization an 8 or 16 bit word is desired. Since the optical processor has limited accuracy, an 8 bit word might indeed be sufficient. However the processor might have better precision for certain specific examples. Also, with an 8 bit word it might be necessary to have a separate block scale factor containing a sign and multiplier or offset. To avoid these problems at this time, we shall simply require a 16 bit output word. The brightness map shall be scanned sampled on a rectangular grid of dimension 4096 x 4096.

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* Corresponding to a source at 40° declination

Dimensions of 2048 x 2048, 1024 x 1024, 512 x 512, 256 x 256 and 128 x 128 shall also be possible by scanning every nth point, n = 2, 4, 8, 16 and 32. A full resolution scan of a subsection of the output plane should also be possible on a similar scale. The output rate shall be approximately 500,000-8bit words/sec. This is selected as a convenient disc I/O rate and may be adjusted if necessary.

Definition of 1%

The optical processor is loosely referred to as a 1% machine. What we mean by 1% can be seen by the following gedanken acceptance test. Generate visibility data corresponding to a point source* in the brightness plane at position $x_o y_o$. This data can best be obtained by simply evaluating the analytical transform at the required (u,v) points. Coordinate information on the data should match the test coordinate positions given in the supplied test data. The generated data is complex (16 bit real and 16 bit imaginary) as is the coordinate information. $(x_o y_o)$ should be within a circle of radius Δ, i.e., $\sqrt{x_o^2 + y_o^2} \leq \Delta$ where $\Delta = 500/\sqrt{u_{max}^2 + v_{max}^2}$ ** or $\Delta \leq 0.05$ in the reciprocal units of the (u,v) data we shall provide.

Then take a two dimensional transform of the sampled visibility function to obtain what we shall consider to be the output of a perfect processor; denote this output B(x,y). B(x,y) should be numerically generated with care so as not to introduce errors since it will form the basis to test the precision of the optical processor output. This

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* Of magnitude unity at all (u,v) points

** This corresponds to what we call a numerical field of view of 2000.

implies that B(x,y) should be generated either using a direct two dimensional classical transform or a very finely gridded two dimensional FFT. In the latter case the grid size should be at least 8,000 on a side.

Define B* (x,y) as the calibrated output of the optical processor. This calibration may be internal to the processor and not visible to the user or it may require some user interaction. If such interaction is required for each output, the interaction must be very simple. If the interaction is required only every few hours, it may be more complex, perhaps requiring 10 or 15 minutes.

Look at $|B^*(x,y) - B(x,y)|$ for $\sqrt{x^2 + y^2} < \Delta$ and define (x_w,y_w) as the point where this is a maximum but such that $\sqrt{(x_w-x_o)^2+(y_w-y_o)^2} \geq P$ where P is the distance from the position (x_o,y_o) to the peak of the first ring sidelobe, i.e., the 1% criteria does not apply to the center of the point spread function. Then the 1% specification is expressed as $|B^*(x_w,y_w - B(x_w,y_w)| < 1\% \; B(x_o,y_o)$ for all allowable (x_o,y_o). Also $|B^*(x_oy_o) - B\ (x_oy_o)| < 1\% \; B(x_o,y_o)$ for all (x_oy_o).

Dynamic Range Test

The required input dynamic range will also be defined by a specification in the output brightness plane, also by means of a gedanken acceptance test.

As in the previous case, generate a visibility function on the supplied (u,v) coordinate points. This time, however, include two sources of unit amplitude in the (x,y) plane, a point source at location (x_o,y_o) and an extended source at (x_1,y_1). That is

$$V(u,v) = \exp - 2\pi i(ux_o + vy_o) \exp - (u^2 + v^2)/R^2$$
$$+(\frac{R^2}{a^2}) \exp - 2\pi i(ux_1 + vy_1) \exp - \frac{1}{2}(u^2+v^2)/a^2$$

where a = 0.1 R.

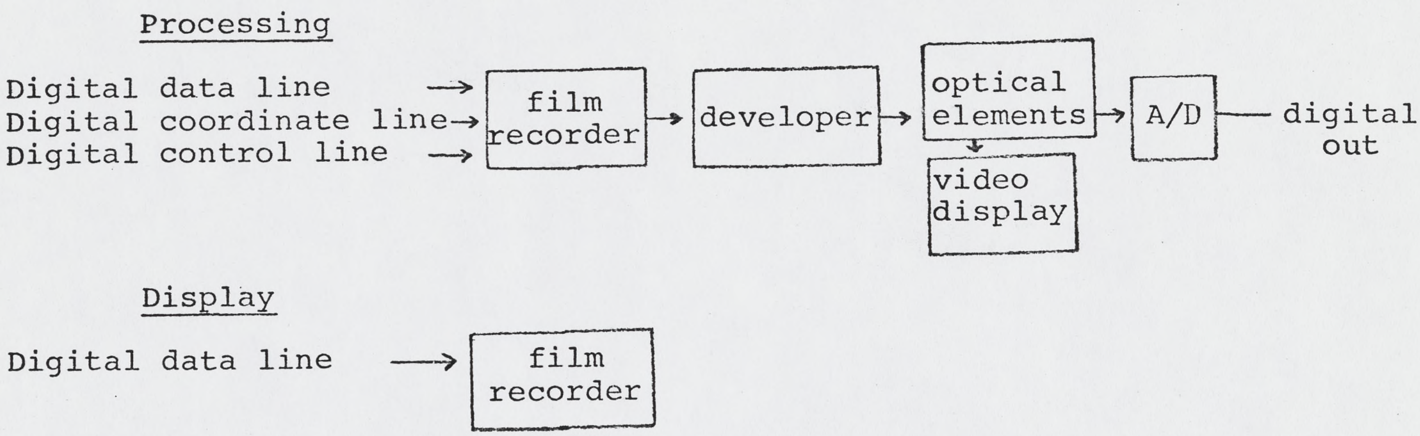
The optical and "calculated" brightnesses B* and B are then calculated as before. We then require

$$\left| \frac{B(x_o,y_o)}{B(x_1,y_1)} - \frac{B^*(x_o,y_o)}{B^*(x_1,y_1)} \right| < 1\%\ B(x_o y_o)$$

This could be done for a reasonable set of (x_o,y_o), (x_1,y_1) such as that tabulated below

(x_o,y_o)	(x_1,y_1)
	$(.15\Delta,0)$
(o,o)	$(0,.15\Delta)$
	$(.1\Delta,.1\Delta)$
	$(.9\Delta,0)$
	$(.6\Delta,.6\Delta)$
	$(0,.9\Delta)$
$(.15\Delta,0)$	$(0,0)$
$(0,.15\Delta)$	$(0,0)$
$(0,.9\Delta)$	$(0,.75\Delta)$
	$(0,0)$
	$(0,-.9\Delta)$
$(.9\Delta,0)$	$(.75\Delta,0)$
	$(0,0)$
	$(.9\Delta,0)$

System Block Diagram



Weighting

Each (u,v) track has approximately 4000 samples. Each track forms an ellipse centered at a point on the v axis. See Attachment A for some examples. Although the ellipses vary in size, the number of sample points on each is the same. The density of points per arc length is high near the origin and in general decreases with increasing radius. If all the sample points are drawn unaltered, we refer to the amplitude weighting (or lack thereof) as Natural Weighting. In certain cases it, however, may be attractive to scale the amplitudes so as to produce a uniform amplitude density. We call this Uniform Weighting. In fact we would like to have either weighting as an option. Natural Weighting, however, probably causes saturation problems and if these prove too difficult, the natural weighting requirement will be dropped. Uniform Weighting can be achieved several different ways. It can be approximated by multiplying the amplitudes by the arc length between sample points in the (u,v) plane. This information could easily be supplied to the optical processor; it could also be internally generated. Uniform weighting can also be approximated by simply multiplying all amplitudes by an envelope function (of radius) which is a minimum at the origin and a maximum at the edge of the (u,v) plane. Whether, in fact, either of these approximations is adequate is not known at this time. The major reason the approximations get into difficulty is that there are areas of the (u,v) plane where several tracks run parallel and very close. This effect produces regions of high sample density which do not vary over the (uv) plane in an easily described way. It is not known if this effect is important. It is felt the effect might cause special problems for the optical processor. Numerical studies are underway and an answer should be in hand by 9/1/76.

If all else fails the desired uniform weighting can be achieved by numerically weighting the amplitudes prior to the optical processing by the use of a stored mask function. This function could be determined exactly for a single frequency and scaled and applied to the 255 other frequencies. (The (uv) plane scales linearly with frequency.)

Controlled Amplitude Taper

A taper function which can be selected by the observer so as to eliminate Gibbs behaviour in the Fourier Transform process is desired. The function may be a simple function of radius - a Gaussian would be suitable. The maximum required attenuation at the edge of the plane results in an amplitude ratio of 10%.

Some Useful Definitions

Brightness Plane - Output plane (real) containing resulting map - positions referred to in (x,y) coordinates.

Visibility Plane - Input plane containing data to be Fourier Transformed, positions referred to in (u,v) coordinates.

Brightness Function - Output map function (real).

Visibility Function - Input function to be Fourier Transformed (complex).

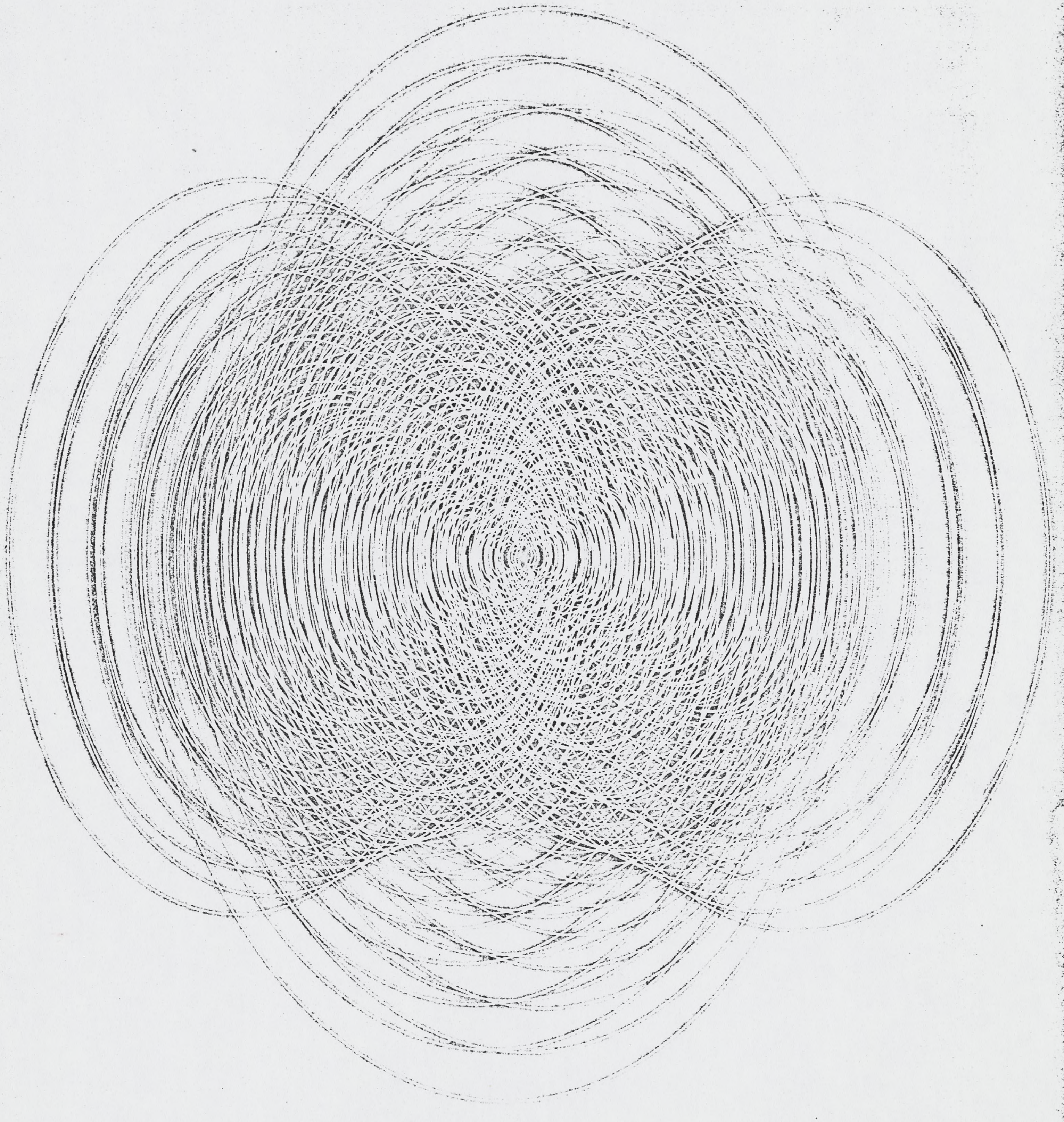
(uv) Track - Single ellipse on (uv) plane resulting from output of single interferometer formed by pair of antennas.

Baseline - Single interferometer output - sometimes synonomous with ellipse or (uv) track.

Uniform Weighting - Multiplication of each visibility sample by the inverse of the local average density of samples; the averaging being done on some convenient scale.

Natural Weighting - No artificial weighting of visibility function. Use of actual value of each visibility sample, regardless of sampling density. In the VLA the resulting density is much larger near the center of the (uv) plane.

ATTACHMENT A



(uv) plane for source at declination = 40°
Resolution = 1 part in 4096
Courtesy of N. Vandenberg