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MEMORANDUM TO: VLA Optical Processor File

FROM: Ivan Cindrich I.C.

SUBJECT: Scanning Beam Model with Scan Position Errors

Consider a pencil-like laser beam having a spatial distribution h(u'). We will treat one spatial dimension here for convenience. The beam power will be taken to be modulated as a function of time, i.e., P(t). Thus, P(t) is an analog of our input time varying signal and the beam intensity may be written as

$$I(u',t) = P(t)\ h(u')$$

for the case when the beam is not moving. The intensity I has dimensions of power per unit area given by P and h is a dimensionless spatial distribution function.

When the light beam is moved relative to a coordinate u the beam is written as

$$I(u',u,t) = P(t)\ h(u - u')$$

That is, the beam center has position u' along the direction of the coordinate u (where u may be envisioned as a coordinate fixed to a recording medium such as film).

If the beam moves so as to change position linearly with time, we have

$$u' = V t \quad , \qquad \text{with } V \text{ a constant velocity.}$$

If the beam moves so as to depart from such a linearly changing position by an amount $\varepsilon(t)$, then beam position becomes

$$u' + \varepsilon(t) = V t + \varepsilon(t)$$

For the latter case where the beam position is $u' + \varepsilon(t)$, the scanned light beam can be written as

$$I(u,t) = P(t)\, h(u - V t - \varepsilon(t))$$

Making the following change of variables

$$t = \frac{u'}{V}$$

we get

$$I(u,u') = P(\frac{u'}{V}) \quad h(u - u' - \varepsilon(\frac{u'}{V}))$$

$$\equiv P(u') \quad h(u - u' - \varepsilon)$$

Further, we can let $u' + \varepsilon = \alpha$ as a change to the dummy variable α giving

$$I(u,\alpha) = P(\alpha - \varepsilon) \quad h(u - \alpha)$$

The equivalence between the variables t, u' and α is that

$$t = \frac{u'}{V} = \frac{\alpha - \varepsilon}{V}$$

from which we have that

$$\frac{dt}{du'} = \frac{1}{V}$$

and

$$\frac{dt}{d\alpha} = \frac{1}{V}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$\approx \frac{1}{V}$$

Exposure of a recording material with the scanned beam can be characterized as a collection of energy being deposited by the writing beam. The physical units for the writing beam $I = P\ h$ are power per unit area as defined by P since h is a dimensionless function. The integration of I over time will give the exposure energy density at the position u on the film. Thus, we may write exposure as

$$E(u) = \int I(u,t) \quad dt$$

$$= \int P(t) \quad h(u - V\ t - \varepsilon) \quad dt$$

In the other two variables used previsouly we have

$$E(u) = \frac{1}{V} \int P(u') \quad h(u - u' - \varepsilon) \quad du'$$

and using the dummy variable $\alpha = u' + \varepsilon$

$$E(u) = \frac{1}{V} \int P(\alpha - \varepsilon) \; h(u - \alpha) \; d\alpha$$

The integral relation is the convolution operation. This relation establishes the linear system theoretic view of the writing beam as an impulse response function.

It should be noted that the scan error ε is seen to be interpretable as position error of the beam location function h, or alternatively as the negative of that error $(-\varepsilon)$ in the signal P.

IC:sd

xc: C. Aleksoff
J. Fienup
A. Klooster
J. Zelenka