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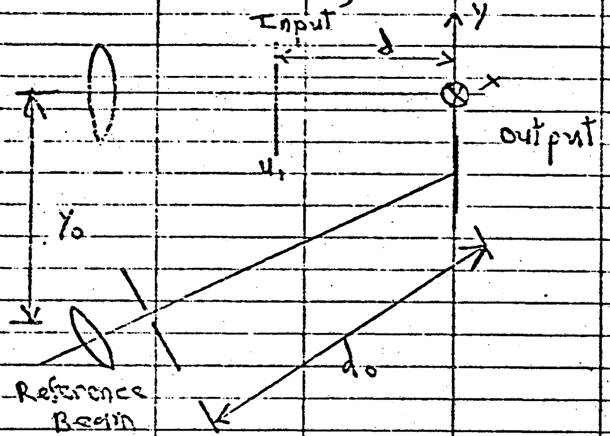
Optical Processor Detection ~~Step~~ Characteristics, R. Harrison

7 June; 13 Jul, 1976

Optical Processor: Geometry and definition of terms used in following discussions.



Monochromatic illumination of wavelength λ.

Processor is assumed to be aberration free.

Ideally:

(1) Input Data (Film): $\tau(u,v) = a_0 + Kb(u,v)$ note $\tau(u,v)$ is real valued

(2) Input Field: $U_1(u,v) = \tau(u,v) A_1 e^{-i\frac{2\pi}{\lambda d}(u^2+v^2)}$

(3) Output Field: $U(x,y) = A e^{i\frac{2\pi}{\lambda d}(x^2+y^2)} \iint_{-\infty}^{\infty} \tau(u,v) e^{-i\frac{2\pi}{\lambda d}(ux+vy)}\,du\,dv$

(4) Reference Beam: $U_0(x,y) = A_0 e^{i\frac{\pi}{\lambda d_0}[x^2+(y+y_0)^2]}$

(5) Output intensity: $I(x,y) = |U(x,y) + U_0(x,y)|^2$

Now, for our purposes it will be sufficient to write the waves in the more general form

(3') $U(x,y) = A\mathcal{F}\{\tau(u,v)\} e^{i\theta(x,y)}$

(4') $U_0(x,y) = A_0 e^{i\theta_0(x,y)}$

A and A_0 are assumed real and > 0

where $\mathcal{F}\{.\}$ denotes Fourier Transform

Since $\tau(u,v) = a_0 + Kb(u,v) = a_0 + Ka(u,v)\cos[\omega_1 v - \phi(u,v)]$

$= a_0 + \frac{1}{2}Ka(u,v)[e^{i(\omega_1 v - \phi(u,v))} + e^{-i(\omega_1 v - \phi(u,v))}$

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$$\therefore \quad \mathcal{F}\{I(u,v)\} = a_0\,\delta(x,y) + B(x,y) + \tilde{B}(x,y)$$

where

$$B(x,y) = \frac{K}{2}\,\mathcal{F}\{a(u,v)\,e^{-i(\omega_1 v - \phi(u,v))}\}$$

$$B(x,y) = \frac{K}{2}\,F(x, y+y_1)$$

where $y_1 = \frac{\omega_1}{2\pi}$ and $\boxed{F(x,y) = \mathcal{F}\{a(u,v)\,e^{i\phi(u,v)}\}}$ ultimate desired result

and

$$\tilde{B}(x,y) = \frac{K}{2}\,\mathcal{F}\{a(u,v)\,e^{i(\omega_1 v - \phi(u,v))}\}$$

$$= \frac{K}{2}\,\tilde{F}(x, y-y_1)$$

with

$$\tilde{F}(x,y) = \mathcal{F}\{a(u,v)\,e^{-i\phi(u,v)}\}$$

$$= \left[\mathcal{F}^{-1}\{a(u,v)\,e^{i\phi(u,v)}\}\right]^{*}$$

* denoting complex conjugate.

The only term of interest is $B(x,y)$, and we assume by appropriate choice of ω_1, the three terms of $\mathcal{F}\{I(u,v)\}$ are separated sufficiently in the output plane, to allow use to detect $B(x,y)$ alone (with reference beam).

Now we calc. intensity in the output plane due to $B(x,y)$ term and the reference beam.

> In the aberration free case we are considering, $B(x,y)$ is real valued.

So, we have over the detector output plane

$$u(x,y) = A\,B(x,y)\,e^{i\theta(x,y)} \longrightarrow A\,\mathcal{F}[\;]\,e^{j\theta}$$

if spatially [illegible] terms are included

$$u_0(x,y) = A_0\,e^{i\theta_0(x,y)}$$

So, $I(x,y) = |u(x,y) + u_0(x,y)|^2$

Dropping the explicit representation of the functional dependence on (x,y) (as we will often do in our discussions)

$$I(x,y) = [AB e^{i\theta} + A_0 e^{i\theta_0}][AB e^{-i\theta} + A_0 e^{-i\theta_0}]$$

$$I(x,y) = A^2B^2 + A_0^2 + 2ABA_0 \cos(\theta - \theta_0)$$

Then shifting the phase of the reference beam by π to produce the second intensity I_π:

i.e. $\theta_0 \to \theta_0 \pm \pi$

and $I \to I_\pi$

where $I_\pi(x,y) = A^2B^2 + A_0^2 - 2ABA_0 \cos(\theta - \theta_0)$

$$\boxed{\text{Thus} \quad I(x,y) - I_\pi(x,y) = 4AA_0B(x,y)\cos[\theta(x,y) - \theta_0(x,y)]}$$

We will find the following additional notation useful:

$$W(x,y) \equiv I(x,y) - I_\pi(x,y)$$

$$S(x,y) \equiv AB(x,y)$$

$$K(x,y) \equiv \frac{S(x,y)}{A_0}$$

$$R(x,y) \equiv |K(x,y)|$$

$$R_m \equiv |K(x,y)|_{max}$$

Thus, we see

$$I = A_0^2[1 + K^2 + 2K\cos(\theta - \theta_0)]$$

$$I_\pi = A_0^2[1 + K^2 - 2K\cos(\theta - \theta_0)]$$

Optical Processor: Complete standard error propagation and discussion of the ratio $K \equiv \frac{S}{A_0}$ for the case of a detecting diode array with no dark current, no nonlinearities, where any Fabry-Pérot effects constitute a fixed pattern in the spatial sensitivity variations, and assuming fixed pattern noise due to switching transients has been compensated for as best as possible using filters or a current integrating amplifier.

Note:

① We ignore dark current because it appears possible to cool array sufficiently ∴ dark current becomes negligible

② Nonlinearities are ignored in this discussion because, if present, they are small (~1% per decade intensity) and will have little bearing on present discussion, assuming they can adaquately be corrected for (adaquate meaning corrected so the error resulting is small compared to allowable uncertainties, which we shall discuss here).

③ Any Fabry-Pérot effects are assumed to be effectively part of the spatial sensitivity variations for a given processor geometrical set-up because if they are not, and if they are too large, the detector is useless for our purposes.

④ We are considering only normally distributed errors, assuming there are no systematic errors in any of the measured quantities due to laser output drifts, etc. Data is assumed to be free of aberrations. Errors in various quantities are assumed to be uncorrelated.

Summary of detailed analysis to follow.

The main ideas and procedures involved are:

I. Clarification of the relationship assumed between actual output of the detector on measuring an intensity (I or I_π) and the actual value of the intensity.

Let $q(x,y)$ be the actual output of the detector / amp. configuration on measuring $I(x,y)$ and likewise let $q_\pi(x,y)$ be th actual output on measuring $I_\pi(x,y)$. [q may be a voltage lev

If t denotes the integration time (the same for both measurements)

Then
$$I(x,y) = \frac{1}{H(x,y)} \frac{q(x,y)}{t}$$
$$I_\pi(x,y) = \frac{1}{H(x,y)} \frac{q_\pi(x,y)}{t}$$

where $H(x,y)$ is the calibration "constant" (constant for each pixel, but varies from pixel to pixel)

For simplicity define $I'(x,y) = \frac{q(x,y)}{t}$ and $I'_\pi(x,y) = \frac{q_\pi(}{t}$

II. Define, for $\cos(\theta-\theta_0) \neq 0$,
$$B(x,y) \equiv \frac{I(x,y) - I_\pi(x,y)}{4AA_0 \cos(\theta-\theta_0)} \equiv \frac{I'(x,y) - I'_\pi(x,y)}{Q(x,y)}$$
i.e., $Q(x,y) \equiv 4AA_0 H(x,y) \cos(\theta-\theta_0)$

State a constraint on how precise $B(x,y)$ must be determin from the detector output.

N.B. For simplification, we consider here only the case $|\cos(\theta-\theta_0)| = 1$. With a little work, we can generalize to the case $\cos(\theta-\theta_0) \neq 0$. It is clear however, the smalle

$|\cos(\theta-\theta_0)|$, the smaller $I-I_\pi$, and the less precise the measurement of $B(x,y)$. Thus, initially, we wish to establish the constraints for the easiest case, namely $|\cos(\theta-\theta_0)| = 1$.

The form taken is as follows, for a given output map:

For all (x,y) such that $10^{-\alpha}|B(x,y)|_{max} \le |B(x,y)| \le |B(x,y)|_{m}$

we require $\sigma_B \le 10^{-\alpha}|B(x,y)|_{max}$

where

α is a small positive number (like 2 or 3)

σ_B is the uncertainty in $B(x,y)$ at (x,y) [i.e., std. deviation in $B(x,y)$]

III. Take the above constraint, and transform it into secondary constraints on the determination of

$$I',\ I'_\pi \text{ and } Q(x,y) = 4AA_0H(x,y)$$

where Q is the result of the actual calibration.

This, then, gives us constraints on $\sigma_{I'}$, $\sigma_{I'_\pi}$ and σ_Q

IV. Model the output of the detector/amp. as follows

For the l^{th} photoelement, which is taken to be measuring at the position (x,y)

$$q_l = F_l N_l + \eta$$

where F_l is a constant (that can vary from photoelement to photoelement)

N_l is the total number of photons recorded by the l^{th} photoelement during the integration time t [Recorded meaning produced a pair(s) which were collected on the storage capacitance.

And η is an irreducible background noise, whose average value is zero.

With the above model, find expressions for σ_I and $\sigma_{I'_\pi}$ in terms of I and I'_π respectively.

Rewrite the 1st two constraints of III using these expressions.

V. Since we have

$$\left.\begin{aligned} I &= A_0^2[1+K^2+2K] \\ I_\pi &= A_0^2[1+K^2-2K] \end{aligned}\right] \text{ for } \cos(\theta-\theta_0)=1$$

and

$$\left.\begin{aligned} I &= A_0^2[1+K^2-2K] \\ I_\pi &= A_0^2[1+K^2+2K] \end{aligned}\right] \text{ for } \cos(\theta-\theta_0)=-1$$

Rewrite the constraints from IV in terms of the parameters

$$Z \equiv tA_0^2 \quad \text{and} \quad R_m \equiv |K(x,y)|_{max}$$

Recall $K(x,y) = \dfrac{AB(x,y)}{A_0}$

The two constraints are seen to reduce into one, of the following form

For all photoelements l, we require

$$PZ[1+R_m]^2 + \Phi_l Z^2[1+R_m]^4 + \Gamma_l \leq \beta Z^2 R_m^2$$

where

P, Φ_l, Γ_l and β are constants depending on detector/amp. properties and α.

VI Write another constraint deriving from the fact the detector can record only a finite number of photons (ie, it saturates).

This constraint takes the form: For all photosites ℓ

$$z(1+R_m)^2 + 3\left[Pz(1+R_m)^2 + \Phi_\ell z^2(1+R_m)^4 + \Gamma_\ell\right]^{1/2} \leq P N_{\ell max}$$

where $N_{\ell max}$ is a constant depending on the device (it is the max. number of photons the device can record in the ℓ^{th} photoelement)

VII Pause, and realize what we have:

By picking a specific device/amp. configuration, and an α,

P, Φ_ℓ, Γ_ℓ, β and $N_{\ell max}$ all become determined, and

we are left with two inequalities in z and R_m one from V and one from VI.

We now wish to find what values of z and R_m satisfy the two inequalities simultaneously.

VIII Note the physical significance of z and R_m:

① $z \equiv t A_0^2$ is the total energy/unit area incident on the output plane, during the integration time t, due to the reference wave alone. By picking a t (consistent with device parameters) we get the required value for the amplitude of the reference wave.

② $R_m \equiv |K(x,y)|_{max} \equiv \frac{A}{A_0}|B(x,y)|_{max}$ is the ratio of the maximum amplitude of the signal wave, to the amplitude of the reference wave.

IX. To accomplish the determination of values for z an R_m, rewrite the inequality of V in the form

$$z \geq g(R_m) \quad \text{and}$$

rewrite the inequality of VI in the form

$$z \leq f(R_m)$$

X. For several specific systems involving Fairchild and Reticon devices, calculate typical values for

$$P, \Phi_l, \Gamma_l, \beta \text{ and } N_{l\,max}$$

Calculate $g(R_m)$ and $f(R_m)$ for various values of R_m and plot them.
Any area on the plot bounded by $g(R_m)$ below and $f(R_m)$ above represent allowable values for z and R_m.

Allowable Values of R_m cluster around 1, with z depending on the device, but generally in the range 1 to 10 $\frac{erg}{cm^2}$

Note that I have done (so far) no more than check to see if typical parameters for several detectors give allowable values of R_m and z. Much more could be done in terms of evaluating the effects of nonuniformities in sensitivity across the array [recall, both inequalities must be satisfied at every photoelement]

If one wanted to include a nonzero dark current it would effectively lower the value for $N_{\ell max}$. Any noise due to the dark current should be negligible compared with other noise sources.

(A) The ultimate numbers we want from the processor detector are

$$B(x,y) = \frac{I(x,y) - I_\pi(x,y)}{4AA_0\cos(\theta-\theta_0)} = \frac{I'(x,y) - I'_\pi(x,y)}{4AA_0\cos(\theta-\theta_0)H(x,y)}$$

for points with $\cos(\theta-\theta_0) \neq 0$

where $I'(x,y) = I(x,y)\,H(x,y)$

$I'_\pi(x,y) = I_\pi(x,y)\,H(x,y)$,

$H(x,y) > 0$,

and I' and I'_π represent the actual quantities derived from the detector output. [More detailed discussion later]

In principle what we do is calculate I' and I'_π from the detector output, measure A and A_0, calculate $\cos(\theta-\theta_0)$, and know $H(x,y)$ from the detector characteristics, enabling us to solve for $B(x,y)$.

In practice, there is a much easier and more efficient way: we calc I' and I'_π from the detector output and by a calibration procedure we have determined the function $Q(x,y) \equiv 4AA_0\cos(\theta-\theta_0)H(x,y)$.

Then for all points where $Q(x,y) \neq 0$,

$$B(x,y) = \frac{I'(x,y) - I'_\pi(x,y)}{Q(x,y)}$$

Note we are only interested in points where $Q(x,y) \neq 0$

Defining $W'(x,y) \equiv I'(x,y) - I'_\pi(x,y)$, we have

$$\boxed{B(x,y) = \frac{W'(x,y)}{Q(x,y)}}$$

Where $W'(x,y)$ is derived from measured quantities and $Q(x,y)$ is a measured quantity.

Thus, W' and Q each have their own uncertainties, so

$$\sigma_B^2 = \frac{\sigma_{W'}^2}{Q^2} + \frac{(W')^2}{Q^4}\sigma_Q^2 = \frac{\sigma_{W'}^2}{Q^2} + B^2\left(\frac{\sigma_Q}{Q}\right)^2$$

Assuming $\left|\frac{\sigma_Q}{Q}\right| << 1$ We will later check to see if it seems reasonable that this expression will be satisfied

The origin of this expression comes from the fact that the equation for σ_B^2 is only valid if B can be accurately approximated by it's Taylor expansion terminated at the linear terms in $\Delta W'$ and ΔQ for a small neighborhood

$$W' - \Delta W' \leftrightarrow W' + \Delta W'$$

$$Q - \Delta Q \leftrightarrow Q + \Delta Q$$

where $\Delta W' \approx \sigma_{W'}$ and $\Delta Q \approx \sigma_Q$

So, if our eq. for σ_B^2 is to be accurate, it must be true that

$$\forall\ n \geq 2 \quad \left|\frac{1}{n!}\frac{\partial^n B}{\partial (W')^n}(\Delta W')^n\right| << \left|\frac{\partial B}{\partial W'}\Delta W'\right|$$

and

$$\forall\ m \geq 2 \quad \left|\frac{1}{m!}\frac{\partial^m B}{\partial Q^m}(\Delta Q)^m\right| << \left|\frac{\partial B}{\partial Q}\Delta Q\right|$$

The first inequality is satisfied trivially because

$$\frac{\partial^n B}{\partial (W')^n} = 0 \quad \forall\ n \geq 2$$

since $B = \frac{W'}{Q}$,

$$\forall\ m \geq 1, \quad \left|\frac{\partial^m B}{\partial Q^m}\right| = \left|\frac{W' m!}{Q^{m+1}}\right|$$

This implies

$$\forall \; m \geq 1, \quad \left| \frac{1}{m!} \frac{\partial^m B}{\partial Q^m} (\Delta Q)^m \right| = |W'| \frac{(\Delta Q)^m}{|Q^{m+1}|} \approx \left| \frac{W'}{Q^{m+1}} \right| (\sigma_Q)^m$$

Therefore,

$$\left| \frac{1}{m!} \frac{\partial^m B}{\partial Q^m} (\Delta Q)^m \right| << \left| \frac{\partial B}{\partial Q} \Delta Q \right| \quad \text{will be satisfied}$$

$\forall \; m \geq 2$ if

$$\left| \frac{\sigma_Q}{Q} \right|^{m-1} << 1 \qquad \forall \; m \geq 2$$

which, in turn, implies $\left| \frac{\sigma_Q}{Q} \right| << 1$.

So, assuming $\left| \frac{\sigma_Q}{Q} \right| << 1$ and $Q(x,y) \neq 0$, we have

$$\sigma_B = \left[\left(\frac{\sigma_{W'}}{Q} \right)^2 + B^2 \left(\frac{\sigma_Q}{Q} \right)^2 \right]^{1/2}$$

But recall, $W' = I' - I'_{\mathrm{II}}$

$$\Rightarrow \quad \sigma^2_{W'} = \sigma^2_{I'} + \sigma^2_{I'_{\mathrm{II}}}$$

$$\Rightarrow \quad \sigma_B = \left[\left(\frac{\sigma_{I'}}{Q} \right)^2 + \left(\frac{\sigma_{I'_{\mathrm{II}}}}{Q} \right)^2 + B^2 \left(\frac{\sigma_Q}{Q} \right)^2 \right]^{1/2} \qquad \text{Eq. I.}$$

Now, the precision of the radio astronomy data is assumed to place the following constraint on how precisely we must produce $B(x,y)$:

Let $|B(x,y)|_{max}$ = the maximum value of $|B(x,y)|$ for a given map, and let α be a small positive number (we expect α to be in the interval [2,3] for our application, but right now we leave our equation in the more general form)

$$\boxed{\begin{aligned} &\text{Then } \forall (x,y) \ni \quad 10^{-\alpha}|B(x,y)|_{max} \le |B(x,y)| \le |B(x,y)|_{max} \\ &\sigma_B \le 10^{-\alpha}|B(x,y)|_{max} \end{aligned}} \quad \text{Eqs.}$$

This, then is our basic constraint, which must be used to pass on constraints to the measured quantities $I'(x,y)$, $I'_\pi(x$ and $Q(x,y)$.

Before we go on, we stop and make a simplifica in our analysis, w.r.t. the "$\cos(\theta-\theta_0)$ fringing" in the output plane. It is clear intuitively that data collected at places where $|\cos(\theta-\theta_0)|$ is small will not be as precise as data collected where $|\cos(\theta-\theta_0)| \approx 1$, because of the smaller signal levels involved. Thus, rather than worry about the whole range of $|\cos(\theta-\theta_0)|$,

$$\boxed{\text{From this point on, we assume } |\cos(\theta-\theta_0)| = 1}$$

for all points (x,y) of interest.

Now, $|B(x,y)| = \left|\frac{W'(x,y)}{Q(x,y)}\right| = \frac{|I(x,y) - I_\pi(x,y)|}{4AA_0}$

Since A and A_0 are real constants > 0

Defining $W(x,y) \equiv I(x,y) - I_{\pi}(x,y)$, we have

$$|B(x,y)| = \frac{|W(x,y)|}{4AA_0}$$

It is then clear

$$|B(x,y)|_{max} = \frac{|W(x,y)|_{max}}{4AA_0}$$

and

$$\sigma_B = \left[\left(\frac{\sigma_{I'}}{Q}\right)^2 + \left(\frac{\sigma_{I'_\pi}}{Q}\right)^2 + \left(\frac{W(x,y)}{4AA_0}\frac{\sigma_Q}{Q}\right)^2 \right]^{1/2}$$

Using these two relations just obtained, we can rewrite the constraint in Eqs. II as:

$\forall\ (x,y) \ni:$ $$10^{-\alpha}|W(x,y)|_{max} \le |W(x,y)| \le |W(x,y)|_{max},$$

$$\left[\left(\frac{\sigma_{I'}}{Q}\right)^2 + \left(\frac{\sigma_{I'_\pi}}{Q}\right)^2 + \left(\frac{W(x,y)\,\sigma_Q}{4AA_0 Q}\right)^2 \right]^{1/2} \le 10^{-\alpha}\frac{|W(x,y)|_{max}}{4AA_0}$$

or multiplying through by $4AA_0$ and squaring the inequality we have

$$\left(\frac{4AA_0\sigma_{I'}}{Q}\right)^2 + \left(\frac{4AA_0\sigma_{I'_\pi}}{Q}\right)^2 + [W(x,y)]^2\left(\frac{\sigma_Q}{Q}\right)^2 \le 10^{-2\alpha}[|W(x,y)|_{max}]^2$$

Recall that we are calculating B from three measured quantities: I', I'_π and Q. Note that the LHS of the above inequality has three terms, one with $\sigma_{I'}$, one with $\sigma_{I'_\pi}$ and one with σ_Q. One equitable means of distributing the burden of satisfying the inequality among the three measurements is to form the following three secondary constraints

$\forall\ (x,y) \ni:$ $$10^{-\alpha}|W(x,y)|_{max} \le |W(x,y)| \le |W(x,y)|_{max},$$

(a) $$\left(\frac{4AA_0}{Q}\right)^2 \sigma_{I'}^2 \le \frac{1}{3}10^{-2\alpha}[|W(x,y)|_{max}]^2$$

Eq. III

(b) $$\left(\frac{4AA_0}{Q}\right)^2 \sigma_{I'_\pi}^2 \le \frac{1}{3}10^{-2\alpha}[|W(x,y)|_{max}]^2$$

and

(c) $$[W(x,y)]^2 \left(\frac{\sigma_Q}{Q}\right)^2 \le \frac{1}{3} 10^{-2\alpha} \left[|W(x,y)|_{max}\right]^2$$ Eqs. III

If these three secondary constraints are all simultaneously satisfied, our principal constraint of Eqs. II will also be satisfied.

We next rewrite these three constraints in a more convenient form.

First note that on page (28) the calibration function $Q(x,y)$ is defined by $Q(x,y) = 4AA_0 H(x,y) \cos(\theta - \theta_0)$. Thus, in the case $|\cos(\theta-\theta_0)| = 1$, we have $[4AA_0/Q(x,y)]^2 = \left[\frac{1}{H(x,y)}\right]^2$

Recall also our notation $K(x,y) \equiv \frac{s(x,y)}{A_0}$;

$$s(x,y) \equiv A\,B(x,y)$$

So,
$$|W(x,y)| = |I(x,y) - I_{\pi}(x,y)| = 4A_0 A|B(x,y)| = 4A_0^2\,|K(x,y)|$$
in the case $|\cos(\theta-\theta_0)| = 1$

It is thus obvious
$$|W(x,y)|_{max} = 4A_0^2\,|K(x,y)|_{max}$$

Note also that the inequality (c) of Eqs. III can be simplified immediately because $\frac{\sigma_Q}{Q}$ is independent of $W(x,y)$, for a given point (x,y). Thus, the L.H.S. of the inequality is a monotonically increasing func. of $|W(x,y)|$.

Taking account of the fact that for any given map, we do not know what values of $|W|$ will appear at which point (x,y), secondary constraint (c) will be satisfied for any possible map if the following is true:

$$\boxed{\forall (x,y) \quad \left[\frac{\sigma_Q}{Q(x,y)}\right]^2 \leq \frac{1}{3}\, 10^{-2\alpha}} \qquad \text{Eq. IV}$$

Now remember back to page (28b) when we had to make the assumption $\left|\frac{\sigma_Q}{Q}\right| << 1$ in order to write the equation for σ_B that we have been using all along. Well, note that since we expect, for our application, $\alpha \in [2,3]$, this assumption is consistent with the constraint in Eq. IV.

Thus, if we make the constraint in Eq. IV mandatory, all the equations used in the discussion up to this point are valid, and we can go about our business.

This is all we will have to say about the constraint in Eq. IV for the time being.

We now go back and rewrite the constraints in (a) and (b) of Eqs. III, using the expressions obtained on the previous page:

$$\forall (x,y) \ni: \ 10^{-\alpha}\, |K(x,y)|_{max} \leq |K(x,y)| \leq |K(x,y)|_{max},$$

(a) $$\frac{\sigma_{I'}^2}{[H(x,y)]^2} \leq \frac{16}{3}\, 10^{-2\alpha}\, A_0^4\, (K(x,y))^2_{max}$$

and

(b) $$\frac{\sigma_{I'_\pi}^2}{[H(x,y)]^2} \leq \frac{16}{3}\, 10^{-2\alpha}\, A_0^4\, (K(x,y))^2_{max}$$

Eqs. V

Before we go further, we must digress to obtain expressions for $\sigma_{I'}$, $\sigma_{I'_\pi}$ and $H(x,y)$.

(B.) We are modeling the output of the detector-amplifier combination for a given photoelement as follows:

After an exposure for a time $t \pm \sigma_t$ to a constant, uniform intensity I_λ of wavelength λ, a photo-element will produc after video current integration and amplification, a voltage, q_ℓ, such that

subscript "ℓ" denotes we are talking about the ℓ^{th} photoel

Eq. VI $$q_\ell = F_\ell N_\ell + \eta$$

where F_ℓ is a constant taking into account the amplificatio and detection of the charge packet representing the recorded phot Note that in general, F_ℓ varies from photoelement to photoelemen

Further, to account for any statistical uncertainty in the amplification process, we give F_ℓ an uncertainty σ_{F_ℓ}.

N_ℓ is the total number of photons recorded (i.e., which ha produced a pair(s) that have deposited charge on the storage capacitanc by the ℓ^{th} photoelement during the integration time, and η is an irreducible background noise, inherent in the array-amp configuration, whose mean value is zero, but whose rms deviation is given by

$$\sigma_\eta = \frac{G}{Y} \qquad \text{Eq. VII}$$

where $G \equiv \overline{q_{\ell max}}$ ≡ output of detector when a photo-eleme is saturated, averaged over all photoelements in the array.

and $Y \equiv$ "dynamic range" of detector.

We assume, for simplicity, that σ_η and Y are the same for all photoelements (thus explaining the lack of a subscript ℓ). Eq. VII is basically understood to be the defining relation for the dynamic range Y.

Let $N_{\ell_0} \equiv$ mean total number of photons recorded by the ℓ^{th} photoelement during an exposure to $\mathcal{I}_\ell$ for time t, averaging over infinitely many trials. Then

$$N_{\ell_0} = \frac{\lambda}{hc} a t \mathcal{I}_\ell \xi_\ell$$

ξ_ℓ being the quantum yield of ℓ^{th} photoelement to photons of wavelength λ,

and N_ℓ is an approximation to the value of N_{ℓ_0}, with uncertainty (std deviation) $N_\ell^{1/2}$.

We pause here to note that the above discussion has idealized the physics of the detector array in the following way:

There is no clear cut, definite area per photo-element over which it receives photons, so rigourously to speak of N_ℓ and N_{ℓ_0} impossible. We use a mean effective area, a, which we shal assume is the same for all photo-elements, and is given by

$$a = \frac{\text{total clear aperture of array}}{\text{number of photo-elements}}$$

Assuming a uniform intensity over the array, this value seems to give the best estimate of the area over which the total photon flux can influence a given photo-element, as based on Fig. 4 of the "Reticon C Series Solid State Line Scanners" data sheet © 1974, for arrays

RL256C RL512C RL768C RL1024C

RL256EC RL512EC RL768EC RL1024EC

a is assumed to be an arbitrarily accurately known constant for a specific array. Recall we have ξ_ℓ, which varies from photo-element to photo-element, to account for any sensitivity differences between photo-elements.

With the above discussion aside, we can use our measured value of $q_{f\ell}$ to calculate $\mathcal{I}'_\ell$ when the detector has just been exposed for nominal time t to intensity $\mathcal{I}_\ell$.

For all the discussion that follows, we assume the ℓ^{th} photo-element is being used to measure $\mathcal{I}'_\ell$ at the position (x,y).

we define $$\mathcal{I}'_\ell \equiv \frac{q^*_{f\ell}}{t} \qquad \text{Eq. VIII}$$

with the nominal integration time t being taken as a well determined constant (dependent on the mean clock frequency)

and $$q^*_{f\ell} \equiv F_\ell N_{\lambda_0} = F_\ell \frac{\lambda a t}{hc} \mathcal{I}_\ell \xi_\ell$$

Recall that $\mathcal{I}'_\ell = H(x,y)\mathcal{I}_\ell$

where $H(x,y)$ depends on detector characteristics and wavelength

Since $$\mathcal{I}_\ell = \frac{hc}{F_\ell \xi_\ell \lambda a t} q^*_{f\ell} = \frac{hc}{F_\ell \xi_\ell \lambda a} \mathcal{I}'_\ell$$

It is thus clear that with the ℓ^{th} photo-element measuring at the position (x,y)

$$H(x,y) = \frac{\lambda a F_\ell \xi_\ell}{hc} \qquad \text{Eq. IX}$$

We now use our measured value of $q_{f\ell}$ to obtain an estimate to the (unknown) value of $\mathcal{I}'_\ell$

i.e.

$$\mathcal{I}'_\ell = \frac{q_{f\ell}}{t} \pm \sigma_{\mathcal{I}'_\ell}$$

where $\sigma_{\mathcal{I}'_\ell} = \dfrac{\sigma_{q_{f\ell}}}{t}$ since t is taken as a well determined constant.

Now $\sigma_{q_{f\ell}}$ must take account of not only σ_η and σ_{F_ℓ}, but the fact that the total number of photons recorded by the ℓ^{th} photo-ele

(N_ℓ) is only an estimate of what the average value, N_{ℓ_0}, should be, if our calculated value for $\mathscr{I}_\ell'$ were to come out exactly true.

To calc. the effective uncertainty in q_ℓ, we note the ℓ^{th} photo-element is counting photons from the flux of $\mathscr{I}_\lambda$ for a period of time $t \pm \sigma_t = \tau$

Let us write τ as

$$\tau = t + \varepsilon$$

where t = the same mean integration time discussed above (assumed to be known infinitely accurately)

while $\varepsilon = 0 \pm \sigma_\varepsilon$ where $\sigma_\varepsilon \equiv \sigma_t$

In this case, we have

$$q_\ell = F_\lambda \mathscr{I}_\lambda \frac{\lambda}{hc} a(t+\varepsilon)\xi_\ell + \eta$$

$$q_\ell = F_\ell N_\lambda \left(1 + \frac{\varepsilon}{t}\right) + \eta$$

When q_ℓ is determined by the device, this is effectively the equation used: N_ℓ is counted and is uncertain by $N_\ell^{1/2}$; ε is found to be zero on the average, but to be uncertain by σ_t. Similarly with η.

$$\Rightarrow \quad \sigma_{q_\ell}^2 = F_\ell^2\left(1+\frac{\varepsilon}{t}\right)^2\sigma_{N_\ell}^2 + N_\ell^2\left(1+\frac{\varepsilon}{t}\right)^2\sigma_{F_\ell}^2 +$$

$$F_\ell^2 N_\ell^2 \frac{1}{t^2}\sigma_\varepsilon^2 + \sigma_\eta^2$$

but since for our circuitry $\frac{\sigma_t}{t} \ll 1$, $\left|\frac{\varepsilon}{t}\right| \ll 1$

so.

$$\sigma_{q_\ell}^2 = F_\ell^2 N_\ell + N_\ell^2 \sigma_{F_\ell}^2 + F_\ell^2 N_\ell^2 \left(\frac{\sigma_t}{t}\right)^2 + \sigma_\eta^2$$

$$\therefore \quad \sigma_{q_\ell}^2 = F_\ell q_\ell + q_\ell^2\left(\frac{\sigma_{F_\ell}}{F_\ell}\right)^2 + q_\ell^2\left(\frac{\sigma_t}{t}\right)^2 + \sigma_\eta^2 \qquad \text{Eq. X}$$

Thus, we can now give an expression for $\sigma_{\mathcal{J}_\ell}$:

$$\sigma_{\mathcal{J}_\ell'} = \frac{\sigma_{q_\ell}}{t} = \left[\frac{F_\ell}{t}\mathcal{J}_\ell' + (\mathcal{J}_\ell')^2 \left\{ \left(\frac{\sigma_t}{t}\right)^2 + \left(\frac{\sigma_{F_\ell}}{F_\ell}\right)^2 \right\} + \left(\frac{\sigma_n}{t}\right)^2 \right]^{1/2}$$

$$\sigma_{\mathcal{J}_\ell'} = \left[\frac{F_\ell}{t}\frac{\lambda_a F_\ell \xi_\ell}{hc}\mathcal{J}_\ell + \left(\frac{\xi_\ell \lambda_a F_\ell}{hc}\mathcal{J}_\ell\right)^2 \Phi_\ell + \left(\frac{\sigma_n}{t}\right)^2 \right]^{1/2}$$

where

$$\Phi_\ell \equiv \left[\left(\frac{\sigma_t}{t}\right)^2 + \left(\frac{\sigma_{F_\ell}}{F_\ell}\right)^2 \right] \qquad \text{Eq. XI.}$$

This, then, is the equation that should be used to calc $\sigma_{I'}$ and $\sigma_{I'_\pi}$ discussed earlier.

So, using Eqs. XI and IX, and assuming the same nominal integration time t is used to measure I' and I'_π we can rewrite the two secondary constraints of Eqs V as

$\forall (x,y)$ measured with ℓ^{th} photo-element $\ni$:

$$10^{-\alpha}|K(x,y)|_{max} \leq |K(x,y)| \leq |K(x,y)|_{max})$$

(a) $$\left(\frac{hc}{\lambda_a F_\ell \xi_\ell}\right)^2 \left[\frac{F_\ell}{t}\frac{\lambda_a F_\ell}{hc}\xi_\ell I + \left(\frac{\lambda_a F_\ell \xi_\ell}{hc}\right)^2 \Phi I^2 + \left(\frac{\sigma_n}{t}\right)^2 \right] \leq \frac{16}{3}A_0^4 10^{-2\alpha}(|K(x,y)|_{max}$$

and

(b) $$\left(\frac{hc}{\lambda_a F_\ell \xi_\ell}\right)^2 \left[\frac{F_\ell}{t}\frac{\lambda_a F_\ell \xi_\ell}{hc} I_\pi + \left(\frac{\lambda_a F_\ell \xi_\ell}{hc}\right)^2 \Phi_\ell I_\pi^2 + \left(\frac{\sigma_n}{t}\right)^2 \right] \leq \frac{16}{3}A_0^4 10^{-2\alpha}(|K(x,y)|_{max}$$

OR

(a) $$\frac{hc}{\xi_\ell \lambda_a} t I + \Phi_\ell t^2 I^2 + \left(\frac{hc\sigma_n}{\lambda_a \xi_\ell F_\ell}\right)^2 \leq \frac{16}{3} t^2 A_0^4 10^{-2\alpha}(|K(x,y)|_{max})^2$$

and

(b) $$\frac{hc}{\xi_\ell \lambda_a} t I_\pi + \Phi_\ell t^2 I_\pi^2 + \left(\frac{hc\sigma_n}{\xi_\ell \lambda_a F_\ell}\right)^2 \leq \frac{16}{3} t^2 A_0^4 10^{-2\alpha}(|K(x,y)|_{max})^2$$

To simplify the expression of these constraints, we make the following definitions and

$$R \equiv |K(x,y)|$$

$$R_m \equiv |K(x,y)|_{max} \qquad \beta \equiv \frac{16}{3} 10^{-2\alpha}$$

$$\Gamma_\ell \equiv \left(\frac{hc\sigma_n}{\xi_\ell \lambda a F_\ell}\right)^2$$

$$z \equiv t A_0^2 \qquad P \equiv \frac{hc}{\lambda a \xi_\lambda}$$

The constraints can be written:

$\forall\,(x,y)$ measured with ℓ^{th} photo-element $\ni$:

$$10^{-\alpha} R_m \leq R \leq R_m ,$$

(a) $$P t I + \Phi_\ell t^2 I^2 + \Gamma_\ell \leq \beta z^2 R_m^2$$

and

(b) $$P t I_\pi + \Phi_\ell t^2 I_\pi^2 + \Gamma_\ell \leq \beta z^2 R_m^2$$

Now, recall that for a given (x,y)

$$\left.\begin{aligned} I &= A_0^2 + A^2B^2 + 2ABA_0 \\ I_\pi &= A_0^2 + A^2B^2 - 2ABA_0 \end{aligned}\right] \text{ for } \cos(\theta-\theta_0) = 1$$

$$\left.\begin{aligned} I &= A_0^2 + A^2B^2 - 2ABA_0 \\ I_\pi &= A_0^2 + A^2B^2 + 2ABA_0 \end{aligned}\right] \text{ for } \cos(\theta-\theta_0) = -1$$

Thus, for the case $|\cos(\theta-\theta_0)| = 1$, the greater of the two values I and I_π is given by

$$M = A_0^2 + A^2B^2 + 2A|B|A_0$$

Note that both inequalities (a) and (b) will be satisfied if the following inequality is satisfied:

$$Pt\,M + \Phi_\ell t^2 M^2 + \Gamma_\ell \le \beta z^2 R_m^2$$

Thus, the two constraints can be combined into one:

$\forall (x,y)$ measured with ℓ^{th} photo-element $\ni$:

$$10^{-\alpha} R_m \le R \le R_m \,,$$

$$Pt\,M + \Phi_\ell t^2 M^2 + \Gamma_\ell \le \beta z^2 R_m^2$$

but

$$M = A_o^2 + A^2 B^2 + 2A|B|A_o = [A_o + A|B|]^2$$
$$= [A_o + |B|]^2$$
$$= A_o^2\left[1 + \frac{|B|}{A_o}\right]^2 = A_o^2[1 + |K|]^2$$

$$\therefore\ M = A_o^2[1 + R]^2$$

Notice M is a monotonically increasing func. of R.

Thus the constraint will be satisfied over the entire range of R if it is satisfied for $R = R_m$.

Finally, taking account of the fact any value of R can occur at any (x,y) for an arbitrary map, we can write our constraint in the following form, so that if the form given below is satisfied, Eqs V will be satisfied for any possible output map:

$\forall$ photo-elements ℓ: we require

$$Pt\,A_o^2[1+R_m]^2 + \Phi_\ell t^2 A_o^4[1+R_m]^4 + \Gamma_\ell \le \beta z^2 R_m^2$$

OR

$\forall$ photo-elements l. we require

$$P z [1+R_m]^2 + \Phi_l z^2 [1+R_m]^4 + \Gamma_l \leq \beta z^2 R_m^2 \qquad \text{Eq.}$$

(C.) There is one property of the detector we have yet to consider, and which will give us another constraint.

Let I_{max} = the maximum intensity the array must measure for a given output map. Since a given map consists of measuring I and I_π, it is clear the maximum intensity is just the M discussed above, evaluated at $R = R_m$

$$I_{max} = M(R_m) = A_0 [1+R_m]^2$$

We must require that no photo-element is saturated by exposure to this intensity, since it can, in general appear at any photo-element.

We state our requirement as follows:

$\forall$ photo-elements l: we require

$$q_l(I_{max}) + 3\sigma_{q_l}(I_{max}) \leq q_{l\,max} \qquad \text{Eq. I}$$

where $q_{l\,max}$ is the saturation output of the l^{th} photoelement

$$q_{l\,max} = F_l N_{l\,max}$$

$N_{l\,max}$ = the nominal value for the smallest number of photons (of wavelength λ) recorded by the l^{th} photo-element necessary to saturate the l^{th} photo-element.

We can calculate $N_{l\,max}$ using numbers often given by detector manufacturers. (Reticon and Fairchild)

We use the expressions developed earlier to rexpress Eq. XIII

Since $q_\ell(I_{max}) = F_\ell I_{max} \frac{\lambda}{hc} a t \xi_\ell$.

and from Eq. X

$$\sigma_{q_\ell}(I_{max}) = \left[F_\ell q_\ell(I_{max}) + \Phi_\ell (q_\ell(I_{max}))^2 + \sigma_\eta^2 \right]^{1/2}$$

we have $\forall$ photo-elements ℓ,

$$F_\ell I_{max} \frac{\lambda a t \xi_\ell}{hc} + 3\left[F_\ell^2 I_{max} \frac{\lambda a t \xi_\ell}{hc} + \Phi_\ell \left(\frac{F_\ell I_{max} \lambda a t \xi_\ell}{hc} \right)^2 + \sigma_\eta^2 \right]^{1/2} \leq F_\ell N_{\ell_{max}}$$

OR, since $I_{max} = A_0^2 [1 + R_m]^2$

$$= (1+R_m)^2 \frac{\lambda a \xi_\ell}{hc} t A_0^2 + 3 \frac{F_\ell \lambda a \xi_\ell}{hc} \left[\frac{hc}{\lambda a \xi_\ell} t A_0^2 (1+R_m)^2 + \Phi_\ell t^2 A_0^4 (1+R_m)^4 + \left(\frac{\sigma_\eta hc}{\xi_\ell F_\ell \lambda a} \right)^2 \right]$$

$$\leq F_\ell N_{\ell_{max}}$$

Recalling $z \equiv t A_0^2$; $P \equiv \frac{hc}{\lambda a \xi_\ell}$ and $\Gamma_\ell \equiv \left(\frac{hc\sigma_\eta}{\xi_\ell \lambda a F_\ell} \right)^2$

our constraint becomes

$\forall$ photo-elements ℓ, Eq. XIV

$$\boxed{z(1+R_m)^2 + 3\left[P z (1+R_m)^2 + \Phi_\ell z^2 (1+R_m)^4 + \Gamma_\ell \right]^{1/2} \leq P N_{\ell_{max}}}$$

(D.) Note that now we have two inequalities constraining our system, Eqs. $\underline{XII}$ and $\underline{XIV}$.

If we pick a particular detector, and choose a value for α, each of the inequalities contain only two parameters of our processor, $z \equiv t\, A_o^2$ and $R_m \equiv |K(x,y)|_{max}$

What we wish to do now, is rexpress Eqs. $\underline{XII}$ and $\underline{XIV}$ in the forms

$$z \leq f(R_m)$$

$$z \geq g(R_m)$$

Thus, when $f(R_m)$ and $g(R_m)$ are plotted for a given detector and α, any area bounded by $f(R_m)$ on the top and $g(R_m)$ on the bottom represent allowable values for z and R_m.

Look at Eq. $\underline{XII}$, and divide it through by z^2. For fixed R_m, the RHS is now constant but the LHS is a monotonically decreasing func. of z.

$\Rightarrow$ Eq. $\underline{XII}$ can be rewritten in the form

$$z \geq g(R_m)$$

Now inspect Eq. $\underline{XIV}$.

For fixed R_m, the RHS. is constant but the L.H.S. is a monotonically increasing func. of z.

$\Rightarrow$ Eq. $\underline{XIV}$ can be rewritten in the form

$$z \leq f(R_m)$$

We now try to determine $f(R_m)$ and $g(R_m)$.

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To find $f(R_m)$, we must solve the following eq for z:

$$z(1+R_m)^2 + 3\left[Pz(1+R_m)^2 + \Phi_\ell z^2(1+R_m)^4 + \Gamma_\ell\right]^{1/2} = PN_{\lambda max} \quad \text{Eq. XI}$$

Let $u = (1+R_m)^2$ Note $u \geq 0$

Rearranging and squaring; we have

$$(PN_{\lambda max})^2 + z^2u^2 - 2zuPN_{\lambda max} = 9\left[Pzu + \Phi_\ell z^2u^2 + \Gamma_\ell\right]$$

$$(1-9\Phi_\ell)u^2z^2 - (2PN_{\lambda max} + 9P)uz + (PN_{\lambda max})^2 - 9\Gamma_\ell = 0$$

$$z^2 - \frac{(2N_{\lambda max}+9)P}{u(1-9\Phi_\ell)} z + \frac{(PN_{\lambda max})^2 - 9\Gamma_\ell}{(1-9\Phi_\ell)u^2} = 0$$

$$\Rightarrow \quad z = \frac{(2N_{\lambda max}+9)P}{2u(1-9\Phi_\ell)} \pm \frac{1}{2}\left[\frac{(2N_{\lambda max}+9)^2P^2}{u^2(1-9\Phi_\ell)^2} - 4\frac{(PN_{\lambda max})^2 - 9\Gamma_\ell}{(1-9\Phi_\ell)u^2}\right]^{1/2}$$

$$z = \frac{1}{u}\mathcal{L}_\pm$$

where

$$\mathcal{L}_\pm \equiv \frac{(2N_{\lambda max}+9)P \pm \left[(2N_{\lambda max}+9)^2P^2 - 4\left(P^2(N_{\lambda max})^2 - 9\Gamma_\ell\right)(1-9\Phi_\ell)\right]^{1/2}}{2(1-9\Phi_\ell)}$$

We, of course, are only interested in real values of $z > 0$.

Look at the quantity under the radical above

we expect $\Phi_\ell << 0.1 \Rightarrow (1-9\Phi_\ell) > 0$

Also, typical values of $PN_{\lambda max}$ and Γ_ℓ will be

$$PN_{\lambda max} = 15 \frac{erg}{cm^2}$$

$$\Gamma_\ell = 10^{-4}(PN_{\lambda max})^2$$

$\therefore$ We expect both $\mathcal{L}_+$ and $\mathcal{L}_-$ to be > 0 (assuming they are not complex).

Still, from the form of equation, we know there can be only one solution for a given value of Rm and for $z > 0$.

Thus, one of the two roots must be extraneous. To eliminate the extraneous root, we plug in typical numbers for R_m, $N_{\lambda max}$, P, Φ_λ and Γ_λ and see which satisfies Eq. XV;

$$z_+ = \frac{\mathcal{L}_+}{(1+R_m)^2} \quad \text{or} \quad z_- = \frac{\mathcal{L}_-}{(1+R_m)^2}$$

Take

$$R_m = 1$$

$$PN_{\lambda max} = 15 \ \frac{erg}{cm^2}$$

$$\Gamma_\lambda = 10^{-4}(PN_{\lambda max})^2$$

$$\Phi_\lambda = 10^{-8}$$

$$P = 4.87 \times 10^{-7} \ \frac{erg}{cm^2}$$

$$\Rightarrow \quad \mathcal{L}_+ = 15.45 \qquad \text{so} \qquad z_+ = 3.863$$

$$\mathcal{L}_- = 14.55 \qquad\qquad z_- = 3.637$$

Evaluating the LHS of Eq. XV with z_+ and z_-, we have

$$z_+ : \quad 15.90 \neq PN_{\lambda max}$$

$$z_- : \quad 15.00 = PN_{\lambda max}.$$

Thus $z = \frac{\mathcal{L}_+}{(1+R_m)^2}$ must be the extraneous root, so we conclude.

$$z \leq f(R_m) = \frac{\mathcal{L}_-}{[1+R_m]^2} \qquad \text{Eq. XVI}$$

where

$$\mathcal{L}_- = \frac{(2N_{\ell max}+9)P - \left[(2N_{\ell max}+9)^2 P^2 - 4(P^2(N_{\ell max})^2 - 9\Gamma_\ell)(1-9\Phi_\ell)\right]}{2(1-9\Phi_\ell)}$$

To find $g(R_m)$, we must solve the following for z:

$$Pz[1+R_m]^2 + \Phi_\ell z^2 [1+R_m]^4 + \Gamma_\ell = \beta z^2 R_m^2$$

$$z^2\left[\beta R_m^2 - \Phi_\ell (1+R_m)^4\right] - P(1+R_m)^2 z - \Gamma_\ell = 0$$

$$z^2 - \frac{P(1+R_m)^2 z}{\beta R_m^2 - \Phi_\ell (1+R_m)^4} - \frac{\Gamma_\ell}{\beta R_m^2 - \Phi_\ell (1+R_n)^4} = 0$$

$$\Rightarrow z = \left(\frac{1}{2}\right)\frac{P(1+R_m)^2}{\beta R_m^2 - \Phi_\ell (1+R_n)^4} \pm \frac{1}{2}\left[\frac{P^2(1+R_m)^4}{[\beta R_m^2 - \Phi_\ell (1+R_n)^4]^2} + \frac{4\Gamma_\ell}{\beta R_m^2 - \Phi_\ell (1+R_m)}\right.$$

Note if $\beta R_m^2 - \Phi_\ell (1+R_m)^4 \equiv S < 0$

both roots are negative, which is of no interest to us, since we require $z > 0$.

If $S > 0$, only the root with the + sign will be positive.

We thus conclude

$$z \geq g(R_m) = \frac{P(1+R_m)^2}{2[\beta R_m^2 - \Phi_\ell (1+R_m)^4]} + \frac{1}{2}\left[\frac{P^2(1+R_m)^4 + 4\Gamma_\ell[\beta R_m^2 - \Phi_\ell (1+R_m)^4]}{[\beta R_m^2 - \Phi_\ell (1+R_m)^4]^2}\right]$$

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(E.) Proposed System

Reticon C Series Solid-State Line Scanner, 1024 element
: RL1024C

as described in © 1974 data sheet.

$\lambda = 6328$ Å (HeNe Laser)

$\alpha = 2.5$

$a = 1\,mil^2 = 6.45 \times 10^{-6}\,cm^2$

We assume $\sigma_{F_\ell} = 0$, as seems reasonable, because with the dynamic range as given, σ_η seems to dominate the uncertainty. Assume $F_\ell = F$ = const. for all diodes.

y = dynamic range $= 10^3$

$\frac{\sigma_t}{t} = 10^{-4}$

Assume $\xi_\ell = \xi$ = const for all diodes.

$\xi \simeq 0.45$ (Guessing, Reticon does not give a value)

For the value of $N_{\ell max}$, we must do the following.

Using Fig. 6 of the data sheet and the sensitivity value given under typical electro-optical characteristics, (at 25°C) we have

$$\text{Sensitivity at } 6328\,\text{Å} = S \equiv \frac{\text{photo current}}{\text{incident intensity of wavelength } 6328\,\text{Å}} = \frac{i_p}{\mathcal{I}}$$

$$= (3.7)(0.7)\,\frac{pA}{\mu watt/cm^2}$$

But if we integrate for a time T, the accumulated charge on the diode is $i_p T = Q$.

Suppose T is just long enough that $Q = Q_{sat}$ = saturation charge when intensity $\mathcal{I}$ is incident on the device ($\lambda = 6328$ Å)

Define $E_{sat}(6328\,\text{Å}) = T\mathcal{L}$

but $Q_{sat} = i_p T = T S \mathcal{L} = S E_{sat}(6328\,\text{Å})$

$$\Rightarrow \quad E_{sat}(6328\,\text{Å}) = \frac{Q_{sat}}{S}$$

but

$N_{\ell max} = N_{max}$ (assumed the same for all diode

$$= E_{sat}(6328\,\text{Å}) \frac{\lambda}{hc} a \xi$$

$$\therefore \quad N_{max} = \frac{\lambda a \xi}{hc} \frac{Q_{sat}}{S} = \frac{1}{P} \frac{Q_{sat}}{S}$$

From the data sheet, we have

$Q_{sat} = 4.0$ p coul

$$\text{since } P = \frac{hc}{\xi \lambda a} = \frac{(6.626 \times 10^{-27}\ \text{erg sec})(3 \times 10^{10}\ \text{cm/sec})}{(6.328 \times 10^{-5}\ \text{cm})(6.45 \times 10^{-6}\ \text{cm}^2)(0.45)}$$

$$P = 1.08 \times 10^{-6} \frac{\text{erg}}{\text{cm}^2}$$

We have

$$N_{max} = \frac{4.0\ \text{pcoul}}{(1.08 \times 10^{-6} \frac{\text{erg}}{\text{cm}^2})(3.7)(0.7\ \frac{\text{pcoul/sec}}{\mu\text{watt/cm}^2})}$$

$$= \frac{4.0 \times 10^{6}}{(1.08)(3.7)(0.7)} \cdot \frac{10^{-6}\ \text{joule}}{\text{erg}}$$

$$N_{max} = \frac{4.0 \times 10^{6} \cdot 10^{-6} \cdot 10^{7}}{(1.08)(3.7)(0.7)} = 1.43 \times 10^{7}$$

Calculating the other constants, we have

$$\beta = \frac{16}{3} 10^{-2(2.5)} = 5.33 \times 10^{-5}$$

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$$\Gamma_\ell = \Gamma = \left(\frac{hc}{\xi\lambda a}\frac{\sigma_n}{F}\right)^2 = p^2\left(\frac{\sigma_n}{F}\right)^2$$

but $G = \overline{q_{\ell max}} = q_{max} = Y\sigma_n$

$$q_{max} = F\,N_{max} \Rightarrow \frac{\sigma_n}{F} = \frac{N_{max}}{Y}$$

So, $$\Gamma = p^2\left(\frac{N_{max}}{Y}\right)^2 = \left(1.08\times10^{6}\ \frac{erg}{cm^2}\right)^2\frac{(1.43\times10^{7})^2}{(10^3)^2}$$

$$\therefore \Gamma = 2.38\times10^{-4}\ \frac{erg^2}{cm^4}$$

Also, $\Phi_\ell = \Phi = 10^{-8}$

Using the constants just obtained, we have

$$\mathcal{L}_- = 15.4\ \frac{erg}{cm^2}$$

Now we tabulate some values for $g(R_m)$ and $f(R_m)$

R_m	$z \geq$ $g(R_m)$	Check	$z \leq$ $f(R_m)$
0.02	214.	✓	14.8
0.05	49.6		14.0
1.0	2.16		3.85
1.5	1.44		2.46
2.0	1.08		1.71
2.5	0.869		1.26
3.0	0.726	✓	0.962
3.5	0.624		0.760
4.0	0.548		0.616
4.5	0.488		0.509
5.0	0.441		0.428
5.5	0.402		0.364

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R_m	$z \geq g(R_m)$		$z \leq f(R_m)$
6.0	0.369		0.314
10.0	0.228	✓	0.127
20.	0.124		0.0349
25.	0.105		0.0228
30.	0.0931		0.0160
40.	0.0826	✓	0.00916
45	0.0822		0.00728
50.	0.0855		0.00592
60.	0.114	✓	0.00414
65	0.172		0.00354
70	0.814	✓	0.00305
0.5	4.34		6.84
0.3	7.27		9.11
0.2	11.0		10.69
0.75	2.88		5.03
1/3	6.53		8.66

$g(R_m)$ and $f(R_m)$ are plotted over the area of interest on the next page.

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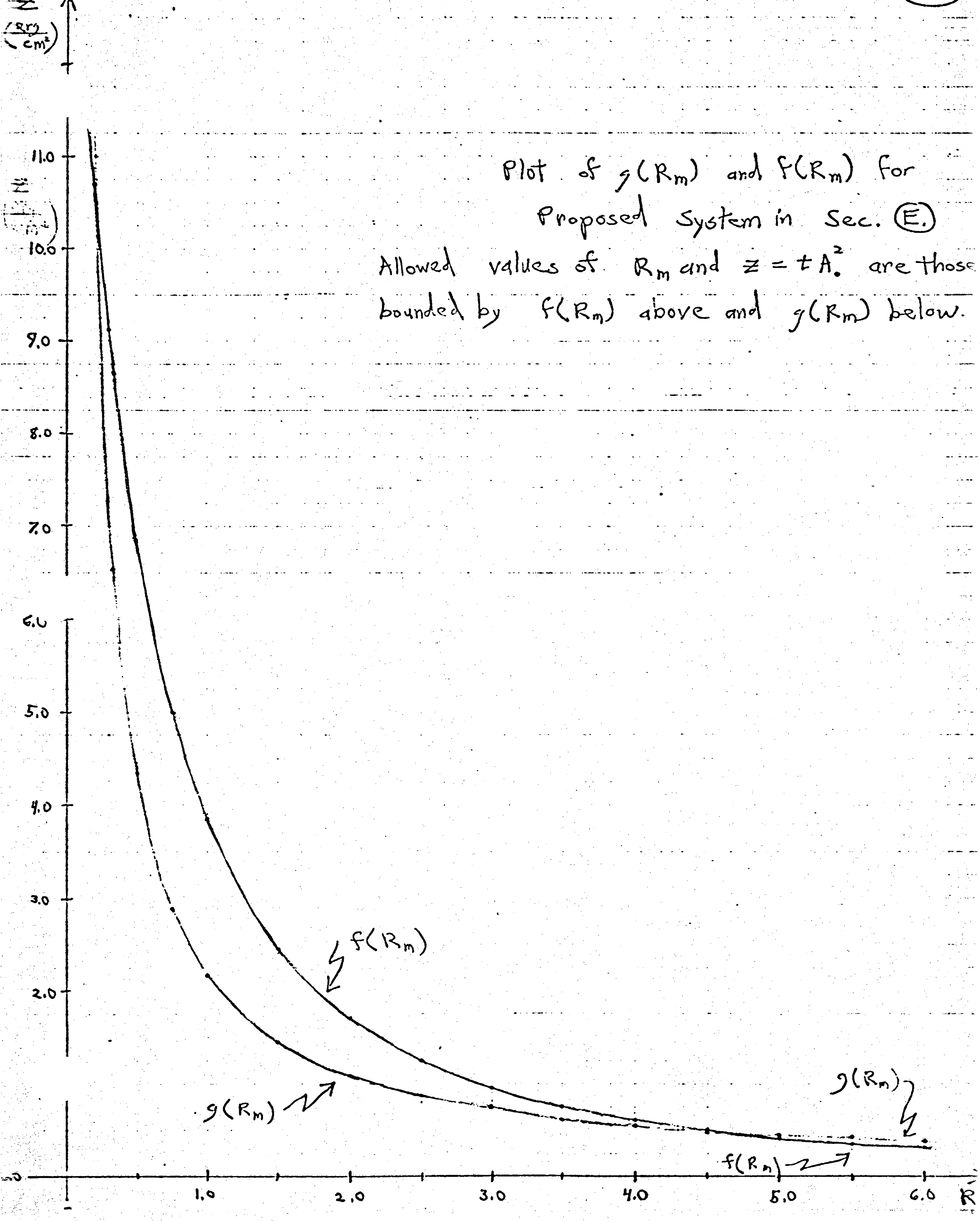

Z (erg/cm²)
Plot of g(Rm) and f(Rm) for Proposed System in Sec. E.
Allowed values of Rm and Z = t A₀² are those bounded by f(Rm) above and g(Rm) below.
f(Rm)
g(Rm)
g(Rm)
f(Rm)
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
2.0
3.0
4.0
5.0
6.0
R

(F) Proposed System:

Reticon RL-1872 F Solid State Line Scanner as described in © 1975 data sheet

$\lambda = 6328\ \text{Å}$

$\alpha = 2.5 \qquad \xi = 0.45$

$\frac{\sigma_E}{E} = 10^{-4}$

Assume all diodes identical (ie., drop subscript ℓ)

Y = dynamic range $= 10^3$

Sensitivity at $6328\ \text{Å} = S = (3.7)\left(0.26\ \frac{pA\ cm^2}{\mu watt}\right)$

As was found in previous section

$$N_{max} = \frac{1}{P}\frac{Q_{sat}}{S}$$

$Q_{sat} = 3.2$ pcoul

$a = 240\ (\mu m)^2 = 2.4 \times 10^{-6}\ cm^2$

$$P = \frac{hc}{\xi \lambda a} = \frac{(6.626 \times 10^{-27})(3 \times 10^{10})}{(0.45)(6.328 \times 10^{-5})(2.4 \times 10^{-6})}\ \frac{erg}{cm^2}$$

$P = 2.91 \times 10^{-6}\ \frac{erg}{cm^2}$

$$N_{max} = \frac{3.2\ pcoul}{\left(2.91 \times 10^{-6}\ \frac{erg}{cm^2}\right)(3.7)\left(0.26\ \frac{pcoul/sec}{\mu watt/cm^2}\right)}$$

$$N_{max} = \frac{3.2 \cdot 10^{-6} \cdot 10^{7}}{(2.91 \times 10^{-6})(3.7)(0.26)} = 1.14 \times 10^{7}$$

$\beta = 5.33 \times 10^{-5}$

Again, we assume $\sigma_{F_\ell} = 0 \Rightarrow \Phi = 10^{-8}$

$$\Gamma = P^2\left(\frac{N_{max}}{Y}\right)^2 = \frac{(2.91 \times 10^{-6})^2 (1.14 \times 10^{7})^2}{10^6}\ \frac{erg^2}{cm^4}$$

$\Gamma = 1.10 \times 10^{-3}\ \frac{erg^2}{cm^4}$

Using the constants just obtained we calc. $\mathcal{L}_-$

$$\mathcal{L}_- = 33.1 \ \frac{erg}{cm^2} \qquad (33.2)$$

We now tabulate some values for $g(R_m)$ and $f(R_m)$

R_m	$z \geq$ $g(R_m)$	$z \leq$ $f(R_m)$
0.02	499.	31.8
0.1	49.6	27.4
0.3	15.7	19.6
0.5	9.35	14.7
1.0	4.66	8.28
1.5	3.11	5.30
2.0	2.34	3.68
2.5	1.88	2.70
3.0	1.57	2.07
4.0	1.18	1.32
4.5	1.06	1.09
5.0	0.954	0.919
6.0	0.800	0.676
9.0	0.546	0.331
15.0	0.346	0.129
25.	0.233	0.0490
35	0.194	0.0255
50	0.199	0.0127
65	0.430	0.00760
70	2.15	0.00657
0.2	23.8	23.0
0.25	18.9	21.2

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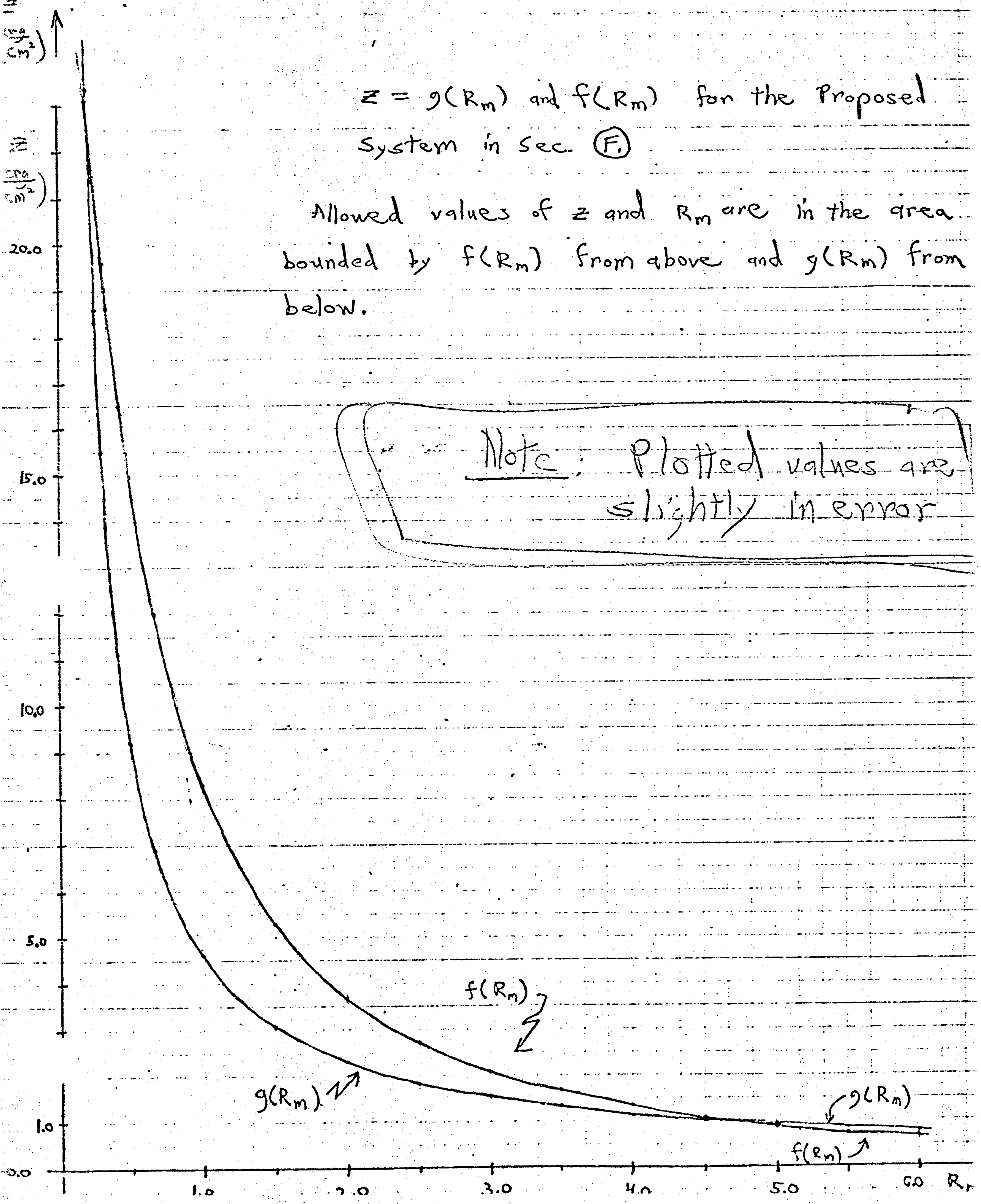
R_m	$z \geq g(R_m)$	$z \leq f(R_m)$
3.5	1.35	1.63
5.5	0.870	0.783
2/3	7.00	11.9
1/3	14.1	18.6
2.5/6	11.2	16.5

$z = g(R_m)$ and $f(R_m)$ for the Proposed System in Sec. F.

Allowed values of z and R_m are in the area bounded by $f(R_m)$ from above and $g(R_m)$ from below.

Note: Plotted values are slightly in error



(G.) Proposed System:

Fairchild CCD 131

1024-element linear image sensor

as described in the data sheet for March 1976

$\lambda = 6328$ Å (HeNe laser)

$\alpha = 2.5$

$\frac{\sigma_t}{t} = 10^{-4}$

We assume all photo-elements are identical, thus dropping subscript ℓ.

Assume $\sigma_F = 0$.

$\Rightarrow \quad \Phi = 10^{-8}$

$\beta = 5.33 \times 10^{-5}$ $\qquad \xi = 0.45$

Y = Dynamic Range $= 1.5 \times 10^{3}$

$a = 169\ (\mu m)^2 = 1.69 \times 10^{-6}\ cm^2$

To find N_{max}, we have to use a different ~~λ~~ proc... than before. For a given test load configuration, Fairchild gives us typical values for q_{max} = saturation output voltage = 1 vol

and typical value for Responsivity per pixel at $\lambda = 6328$ Å

Responsivity/pixel (6328 Å) $= 2.5 \frac{V}{\mu J/cm^2} = \zeta$

It is clear

$$q = \zeta \frac{hc}{\xi \lambda a} N = \zeta P N$$

$$\Rightarrow \quad N_{max} = \frac{q_{max}}{\zeta P}$$

Now $P = \frac{hc}{\xi \lambda a} = \frac{(6.626\times10^{-27})(3\times10^{10})}{(0.45)(6.328\times10^{-5})(1.69\times10^{-6})} \frac{erg}{cm^2}$

$$P = 4.13\times10^{-6}\ \frac{erg}{cm^2}$$

$$\therefore \quad N_{max} = \frac{(1\ \text{volt})}{(2.5\ \frac{V}{\mu J/cm^2})(4.13\times10^{-6}\ \frac{erg}{cm^2})} = \frac{1}{(2.5)(4.13\times10^{-6})}\ \frac{\mu}{e}$$

$$= \frac{10}{(2.5)(4.13\times10^{-6})}$$

$$N_{max} = 9.69\times10^{5}$$

Finally,

$$\Gamma = \left(p\frac{N_{max}}{Y}\right)^2 = 7.12\times10^{-6}$$

Calculating $\mathcal{L}_-$, we find

$$\mathcal{L}_- = 3.99\ \frac{erg}{cm^2}$$

Below are tabulated some values for $g(R_m)$ and $f(R$

R_m	$z \geq$ $g(R_m)$	$z \leq$ $f(R_m)$
0.02	411.	3.84
0.05	39.1	3.62
0.1	10.9	3.30
0.2	3.72	2.77
0.5	1.16	1.77
1.0	0.553	0.998
2.0	0.290	0.443
3.0	0.210	0.249
4.0	0.171	0.160
6.0	0.135	0.0814

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R_m	$z \geq$ $g(R_m)$	$z \leq$ $f(R_n)$
10.0	0.109	0.0330
15.0	0.0986	0.0156
20.	0.0978	0.00905
30.	0.104	0.00415
40.	0.123	0.00237
55	0.206	0.00127
70.	2.95	0.000792
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3.5	0.187	0.197
3.9	0.174	0.166
2.5	0.241	0.326
1.5	0.375	0.638
1.25	0.445	0.788
1.75	0.326	0.528
0.75	0.744	1.30
0.3	2.16	2.36
0.4	1.51	2.04

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$z = g(R_m)$ and $f(R_m)$ for the Proposed System in sec (G.)

Allowed values of z and R_m are in the area bounded by $f(R_m)$ from above and $g(R_m)$ from below.

Note: Plotted values are slightly in error