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MEMORANDUM TO: VLA Optical Processor File

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SUBJECT: Phase Error Plots

ABSTRACT

In this memo we present plots of the error between the ideal spread function and the spread function produced when the input contains phase errors.

INTRODUCTION

For the VLA system the input signal is the (hermetian) visibility function V(u,v) and the output is the (real) brightness function B(x,y) given as the Fourier Transform (FT) of the visibility, namely

$$B(x,y) = \mathfrak{F}_{uv}\{AV\}$$

$$= \int_{-\infty}^{\infty} A(u,v)V(u,v)e^{-2\pi i(xu+yv)}dudv \tag{1}$$

where A is the aperture function. With phase error $\phi(u,v)$ the output is

$$B(x,y) = \mathfrak{F}_{uv}\{AVe^{i\phi}\} \tag{2}$$

The spread function $S(\phi)$ is given by (2) when V = 1, i.e.,

$$S(\phi) = \mathfrak{F}_{uv}\{Ae^{i\phi}\}$$

The quantity of interest is the spread function error $\varepsilon(x,y)$ between the spread function degraded by the phase error and that with no phase error:

$$\varepsilon(x,y) = |S(\phi) - S(0)|$$

$$= |\mathfrak{F}_{uv}\{A(1 - e^{i\phi})\}| \qquad (3)$$

We assume the aperture is normalized such that the ideal spread function is maximum at x,y = 0 and that this maximum is unity:

$$S(0)|_{x,y=0} = 1 \qquad (4)$$

The plots in this memo are one-dimensional versions of (3). Let $\phi$ be a one-dimensional error given by

$$\phi(u,v) = \phi(u) = \phi_u$$

and let the aperture function be separable, i.e.,

$$A(u,v) = A_u(u)A_v(v) = A_uA_v \tag{6}$$

Then (3) becomes

$$\varepsilon(x,y) = \varepsilon_x(x)\varepsilon_y(y) \tag{7}$$

where

$$\varepsilon_y = |S_y(0)| = |\mathcal{F}_v\{A_v\}|$$

$$\varepsilon_x = |S_x(\phi_u) - s_x(0)| = |\mathcal{F}_u\{A_u(e^{i\phi_u} - 1)\}| \tag{8}$$

Using a bar over a quantity to indicate 10 log of the quantity we can write

$$\overline{\varepsilon(x,y)} = \bar{\varepsilon}_x + \bar{\varepsilon}_y$$

$$= 10 \log |S_y(0)| + 10 \log |S_x(\phi_u) - S_x(0)| \tag{9}$$

The one-dimensional quantity $\overline{\varepsilon_x}$ is being plotted in this memo. We note that since $S_y(0) \leq 1$ the error due to $\overline{\varepsilon_y}$ is always negative and maximum at y = 0, i.e.,

$$S_y(0)\big|_{y=0} = 1 \rightarrow \overline{\varepsilon}_y\big|_{y=0} = 0$$

Thus, $\overline{\varepsilon_x}$ is the line of maximum spread function error. We note that maximum error along the y-axis is obtained only for a one-dimensional phase error $\phi_u$ and would not be true, in general, for a two-dimensional phase error function.

THE PLOTS

In this section the subscripts will be dropped since it is to be understood that we are working only in one-dimension.

The functions and parameters are:

$A(u) = \frac{1}{W}\,\text{rect}\left(\frac{u}{W}\right) \cdot T(u)$ - aperture function or window

T(u) - taper or weighting on window

Amp - peak to peak phase error over window

$\phi(u)$ - phase error

$S(\phi) = \mathfrak{F}\{Ae^{i\phi}\}$ - spread function

$S(0) = \mathfrak{F}\{A\}$ - ideal spread function

$\text{Error} = |S(\phi) - S(0)|$ - spread function error

Re Error = $|Re[S(\phi) - S(0)]|$ - Real part of spread function error

Im Error = $|Im[S(\phi) - S(0)]|$ - Imaginary part of spread function error

The error plots are generated using a 1024 FFT with 129 points defining the input window A. For example, if $A = \frac{1}{W}$ rect $\frac{u}{W}$ then its FT is sinc (Wx). For our FFT plots unity distance represents a width of $\frac{129}{128}\frac{1}{W}$. The nulls of the sinc function are at positions $n\,\frac{129}{128}$ where n is an integer, positive or negative. There are 8 samples per unity width on the output plot. See Appendix A also.

Plots 1-36 - Uniform Taper

$T(u) = 1$

Plots 2 - No Phase Error

Plot of 10 log S(0) = 10 log sinc (Wx)

Plots 3-7: Linear Phase Error (positional error)

$$\phi = 2\pi \cdot \frac{Amp}{2} \cdot \frac{u}{W/2}$$

Plots 8-26: Quadratic Phase Error (focusing error)

$$\phi = 2\pi\left(\frac{u}{W/2}\right)^2 \cdot Amp$$

Plots 27-32: Cubic Phase Error

$$\phi = 2\pi \frac{Amp}{2} \left(\frac{u}{W/2}\right)^3$$

Plots 33-36: Fourth Order Phase Errors

$$\phi = 2\pi \cdot Amp \cdot \left(\frac{u}{W/2}\right)^4$$

Plots 37-45: 10% Gaussian Taper

$$T(u) = \exp\left[-\left(\frac{u}{W/2}\right)^2 \ln(.9)\right]$$

Plots 39-45: Quadratic Phase Error

$$\phi = 2\pi \cdot \left(\frac{u}{W/2}\right)^2 \cdot Amp$$

CCA/pw

# APPENDIX A
# THE FFT AS AN APPROXIMATION TO THE FT

In this appendix we check the accuracy of using an FFT spread function to approximate a FT spread function.

Consider a one-dimensional function V(u) and its FT given by B(x). Let $V_n = V(n\Delta u)$, n = 1,2,···, N, be the indicated sampled values of V at equally spaced intervals of $\Delta u$. Then the FFT of the sequence $[V_n]$ is the sequence $[T_k]$, k = 1,2,···,N, where

$$T_{k+1} = \sum_{n=1}^{N} V_n e^{-2\pi i\left(\frac{n-1}{N}\right)\cdot k} \qquad (1)$$

Here, N is always some integer power of 2.

For a uniform rect function input, i.e.

$$V(u) = \frac{1}{W}\,\mathrm{rect}\left(\frac{u}{W}\right) \qquad (2)$$

and its FT

$$B(x) = \mathrm{sinc}(Wx) = \frac{\sin(\pi x W)}{\pi x W} \qquad (3)$$

the corresponding sampled input is

$$V_n = \begin{cases} \dfrac{1}{2P+1} & \text{for } n = \begin{cases} 1 \text{ to } P+1 \\ N-P+1 \text{ to } N \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where we note

$$W = (2P + 1)\Delta u. \tag{5}$$

Then the FFT gives

$$T_{k+1} = \sum_{n=1}^{P+1} \exp\left(-2\pi ik\frac{n-1}{N}\right) + \sum_{n=N-P+1}^{N} \exp\left(-2\pi ik\frac{n-1}{N}\right)$$

$$= \frac{\sin\left[\pi k\left(\frac{2P+1}{N}\right)\right]}{(2P+1)\sin\left(\frac{\pi k}{N}\right)} \tag{6}$$

The error is given by the difference of (6) and an appropriately sampled (3) to give

$$\varepsilon_{k+1} = B\left(\frac{k}{N}\right) - T_{k+1} \tag{7}$$

$$= E_k \sin\left(\pi k \frac{2P+1}{N}\right)$$

where the error envelope $E_k$ is given by

$$E_k = \frac{1}{2P+1}\left[\frac{1}{\frac{\pi k}{N}} - \frac{1}{\sin\frac{\pi k}{N}}\right] \tag{8}$$

We note

$$\varepsilon_{k+1} \leq E_k \tag{9}$$

For small k/N the error envelope is given by

$$E_k \simeq \frac{\pi k}{6N(2P+1)} \tag{10}$$

Hence we note that the error is directly proportional to iteration k and inversely proportional to N(2P + 1).

For N = 1024, P = 64, and k = 64 we find that

$$E_{64} = 2.5 \times 10^{-4}$$

Thus, the error is less the 0.1% over the 1 to 64 range of k values.

# APPENDIX B
# ANALYTIC PHASE ERROR CALCULATIONS

In this appendix some analytic phase error expressions are developed. Numerical calculations from these expressions are compared to the values obtained via the FFT.

The brightness function B(x) is given by

$$B(x) = \int \frac{1}{W} \operatorname{rect}\left(\frac{u}{W}\right) e^{i\phi} e^{-2\pi i u x}\, du \tag{1}$$

where $\phi = \phi(u)$ is the phase error. The central value is thus given by

$$B(0) = \frac{1}{W} \int_{-W/2}^{W/2} e^{i\phi}\, du \tag{2}$$

The output error $\varepsilon$ is given by

$$\varepsilon(x) = B(x) - B_o(x) \tag{3}$$

where $B_o$ is the ideal ($\phi = 0$) case, i.e.,

$$B_o = \operatorname{sinc}(Wx) \tag{4}$$

Thus, the central error is given by

$$\varepsilon(0) = B(0) - 1 \qquad (5)$$

For small $\phi$ we can expand the exponent in a power series to get

$$B(0) \approx 1 + iB_1(0) - B_2(0) + \ldots \qquad (6)$$

where

$$B_1(0) = \frac{1}{W} \int_{-W/2}^{W/2} \phi \, du \qquad (7)$$

and

$$B_2(0) = \frac{1}{W} \int_{-W/2}^{W/2} \phi^2 \, du \qquad (8)$$

thus

$$\varepsilon(0) \approx +iB_1(0) - B_2(0) \qquad (9)$$

Polynomial Error

Let

$$\phi = au^n \tag{10}$$

Then, if n = odd, n = 1, 3, 5, ...

$$\varepsilon(0) \approx -\frac{\pi^2 h^2}{2(2n+1)} \tag{11}$$

and if n = even, n = 2, 4, 6, ...

$$\varepsilon(0) \approx \frac{2\pi i h}{n+1} + \frac{2\pi^2 h^2}{2n+1} \tag{12}$$

where h is the peak-to-peak deviation in terms of wavelengths. Here integrals (7) and (8) were evaluated to obtain the above expressions. Thus, we see that the real part of the error is proportional to $h^2$ while the imaginary part is proportional to h or is zero.

Exact Quadratic Error

The quadratic phase error can be written in terms of cosine and sine Fresnel integrals:

$$C(z) \triangleq \int_0^z \cos\left(\frac{\pi}{2} t^2\right) dt \tag{13}$$

$$S(z) \triangleq \int_0^z \sin\left(\frac{\pi}{2} t^2\right) dt \tag{14}$$

Namely:

$$B(0) = \int_{-\infty}^{\infty} \frac{1}{W} \text{rect}\left(\frac{u}{W}\right) \exp\left[2\pi i h\left(\frac{2u}{W}\right)^2\right] du$$

$$= \frac{1}{2\sqrt{h}} \left[C(2\sqrt{h}) + iS(2\sqrt{h})\right] \tag{15}$$

Note that for small h*

$$\frac{C(2\sqrt{h})}{2\sqrt{h}} \approx 1 - \frac{2\pi^2 h^2}{5} \tag{16}$$

and

$$\frac{S(2\sqrt{h})}{2\sqrt{h}} \approx \frac{2\pi h}{3} \tag{17}$$

in agreement with (12).

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*Handbook of Tables of Functions for Applied Optics, Leo Levi, CRC Press, Cleveland, Ohio, 1974.

A Comparison & Check

The following table was formed for quadratic phase errors using three techniques:

(1) exact - as given by Eq. (15)
(2) approximate - as given by Eq. (12)
(3) FFT - as given by FFT plots

| | -10 log[Re ε(0)] | | | -10 log [Im ε(0)] | | |
|---|---|---|---|---|---|---|
| h | exact | approx | FFT | exact | approx | FFT |
| .5 | 2.04 | 0.06 | 1.96 | 2.97 | 0.20 | 3.00 |
| .1 | 14.05 | 14.04 | 13.98 | 6.91 | 6.79 | 6.85 |
| .02 | 27.68 | 28.02 | 27.88 | 13.78 | 13.78 | 13.72 |
| .01 | 34.03 | 34.04 | 33.90 | 16.79 | 16.79 | 16.72 |

It is seen that the FFT gives answers very close to the exact calculation over the entire range of h in the table. The approximate solution is very good for h $\leq$ .1.

## Appendix C

## ODD AND EVEN PHASE ERROR CONSIDERATIONS

In this appendix we consider how oddness and eveness of the phase error influences the brightness.

We take

$$B(x) = \mathcal{F}\left\{e^{i\phi(u)}\right\}$$

as our basic equation where $\phi$ is the phase error (which is always real). In general

$$\phi = \phi_o + \phi_E$$

where $\phi_o$ is the odd part and $\phi_E$ the even part of the phase. Then if we let

$$e^{i\phi} = E + \mathcal{O}$$

where E is even part and $\mathcal{O}$ is the odd part of the exponent function, it follows that

$$E = \cos\phi_o \cos\phi_E + i \sin\phi_E \cos\phi_o$$

$$\mathcal{O} = -\sin\phi_o \sin\phi_E + i \sin\phi_o \cos\phi_E$$

and that

$$\mathcal{F}\{E + \mathcal{O}\} = 2\int_0^\infty E_\infty \cos(2\pi xu)\,du$$
$$-2i\int_0 \mathcal{O} \sin(2\pi xu)\,du$$

Even Phase

Let $\phi = \phi_E$ , i.e. $\phi_o = 0$ , thus $\mathcal{O} = 0$ and

$$\mathcal{F}\{e^{i\phi}\} = \mathcal{F}\{e^{i\phi_E}\} = \mathcal{F}\{E\}$$

$$= 2\int_0^\infty (\cos\phi_E + i \sin\phi_E)\cos(2\pi \cdot ux)\,du$$

which give, in general, an even and complex B, i.e.

| Even phase | ⟷ | Even and complex brightness |
|---|---|---|

Odd Phase

Let $\phi = \phi_o$ , i.e. $\phi_E = 0$. Then

$$E = \cos\phi_o$$

$$\mathcal{O} = i \sin\phi_o$$

$$\mathcal{F}\{e^{i\phi}\} = \mathcal{F}\{\exp(i\phi_o)\}$$
$$= 2\int_0^\infty \cos\phi_o \cos(2\pi ux)\,du$$
$$+ 2\int_0^\infty \sin\phi_o \sin(2\pi ux)\,du$$

which is real and even. This is to be expected since $e^{i\phi_o}$ is hermetian.

Odd Phase ⟷ Real Brightness with even and odd parts

# APPENDIX D
# 1% ERROR CRITERIA IN dB

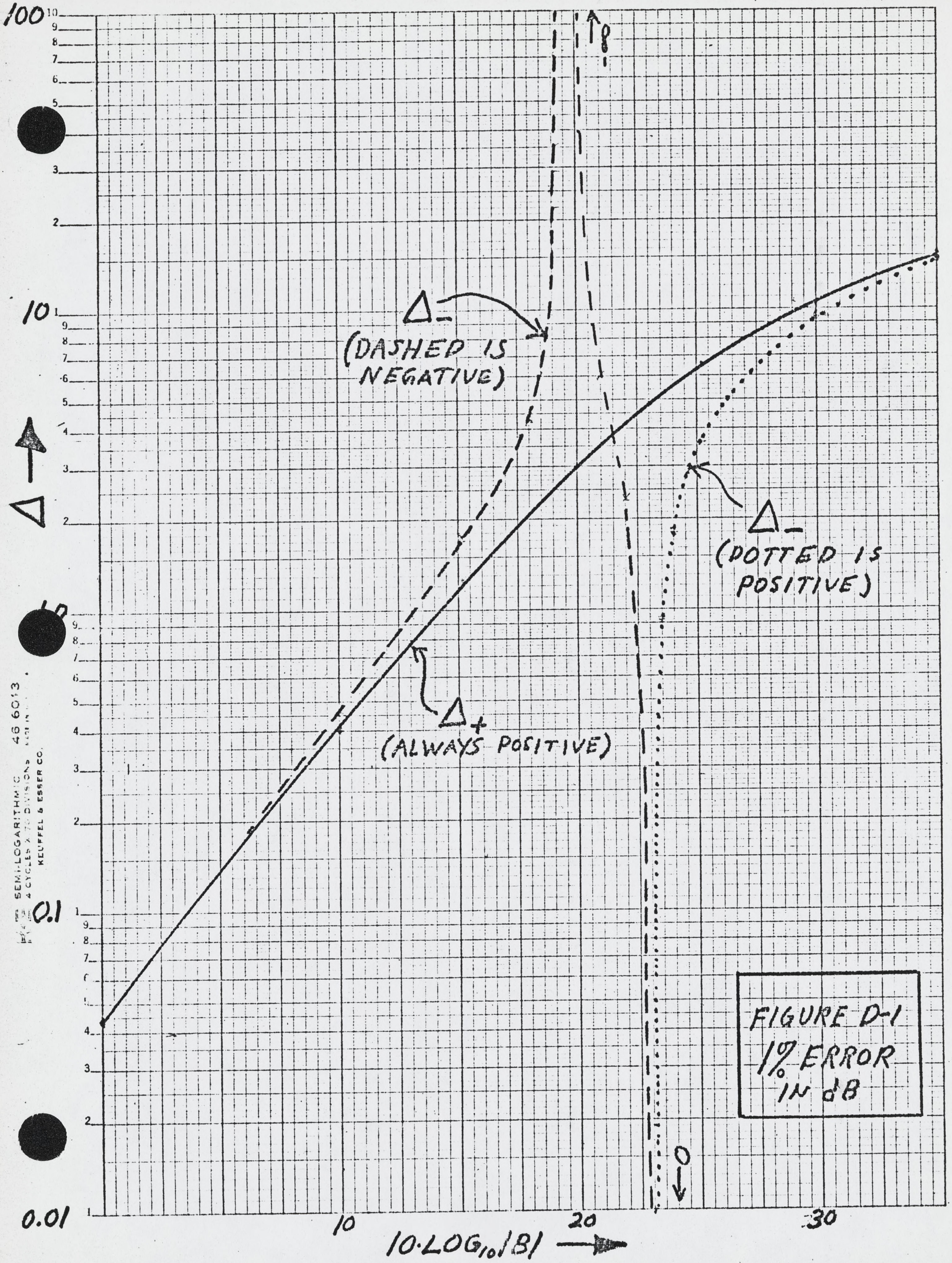
In this appendix we calculate the boundaries of 1% errors as given in dB. That is, given a function $\overline{B}$ where

$$\overline{B} = 10 \log |B| \qquad |B| \leq 1$$

then the boundaries of 1% error about this ideal value of $\overline{B}$ are given by

$$\Delta_{\pm} = 10 \log \left| |B| \pm .01 \right| - \overline{B}$$

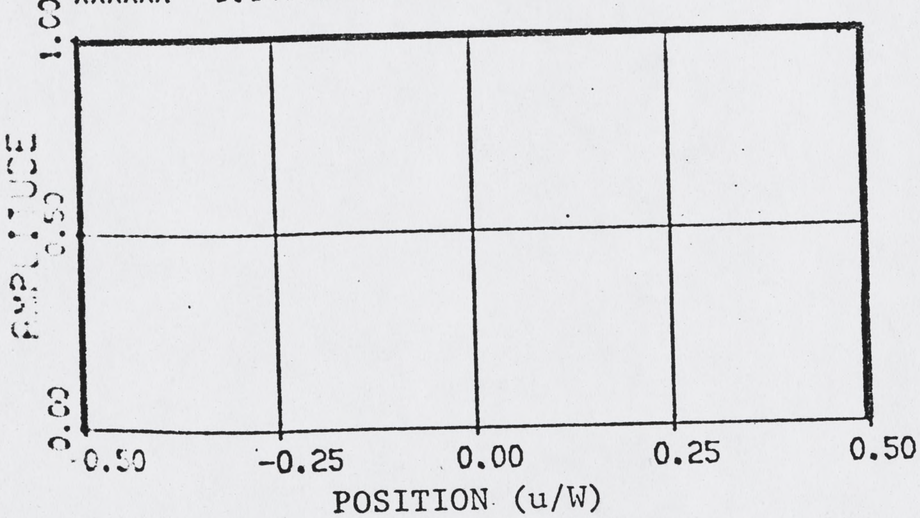
where we assume max $|B|$ is normalized to unity. These boundaries are plotted in Figure D-1 as a function of $\overline{B}$.



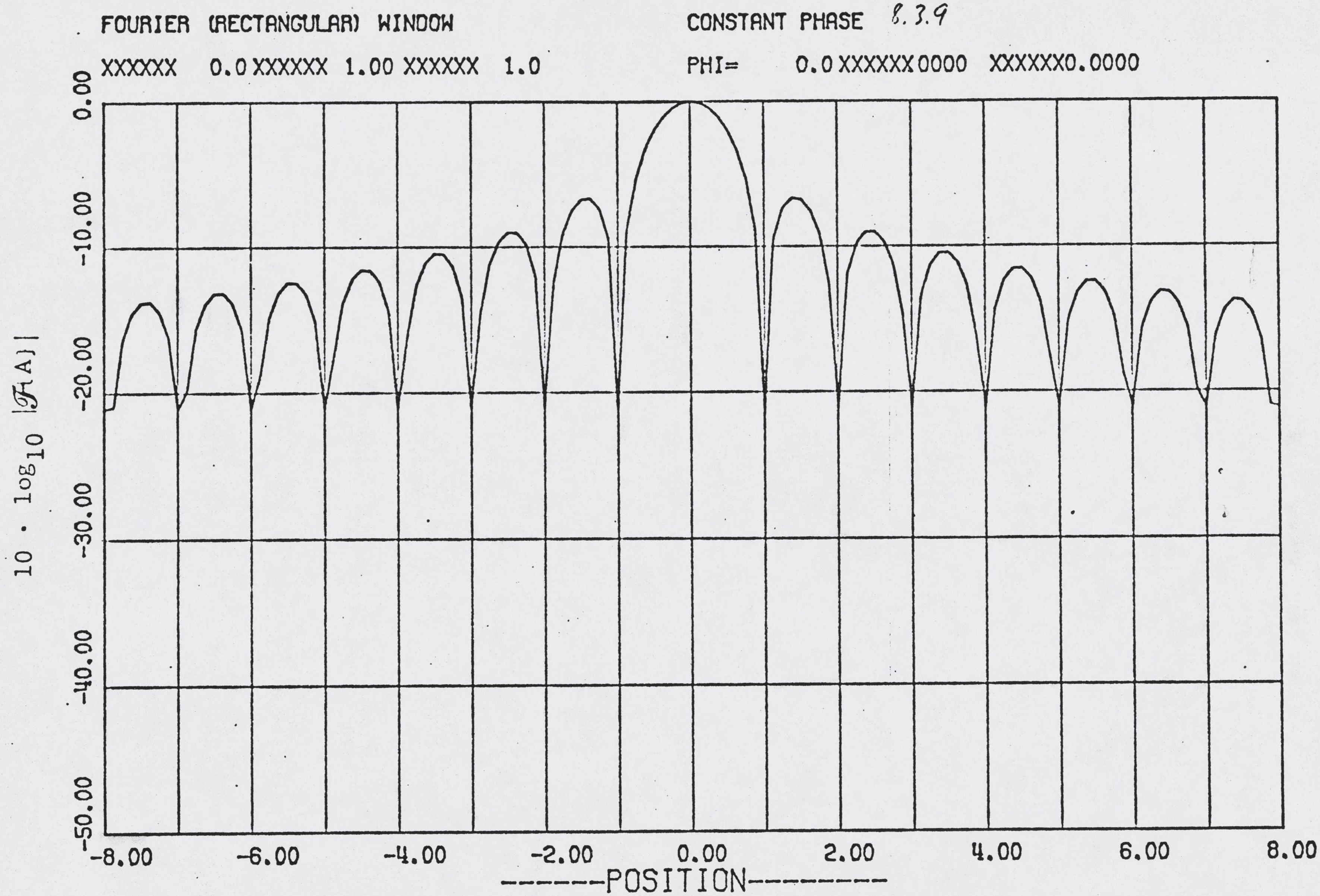
100
10
0.1
0.01
Δ
(DASHED IS NEGATIVE)
(DOTTED IS POSITIVE)
Δ+
(ALWAYS POSITIVE)
∞
0
10
20
30
10·LOG10|B|
FIGURE D-1
1% ERROR IN dB
SEMI-LOGARITHMIC 46 6013
4 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO.

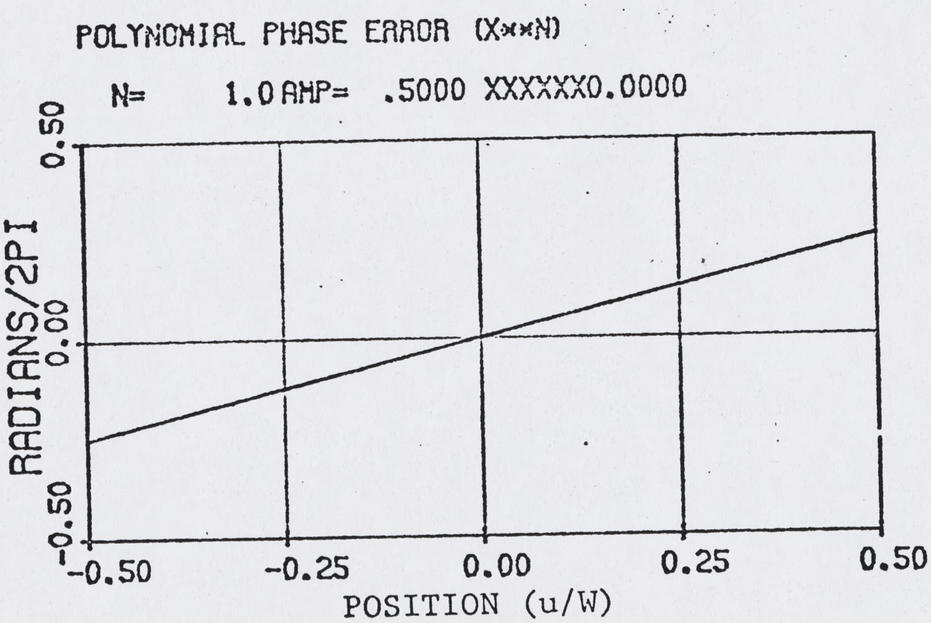
FOURIER (RECTANGULAR) WINDOW

XXXXXX 0.0 XXXXXX 0.00 XXXXXX 0.00

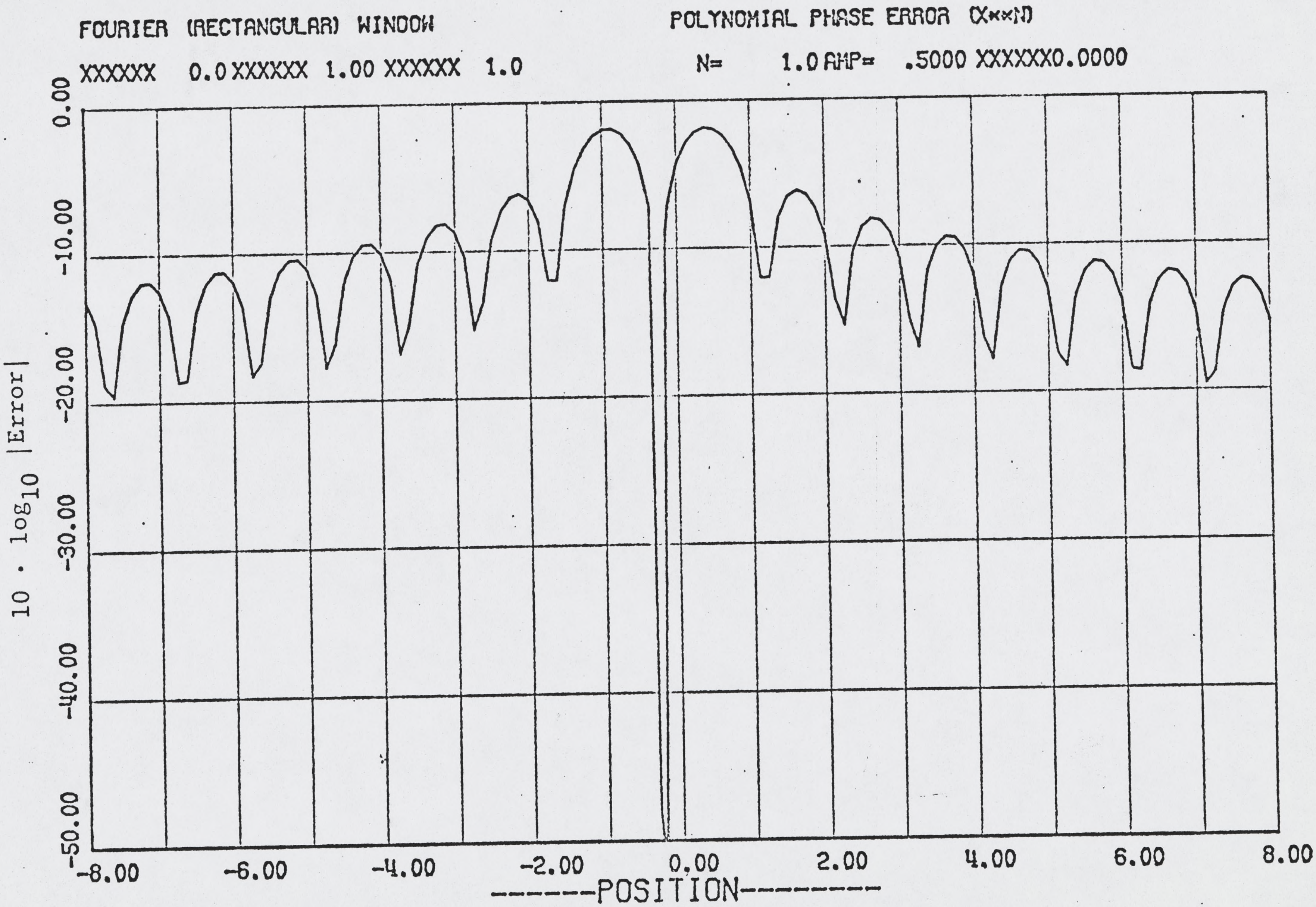



PLOT 1

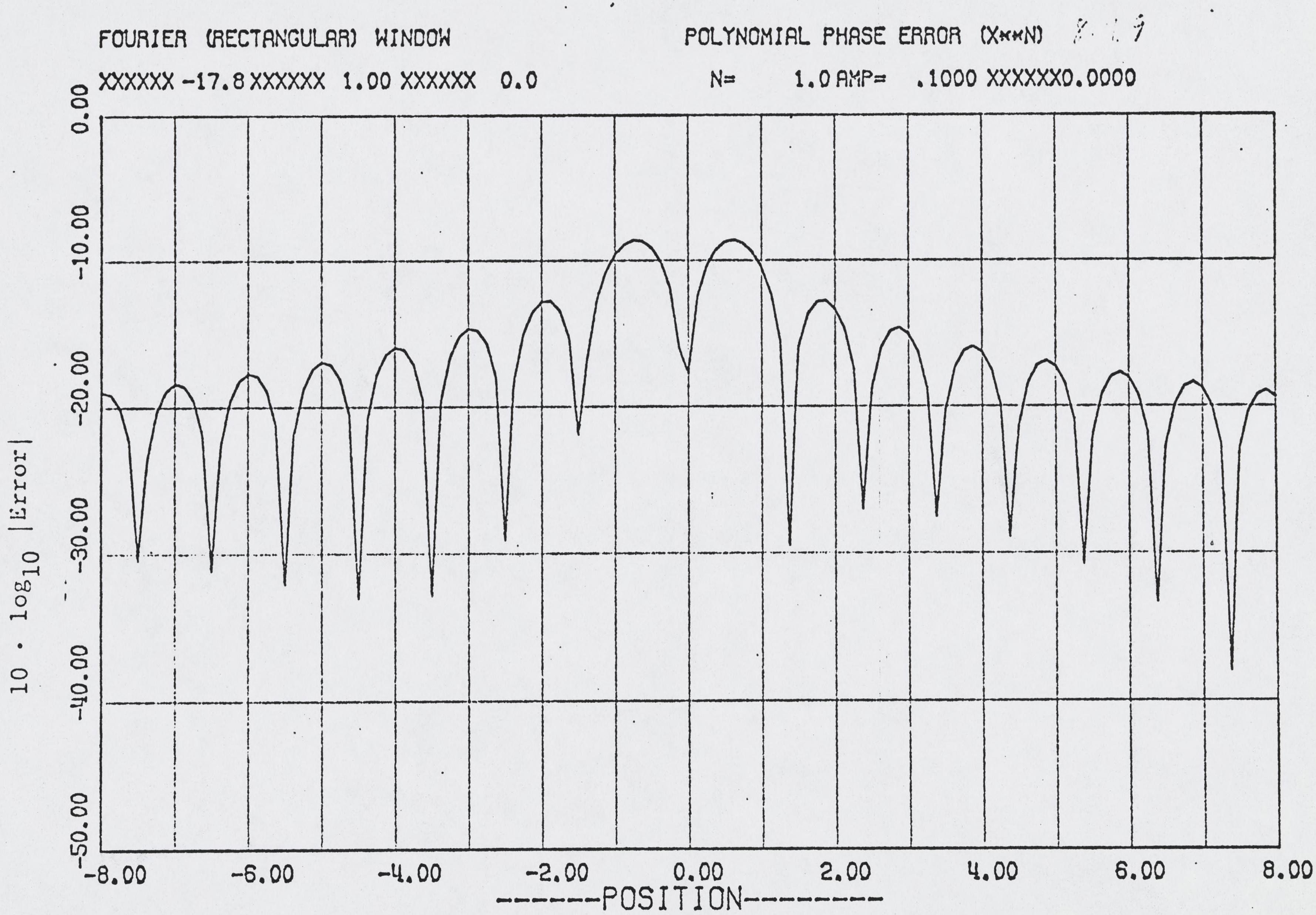

FOURIER (RECTANGULAR) WINDOW
CONSTANT PHASE
8.3.9
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 1.0
PHI= 0.0 XXXXXX 0000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION---------
PLOT 2


POLYNOMIAL PHASE ERROR (X**N)
N= 1.0 AMP= .5000 XXXXXX0.0000
0.50
0.00
-0.50
RADIANS/2PI
-0.50
-0.25
0.00
0.25
0.50
POSITION (u/W)

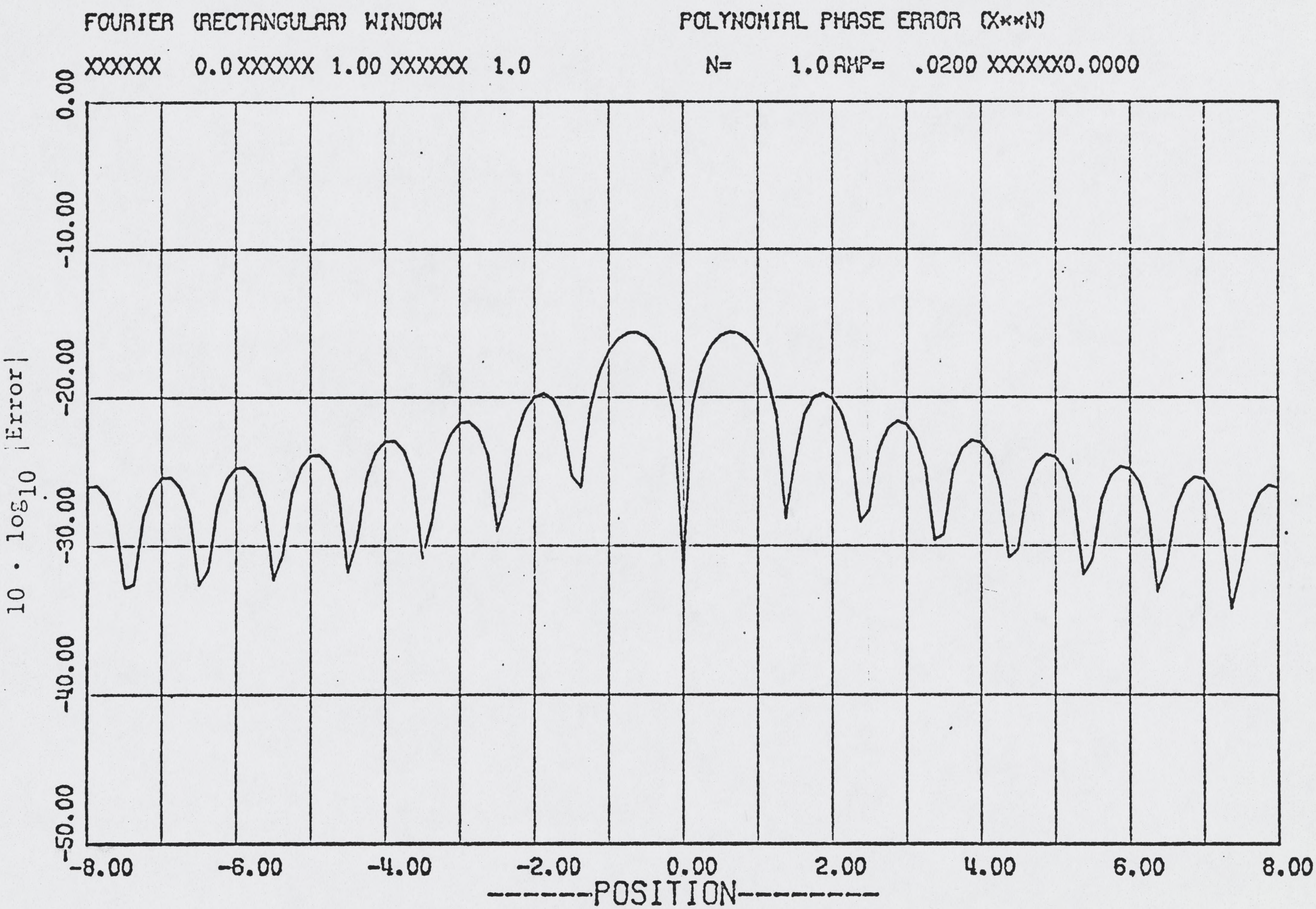
PLOT 3


FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 1.0 AMP= .5000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

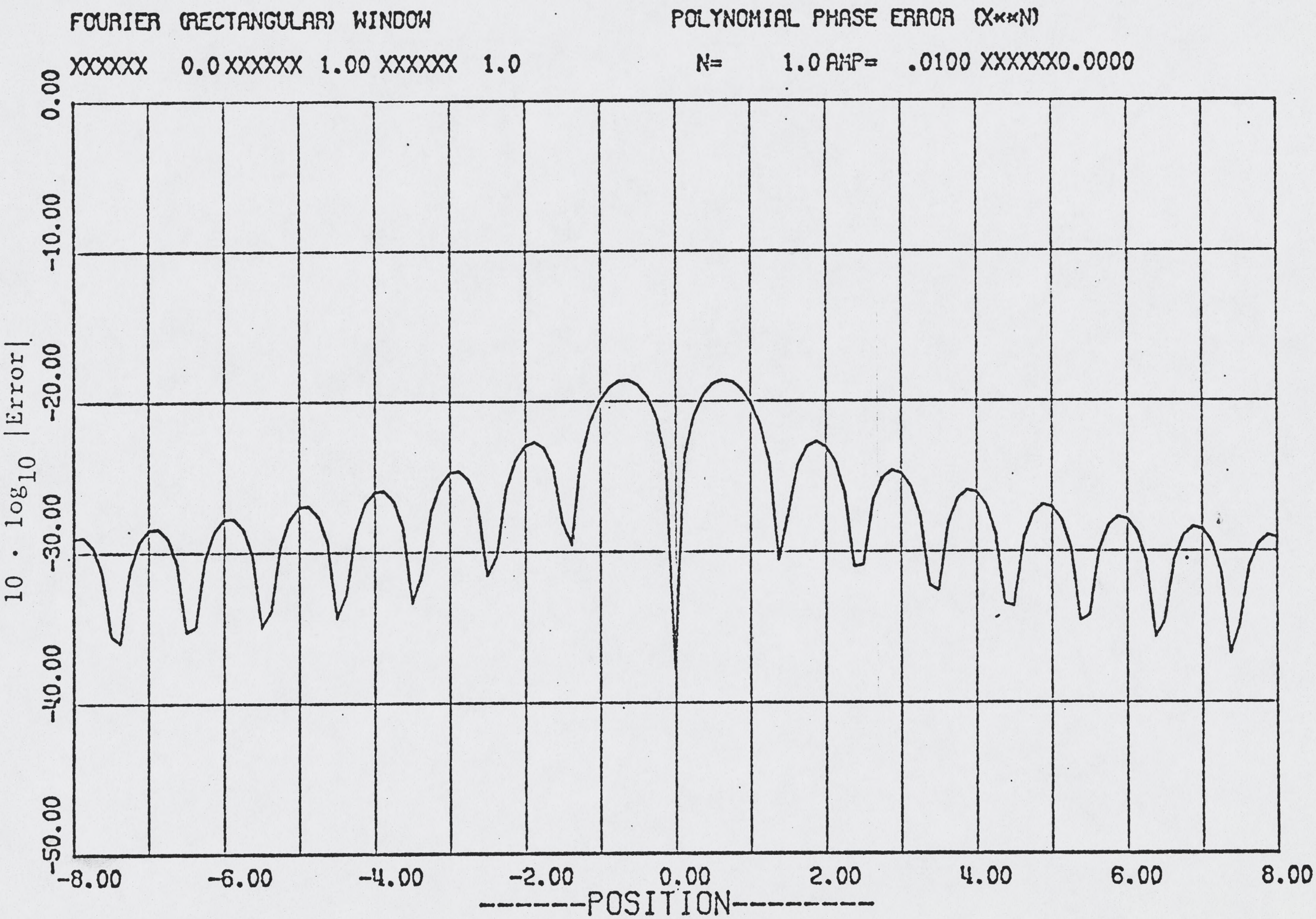
PLOT 4


FOURIER (RECTANGULAR) WINDOW
XXXXXX -17.8 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 1.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
$10 \cdot \log_{10} |\text{Error}|$
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

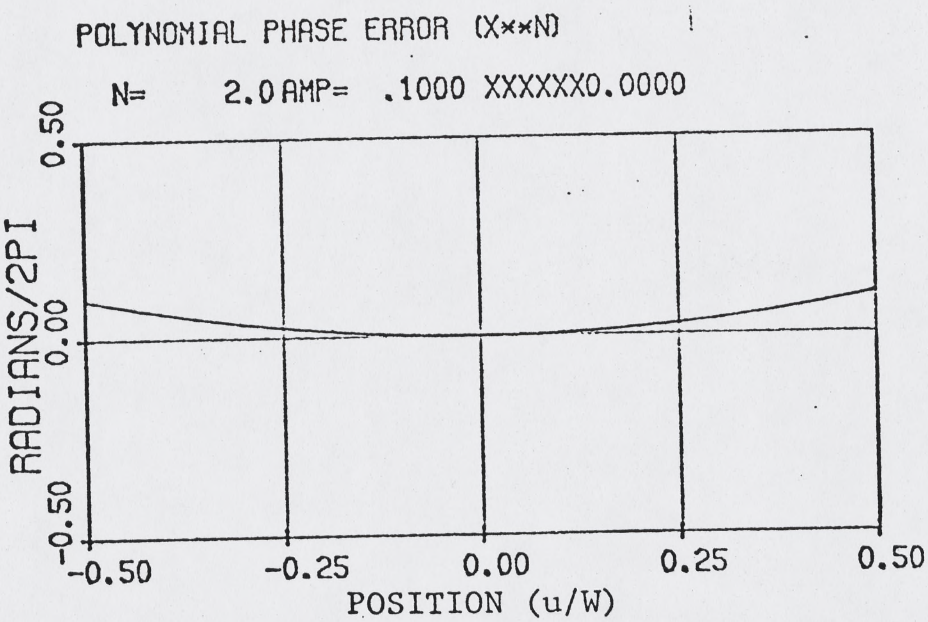
PLOT 5


FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 1.0 AMP= .0200 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

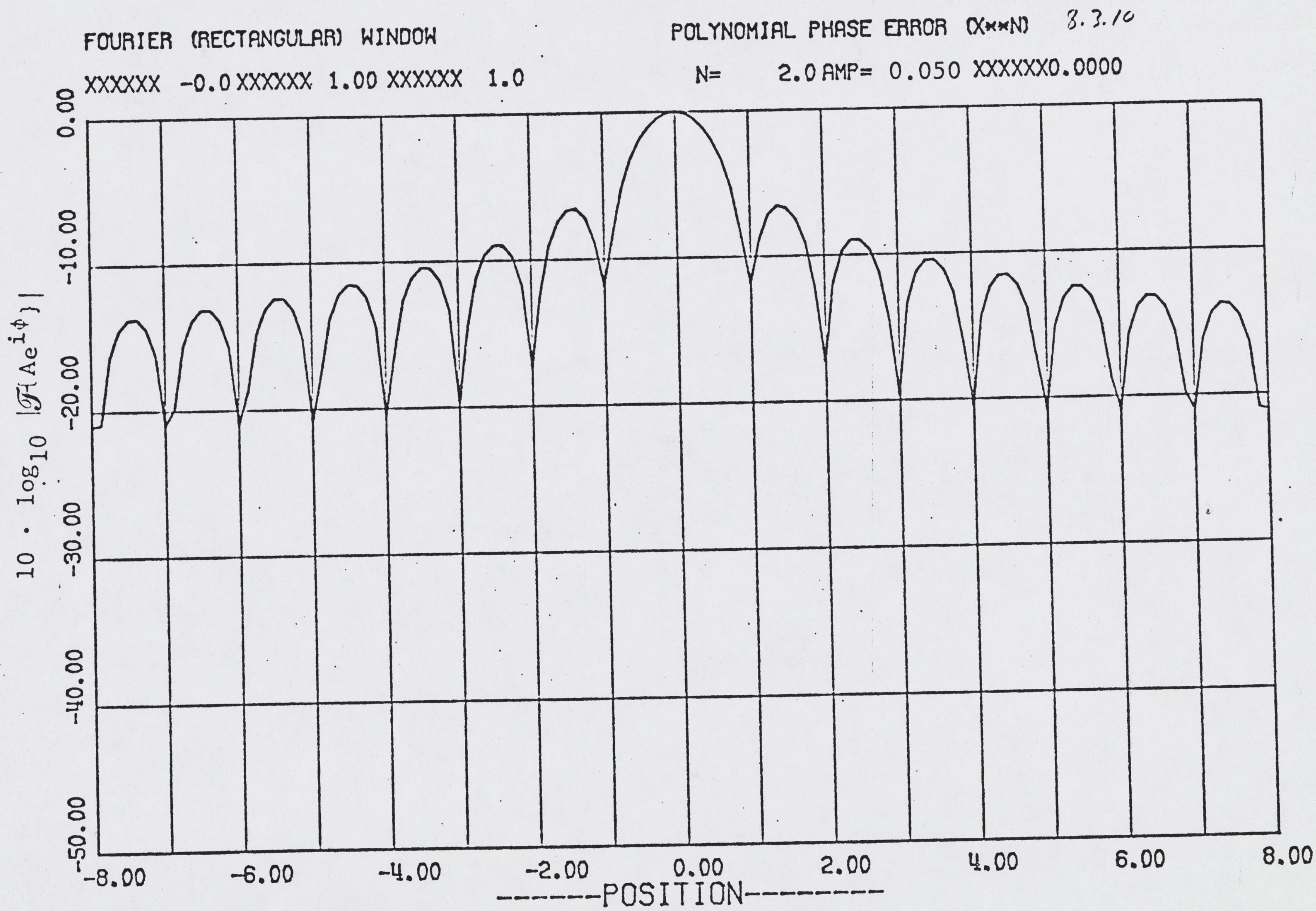
PLOT 6


FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 1.0 AMP= .0100 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 7


POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.50
0.00
-0.50
RADIANS/2PI
-0.50
-0.25
0.00
0.25
0.50
POSITION (u/W)

PLOT 8


FOURIER (RECTANGULAR) WINDOW
XXXXXX -0.0 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N) 8.3.10
N= 2.0 AMP= 0.050 XXXXXX0.0000
10 · log10 |ℱ{Ae^{iφ}}|
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

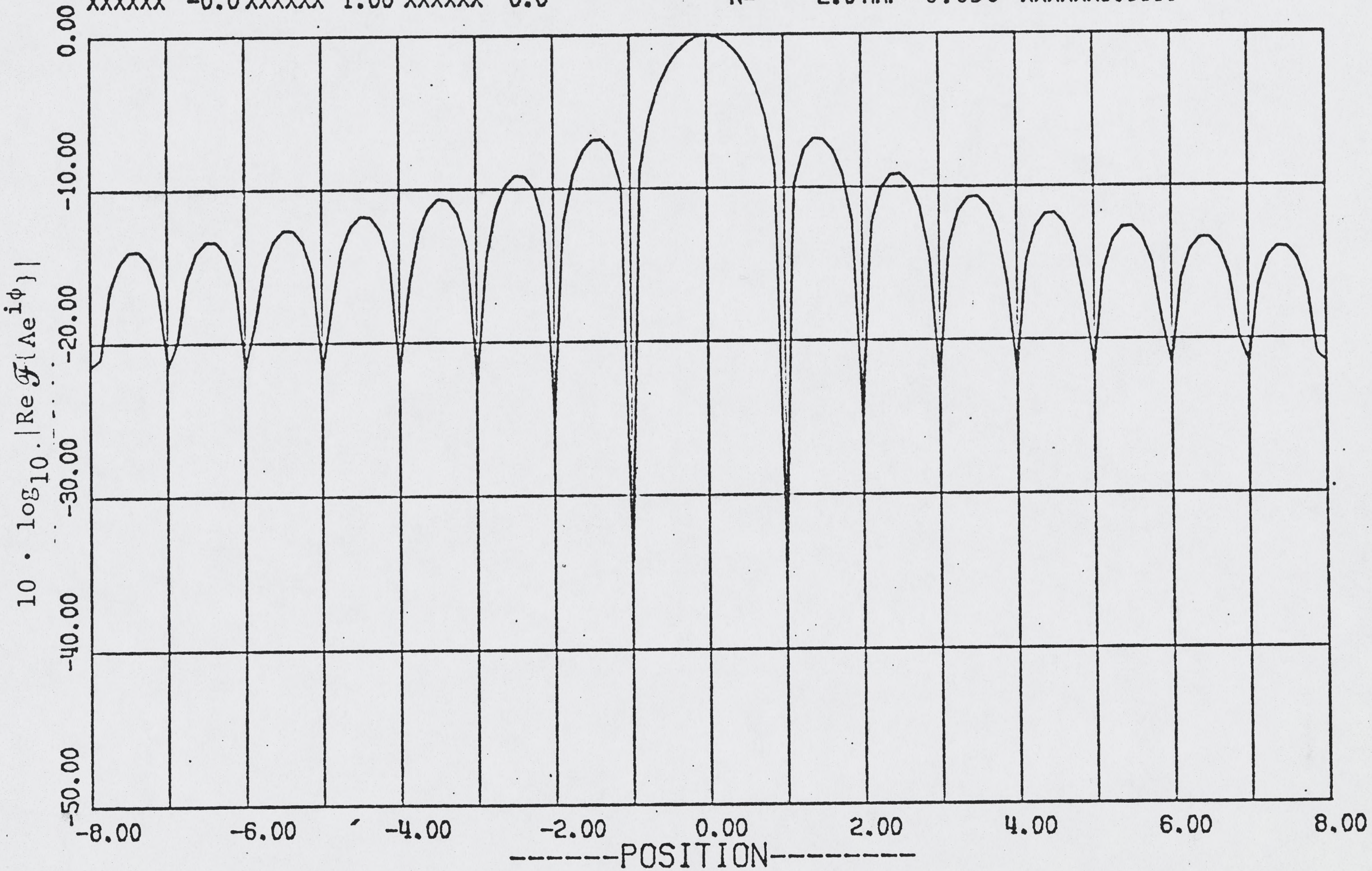
PLOT 9

FOURIER (RECTANGULAR) WINDOW

XXXXXX -0.0 XXXXXX 1.00 XXXXXX 0.0

POLYNOMIAL PHASE ERROR (X**N) 5.3.11

N= 2.0 AMP= 0.050 XXXXXX0.0000



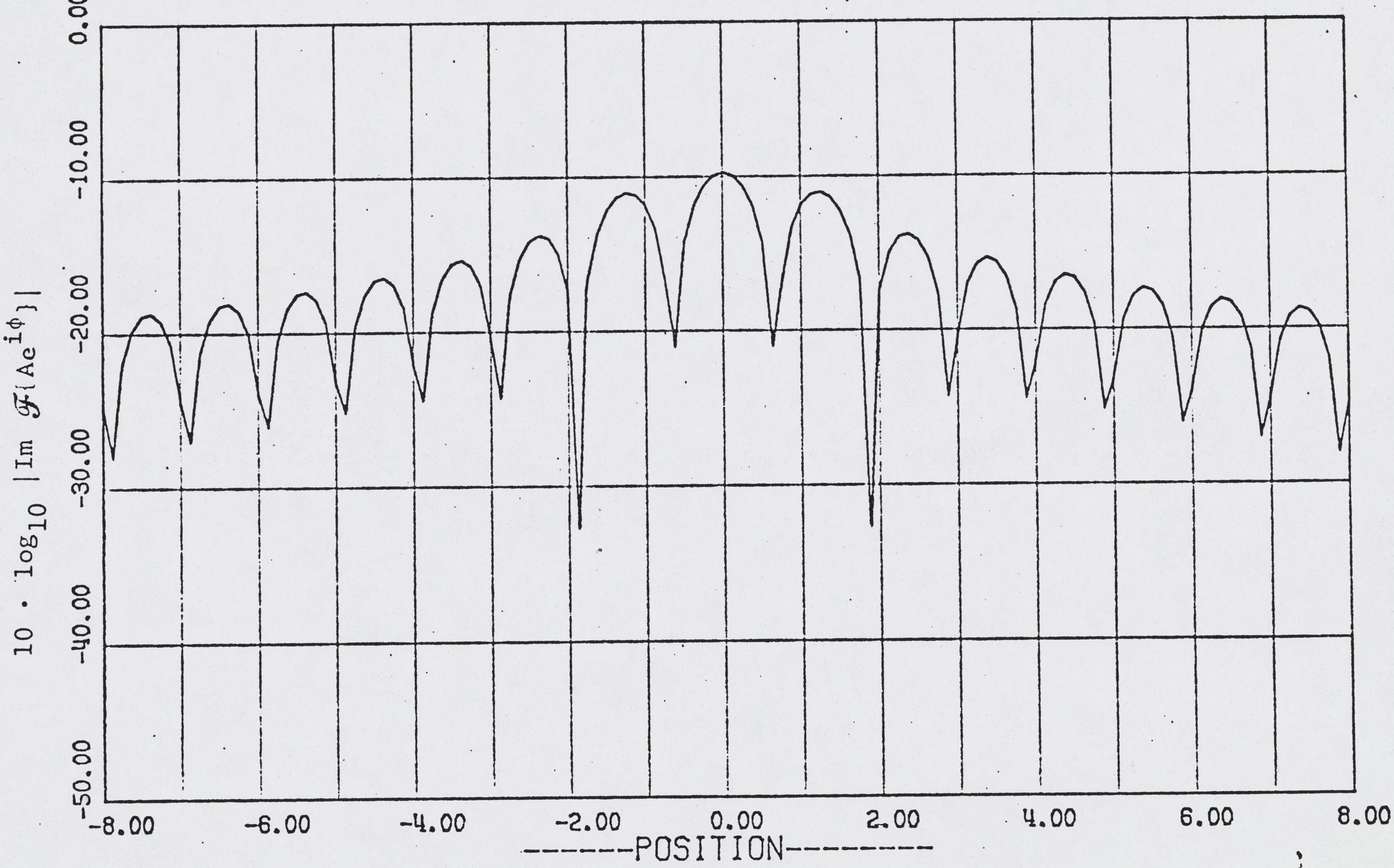

PLOT 10

FOURIER (RECTANGULAR) WINDOW

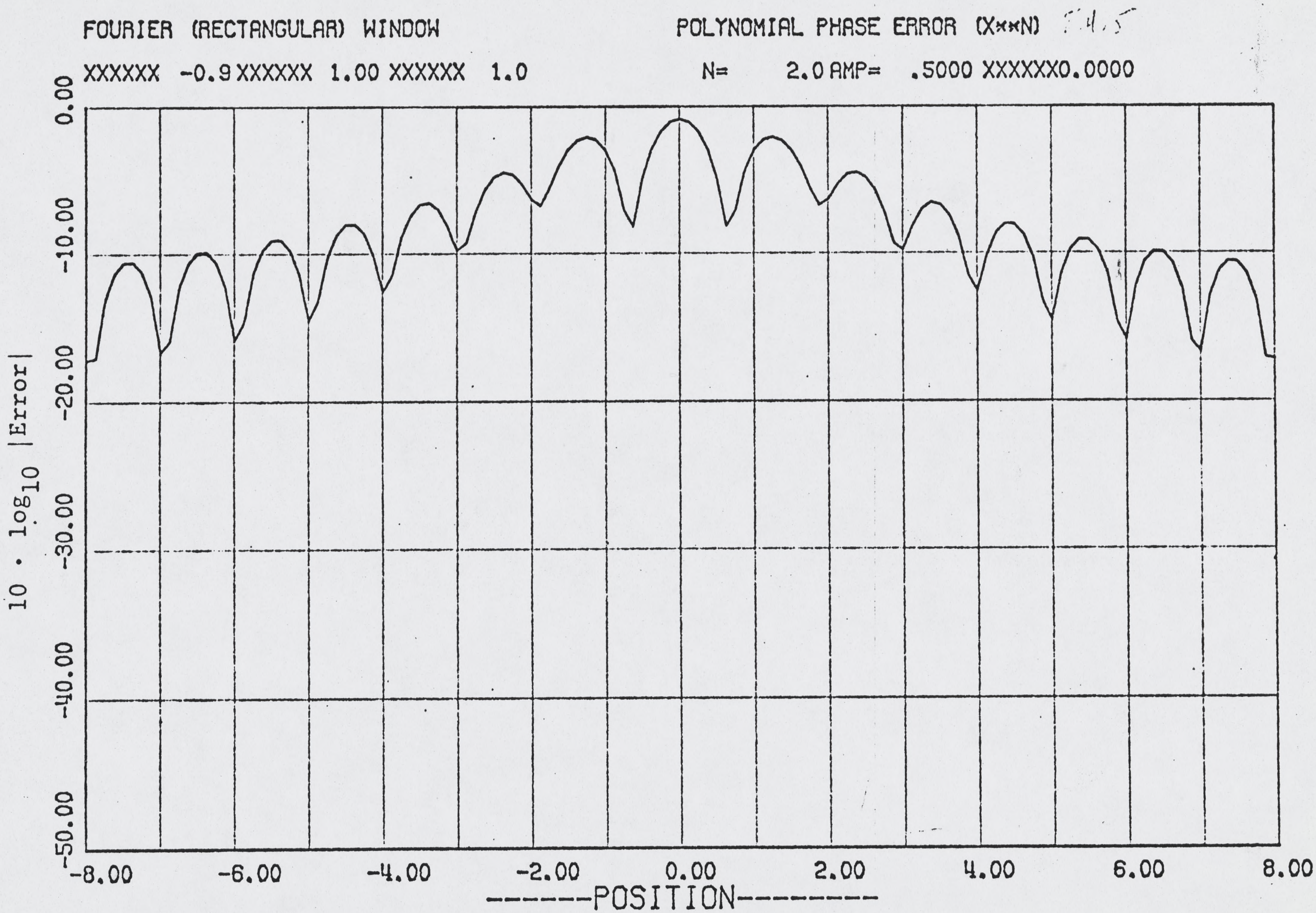
XXXXXX -9.8 XXXXXX 0.00 XXXXXX 1.0

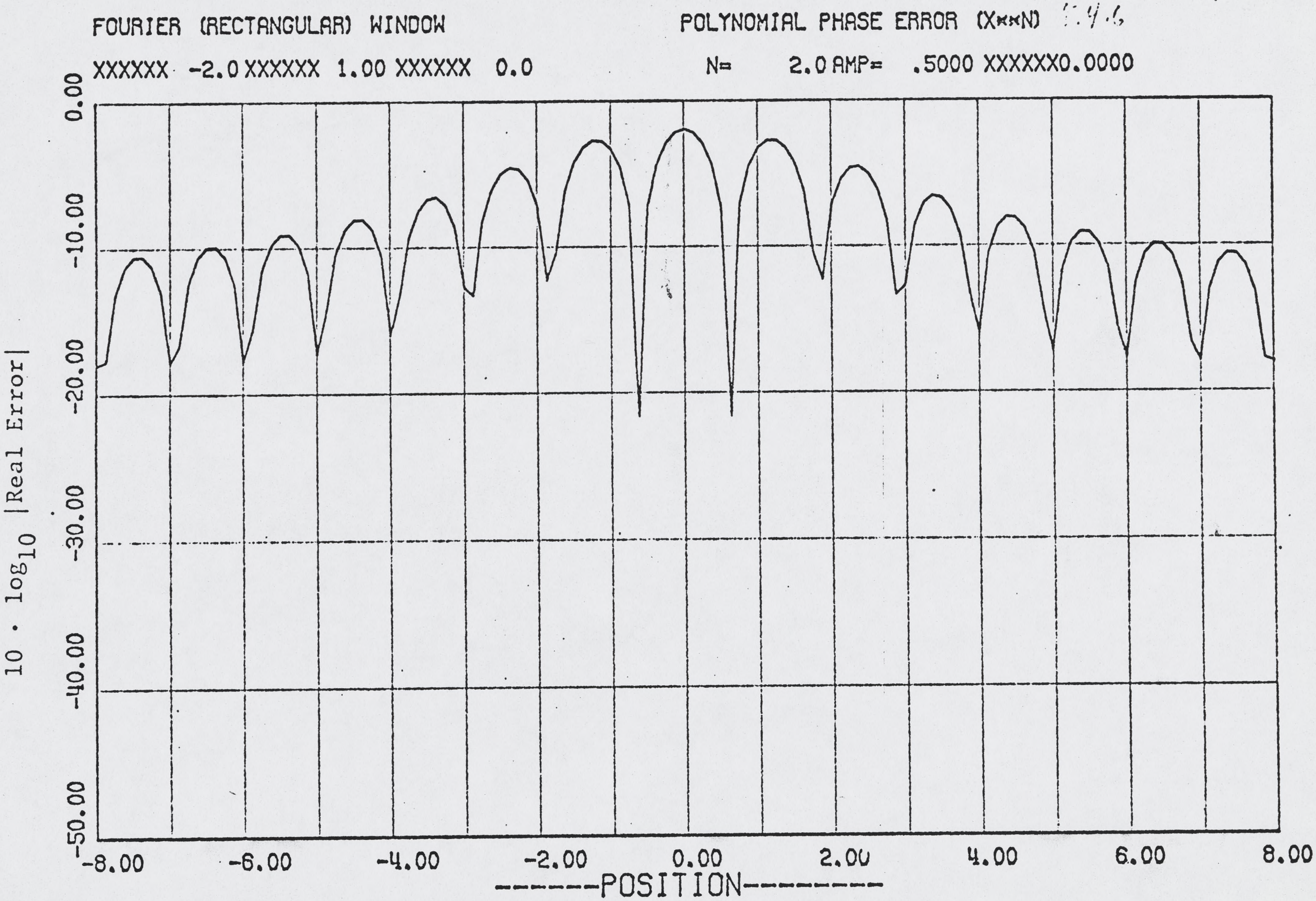
POLYNOMIAL PHASE ERROR (X**N) 5.7.12

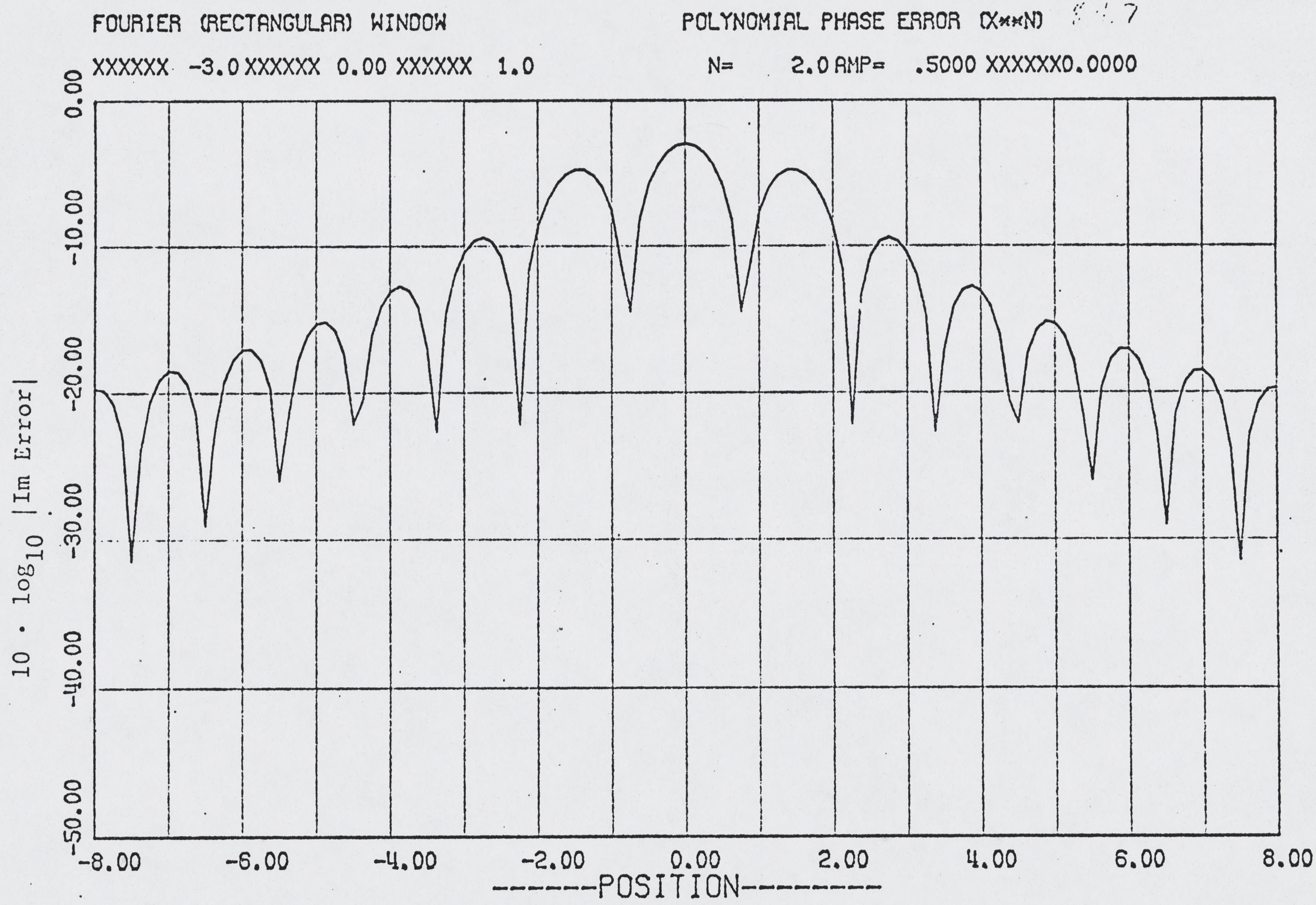
N= 2.0 AMP= 0.050 XXXXXX0.0000



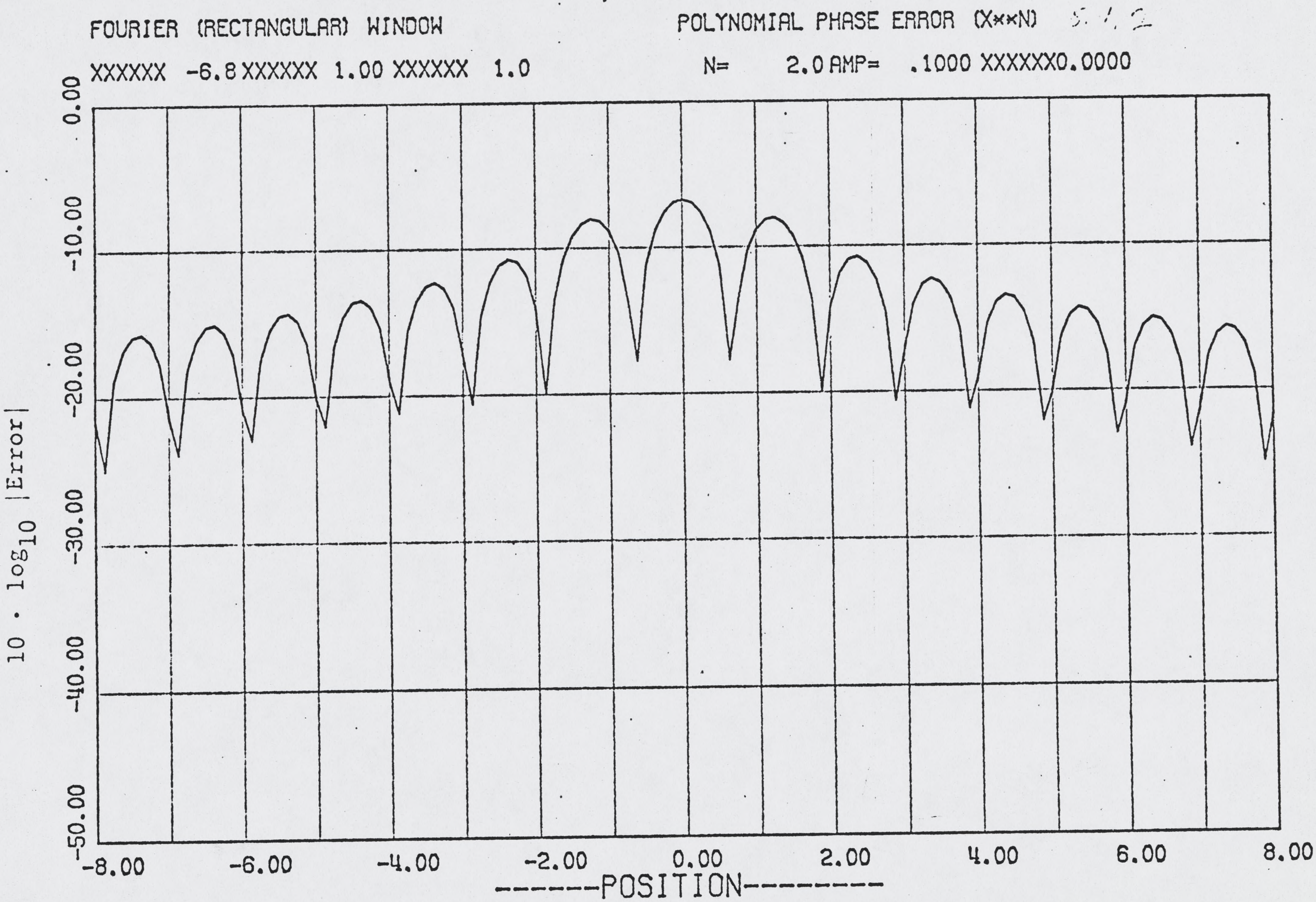

PLOT 11


FOURIER (RECTANGULAR) WINDOW
XXXXXX -0.9 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .5000 XXXXXX0.0000
10 · log10 |Error|
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------
PLOT 12


FOURIER (RECTANGULAR) WINDOW
XXXXXX -2.0 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .5000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Real Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
-------POSITION--------
PLOT 13

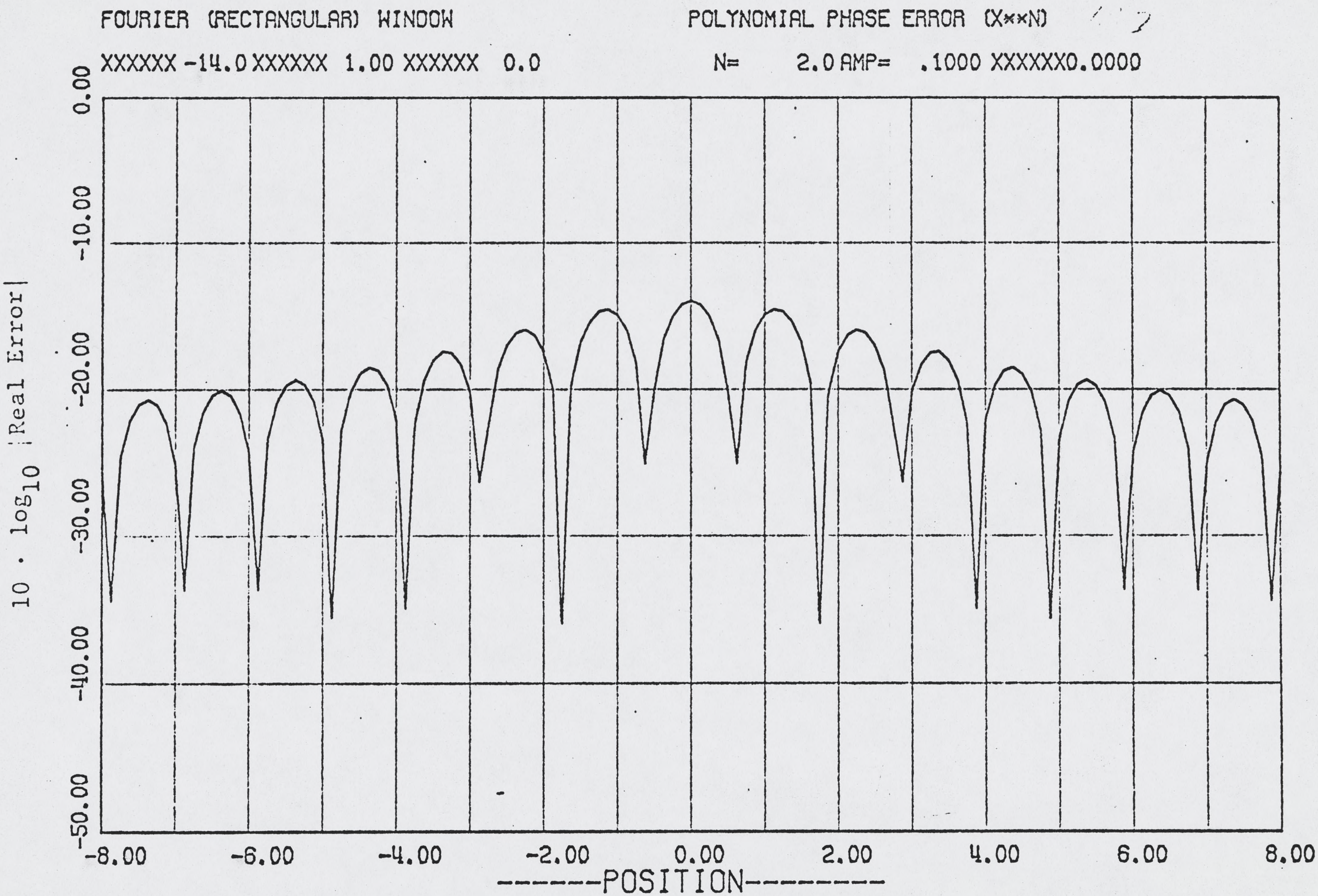

FOURIER (RECTANGULAR) WINDOW
XXXXXX -3.0 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .5000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Im Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 14

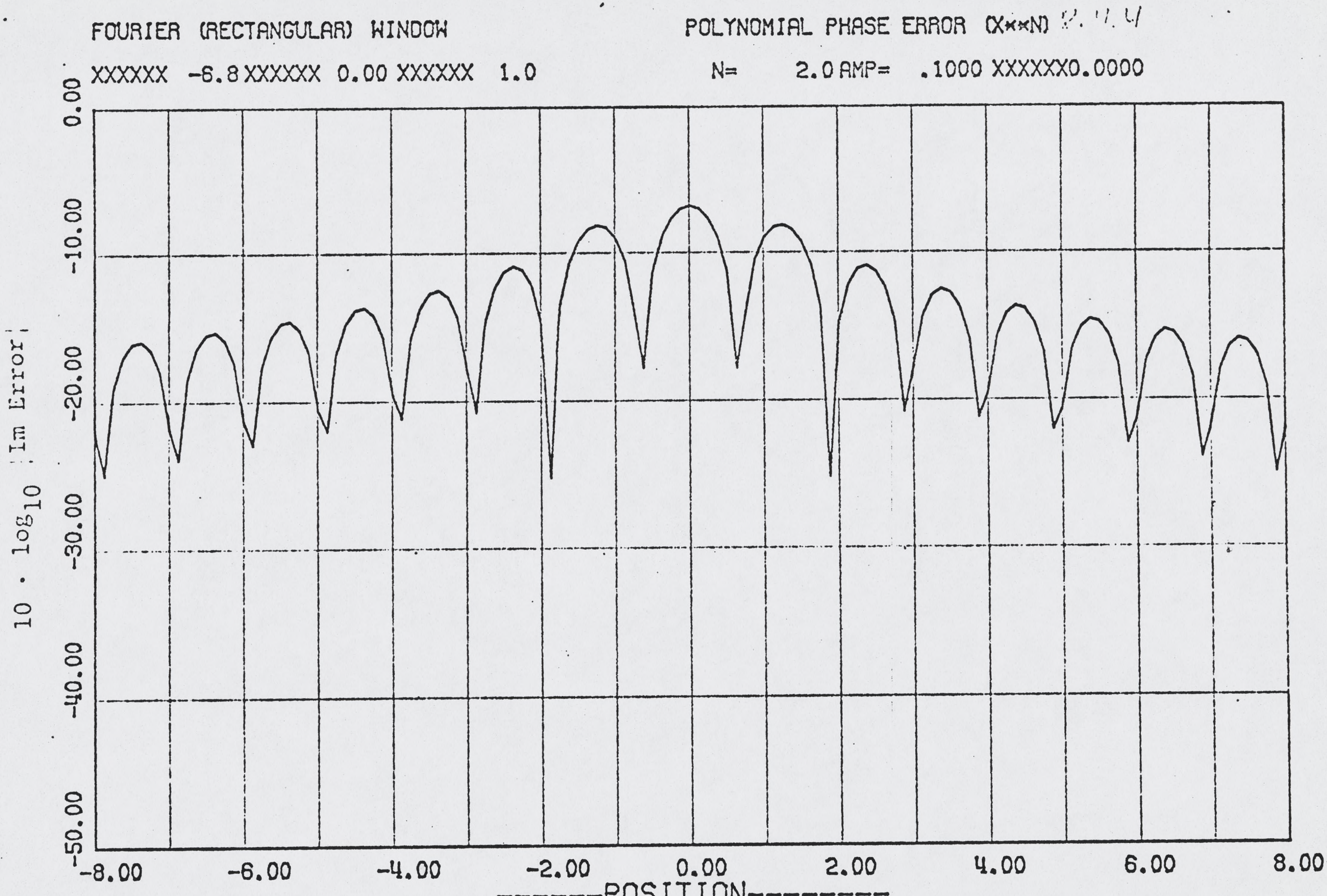


FOURIER (RECTANGULAR) WINDOW
XXXXXX -6.8 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------


PLOT 15

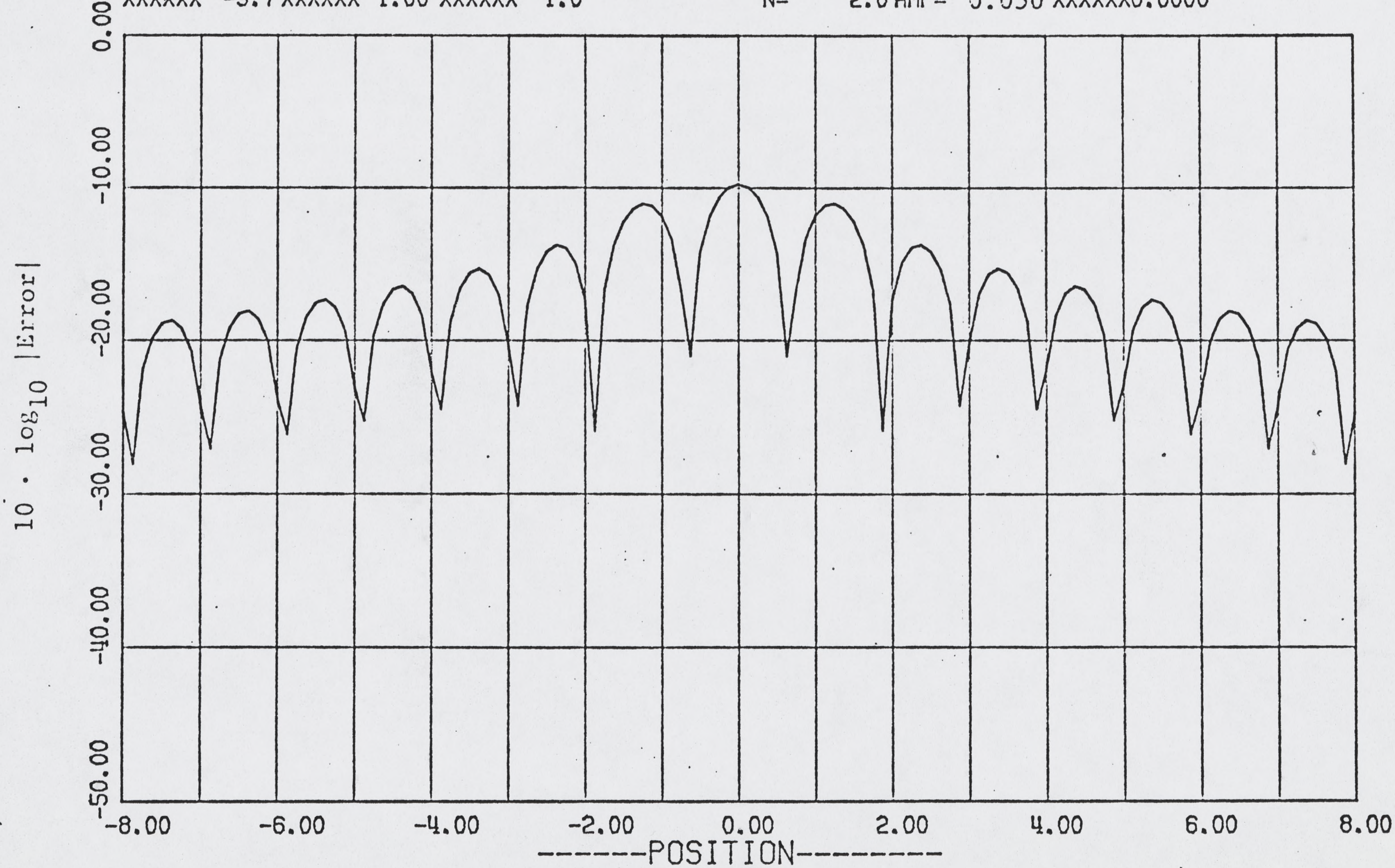

FOURIER (RECTANGULAR) WINDOW
XXXXXX -14.0 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Real Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 16

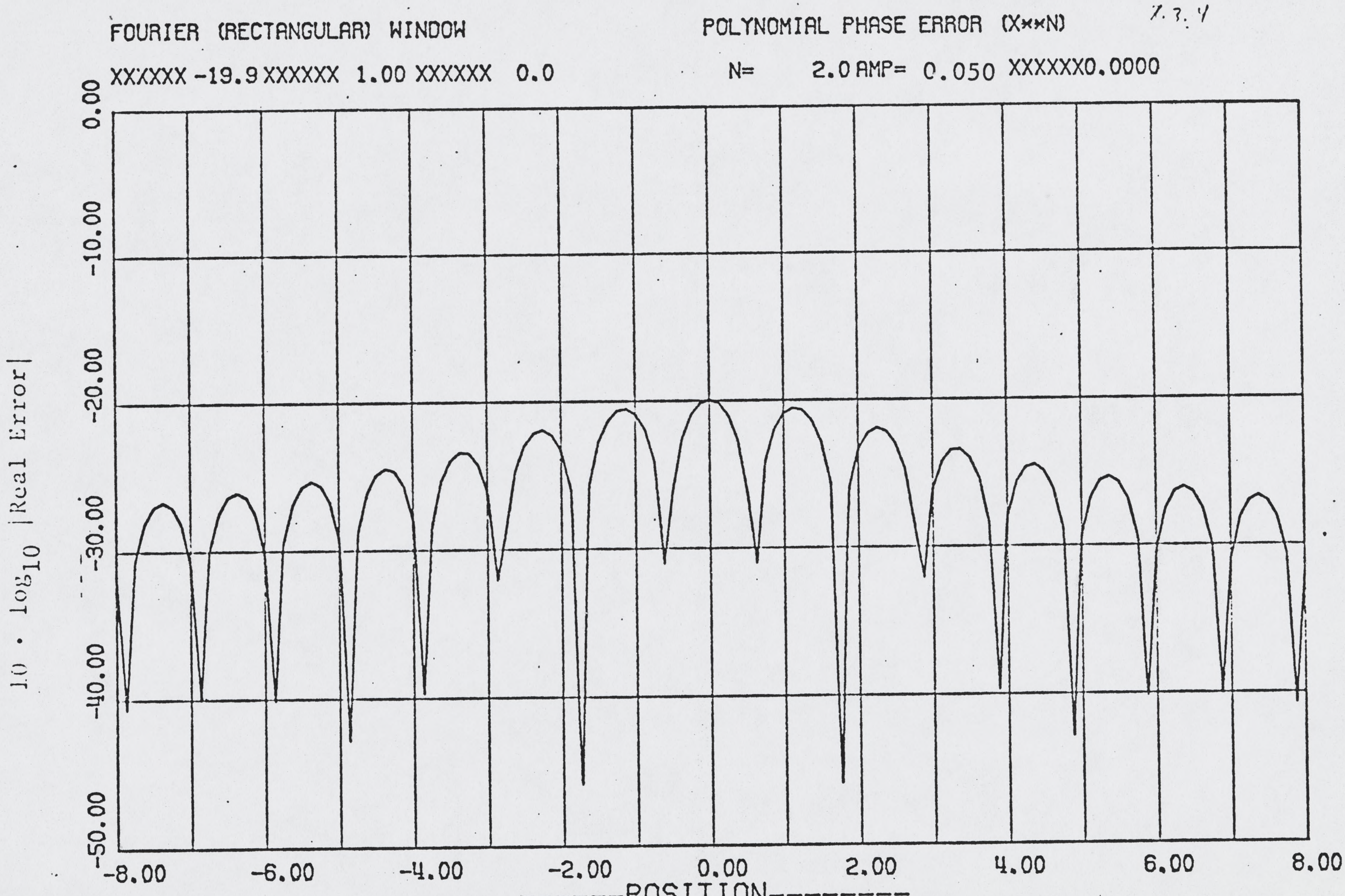

FOURIER (RECTANGULAR) WINDOW
XXXXXX -6.8 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Im Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------
PLOT 17

FOURIER (RECTANGULAR) WINDOW    POLYNOMIAL PHASE ERROR (X**N)    5.3.3

XXXXXX -9.7 XXXXXX 1.00 XXXXXX 1.0    N= 2.0 AMP= 0.050 XXXXXX0.0000




PLOT 18

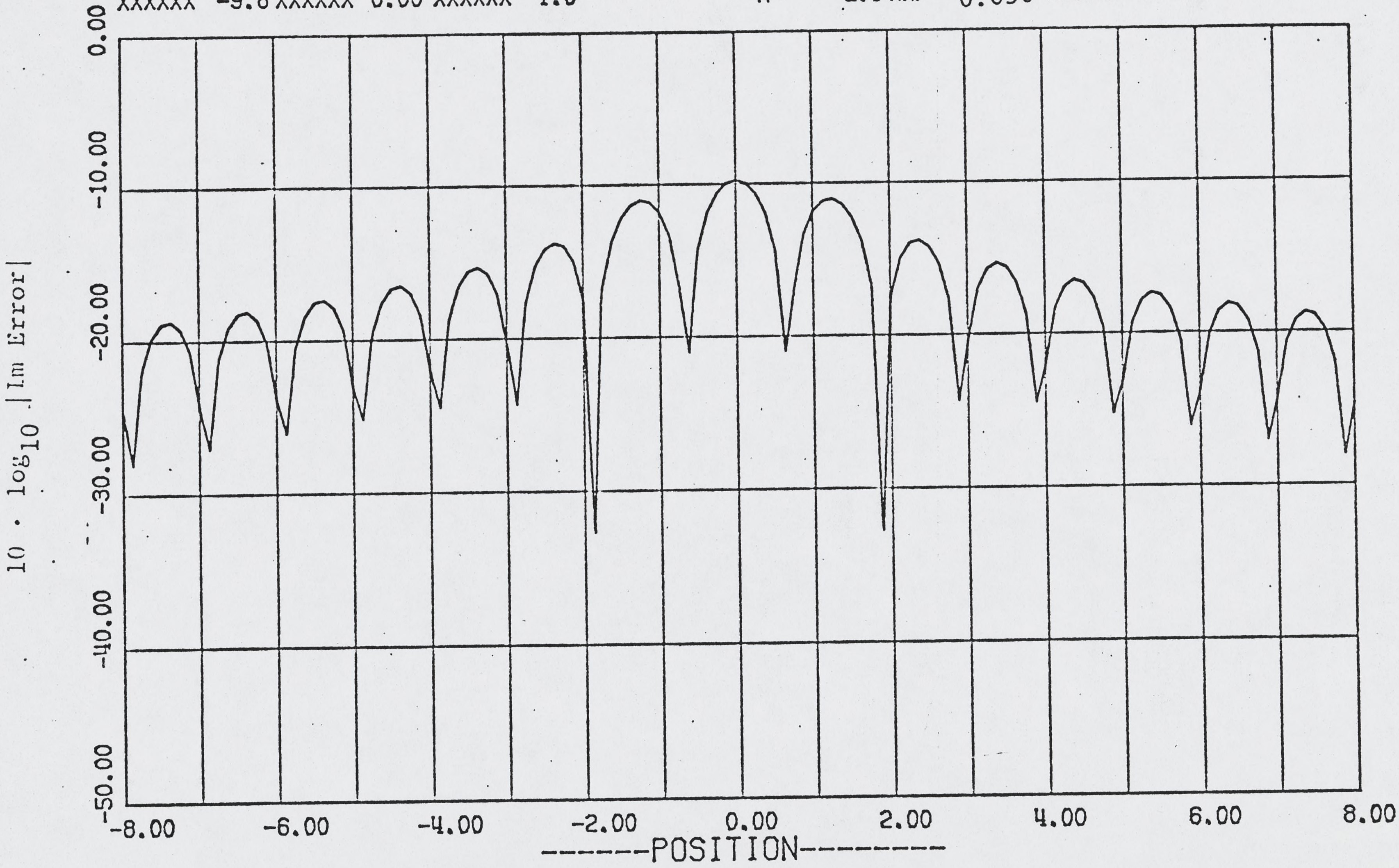

FOURIER (RECTANGULAR) WINDOW
XXXXXX -19.9 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= 0.050 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Real Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------
PLOT 19

FOURIER (RECTANGULAR) WINDOW

XXXXXX -9.8 XXXXXX 0.00 XXXXXX 1.0

POLYNOMIAL PHASE ERROR (X**N)

N= 2.0 AMP= 0.050 XXXXXX0.0000



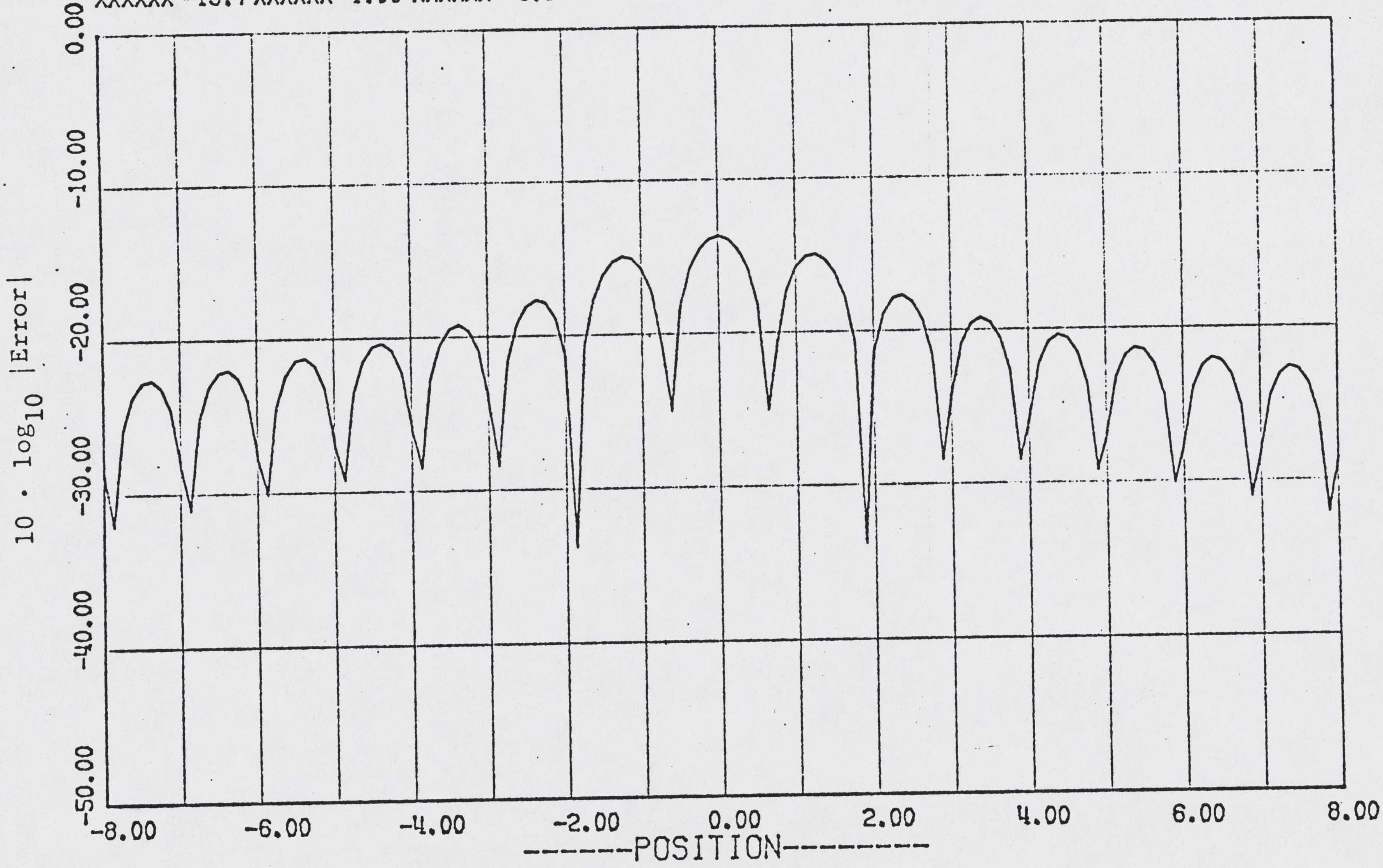

PLOT 20

FOURIER (RECTANGULAR) WINDOW

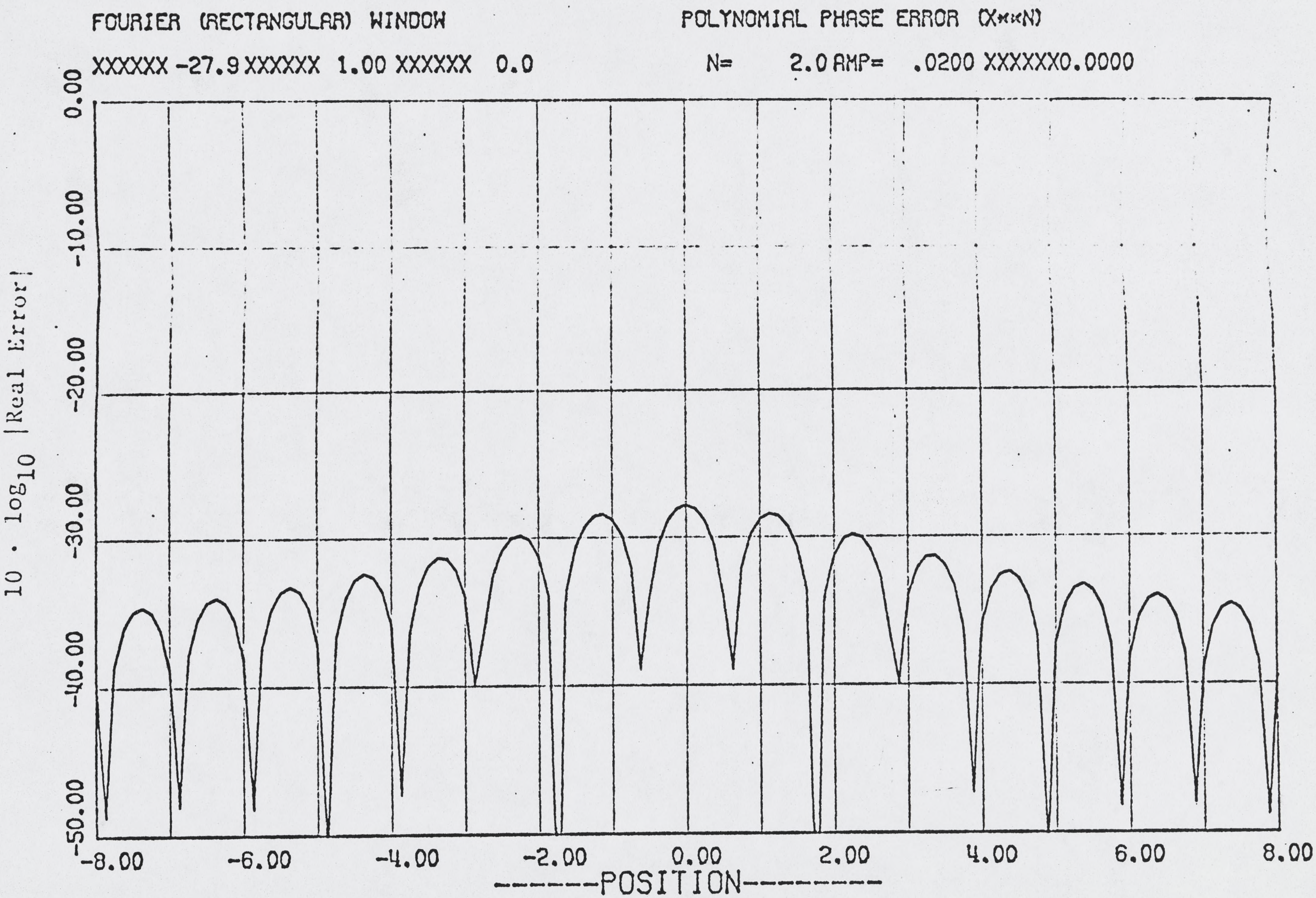
XXXXXX -13.7 XXXXXX 1.00 XXXXXX 1.0

POLYNOMIAL PHASE ERROR (X**N)

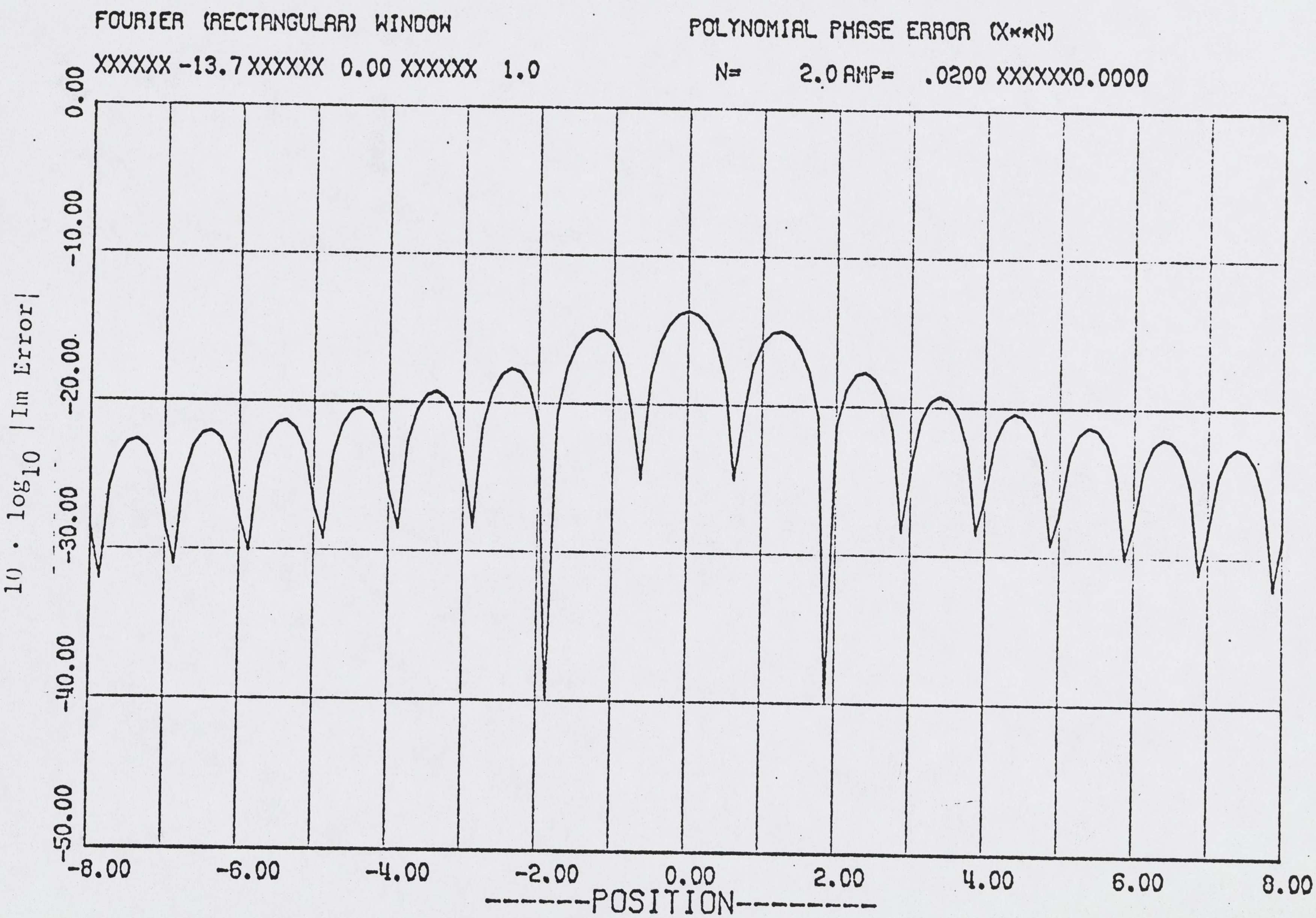
N= 2.0 AMP= .0200 XXXXXX0.0000




PLOT 21


FOURIER (RECTANGULAR) WINDOW
XXXXXX -27.9 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .0200 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Real Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 22


FOURIER (RECTANGULAR) WINDOW
XXXXXX -13.7 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .0200 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Im Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
-------POSITION---------

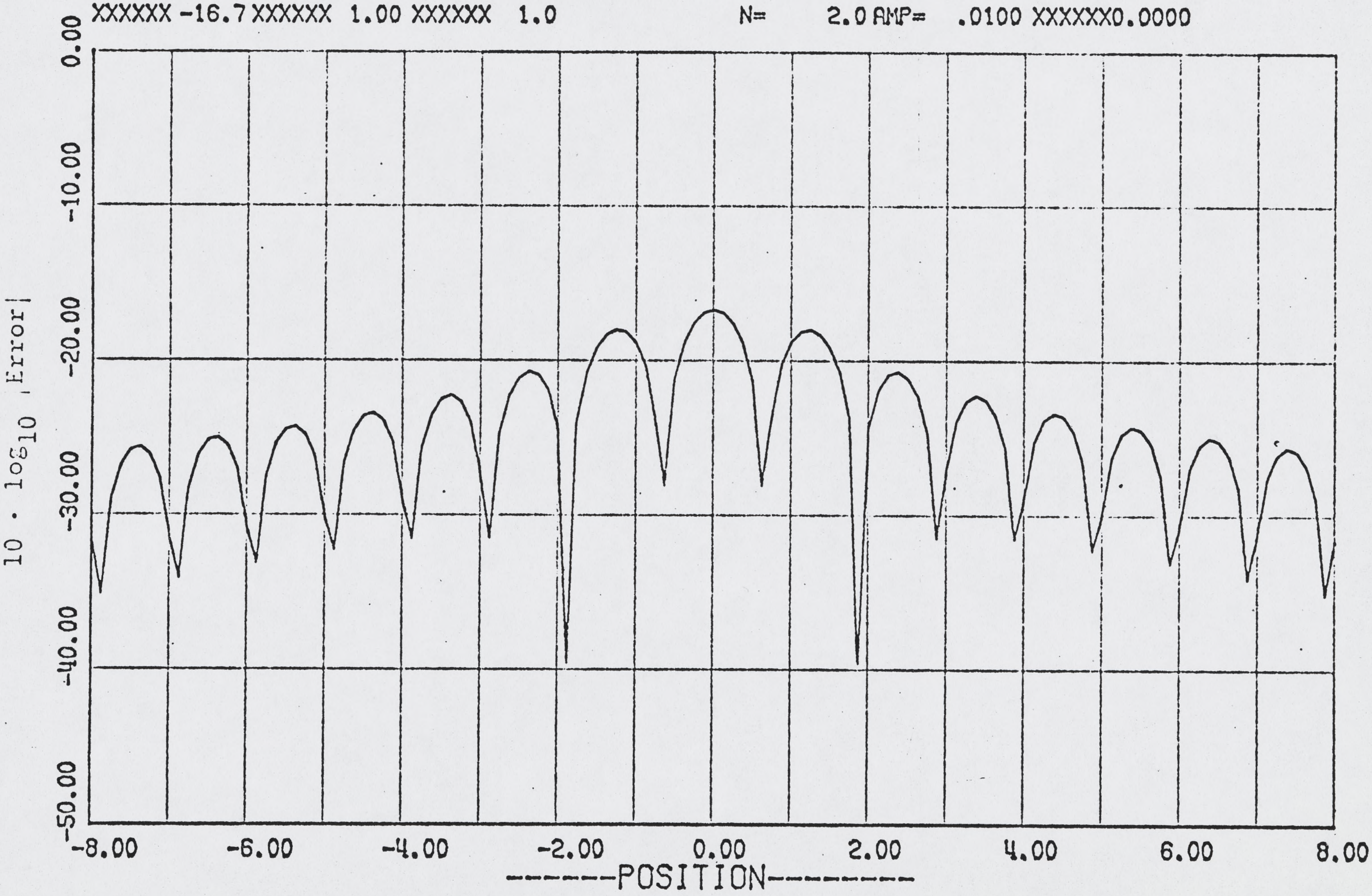
PLOT 23

FOURIER (RECTANGULAR) WINDOW

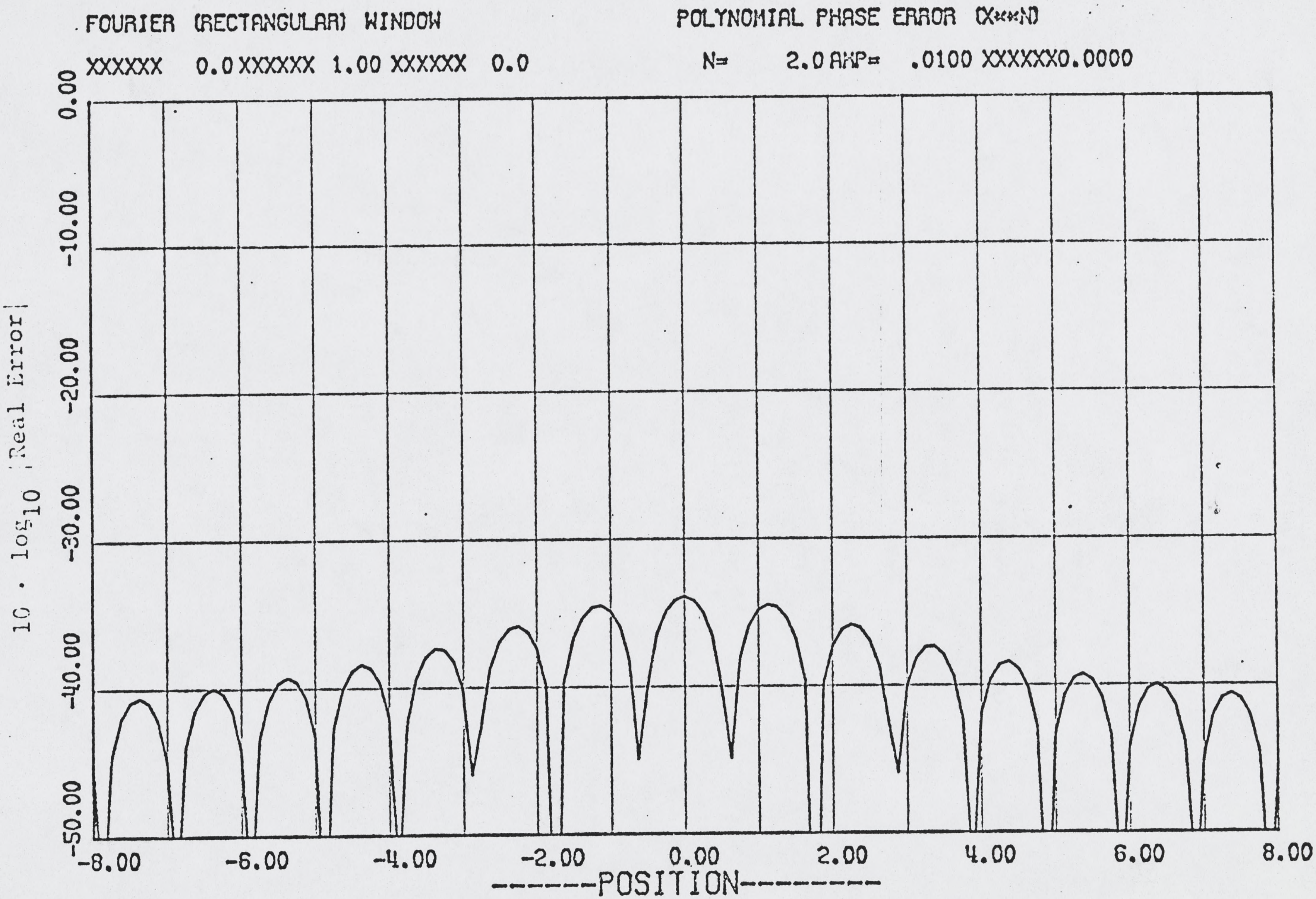
XXXXXX -16.7 XXXXXX 1.00 XXXXXX 1.0

POLYNOMIAL PHASE ERROR (X**N)

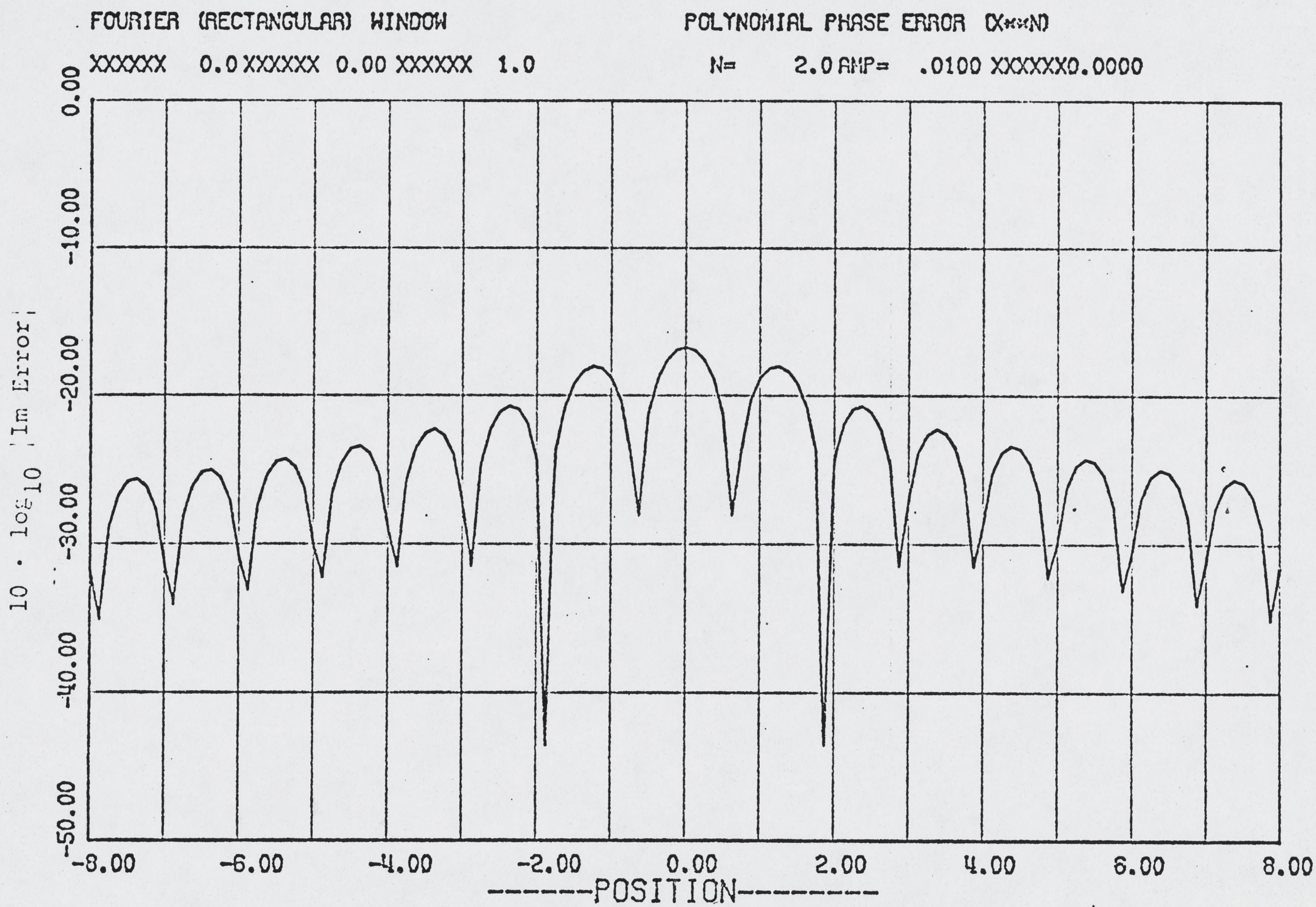
N= 2.0 AMP= .0100 XXXXXX0.0000




PLOT 24

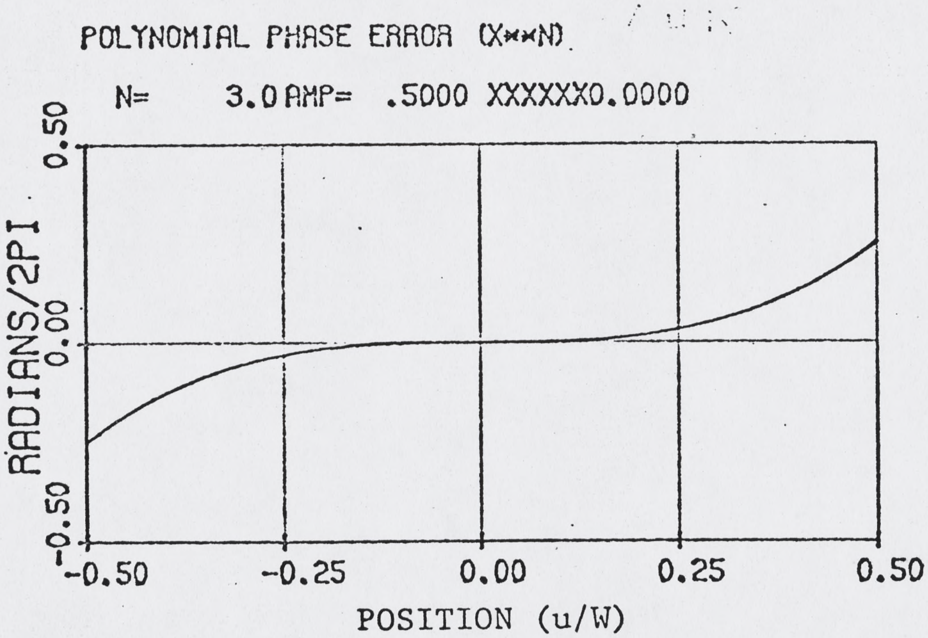

FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .0100 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Real Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 25

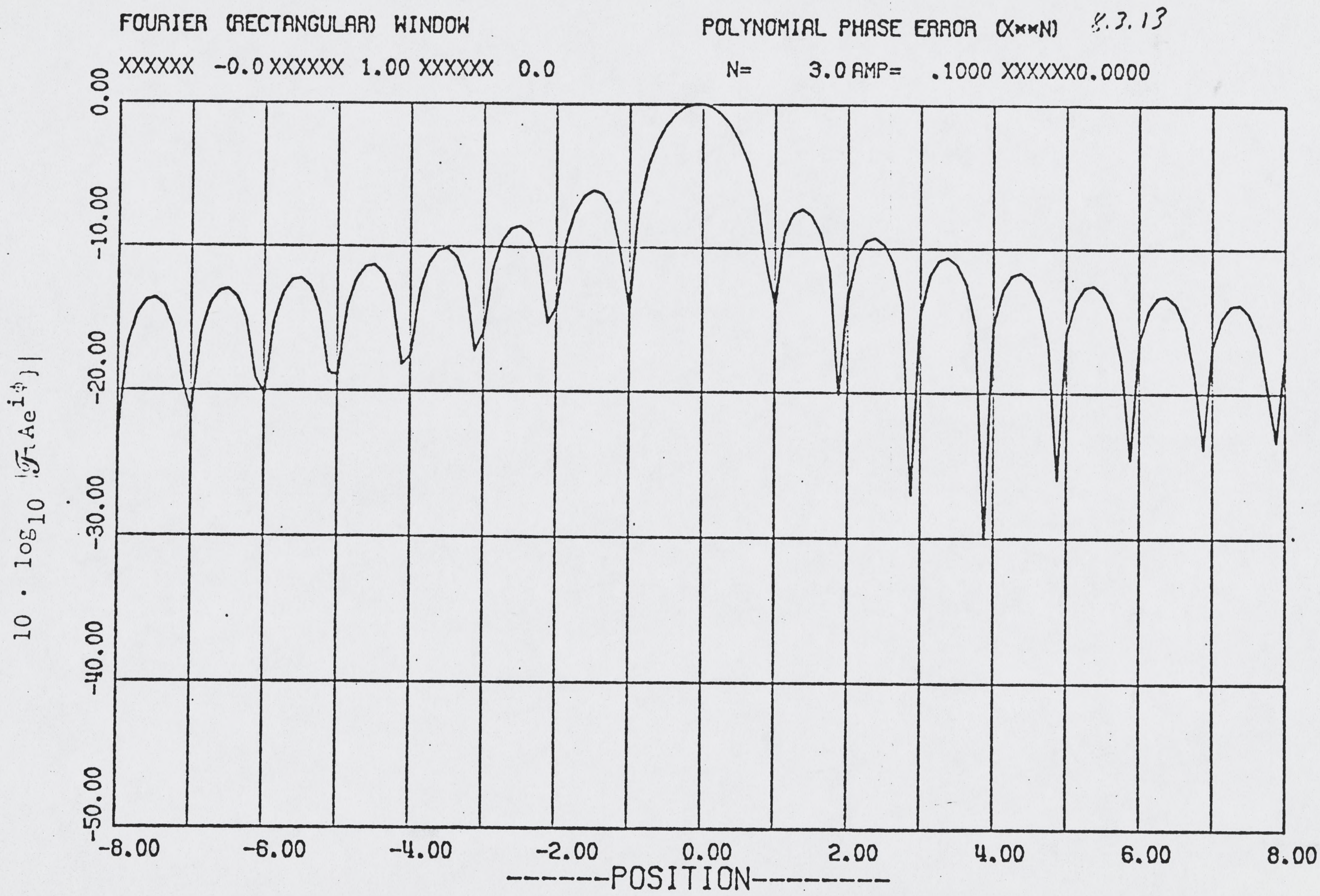


FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .0100 XXXXXX0.0000
10 · log10 |Im Error|
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

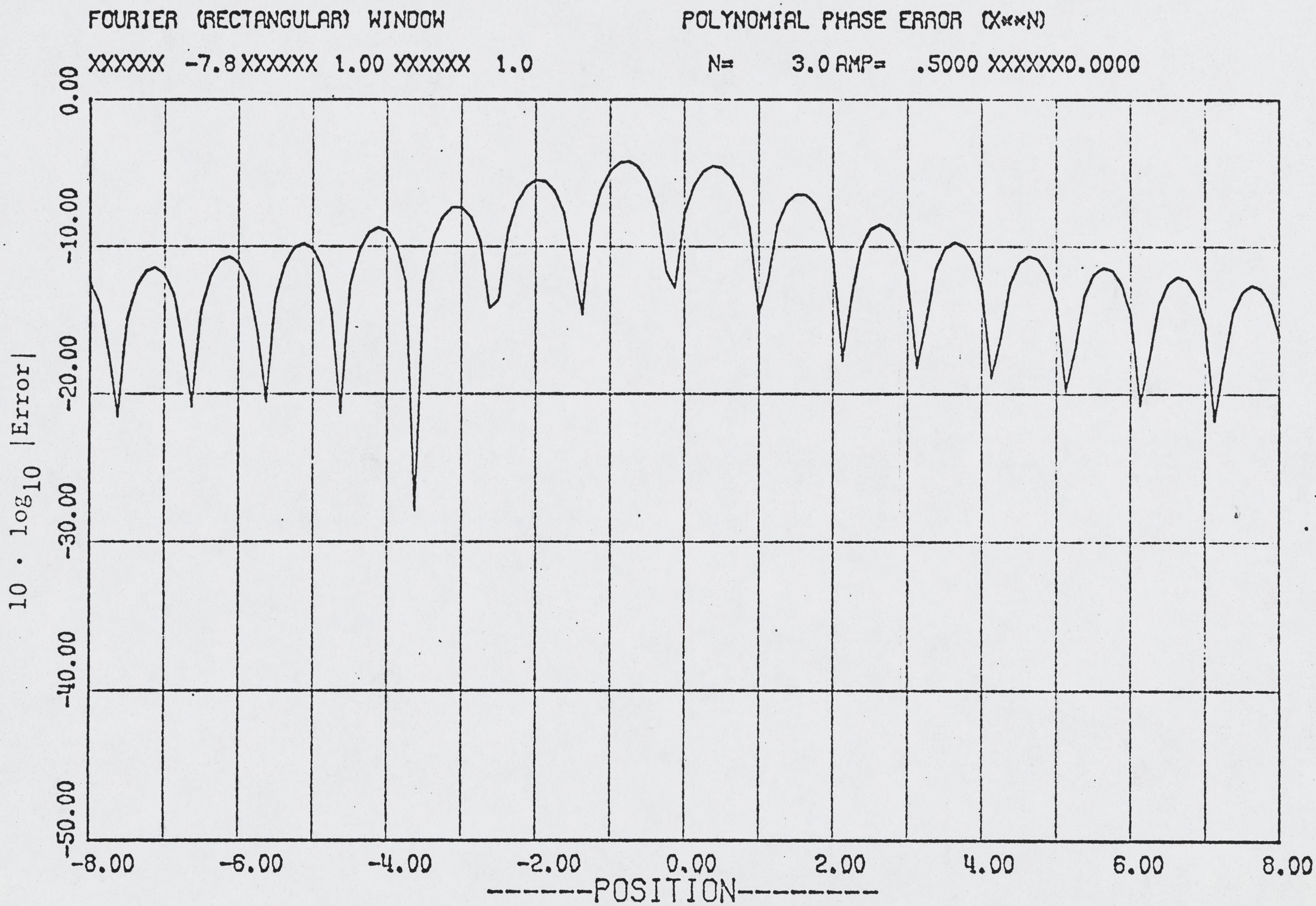

PLOT 26


POLYNOMIAL PHASE ERROR (X**N)
N= 3.0 AMP= .5000 XXXXXX0.0000
RADIANS/2PI
0.50
0.00
-0.50
-0.50
-0.25
0.00
0.25
0.50
POSITION (u/W)

PLOT 27


FOURIER (RECTANGULAR) WINDOW
XXXXXX -0.0 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
8.3.13
N= 3.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 28

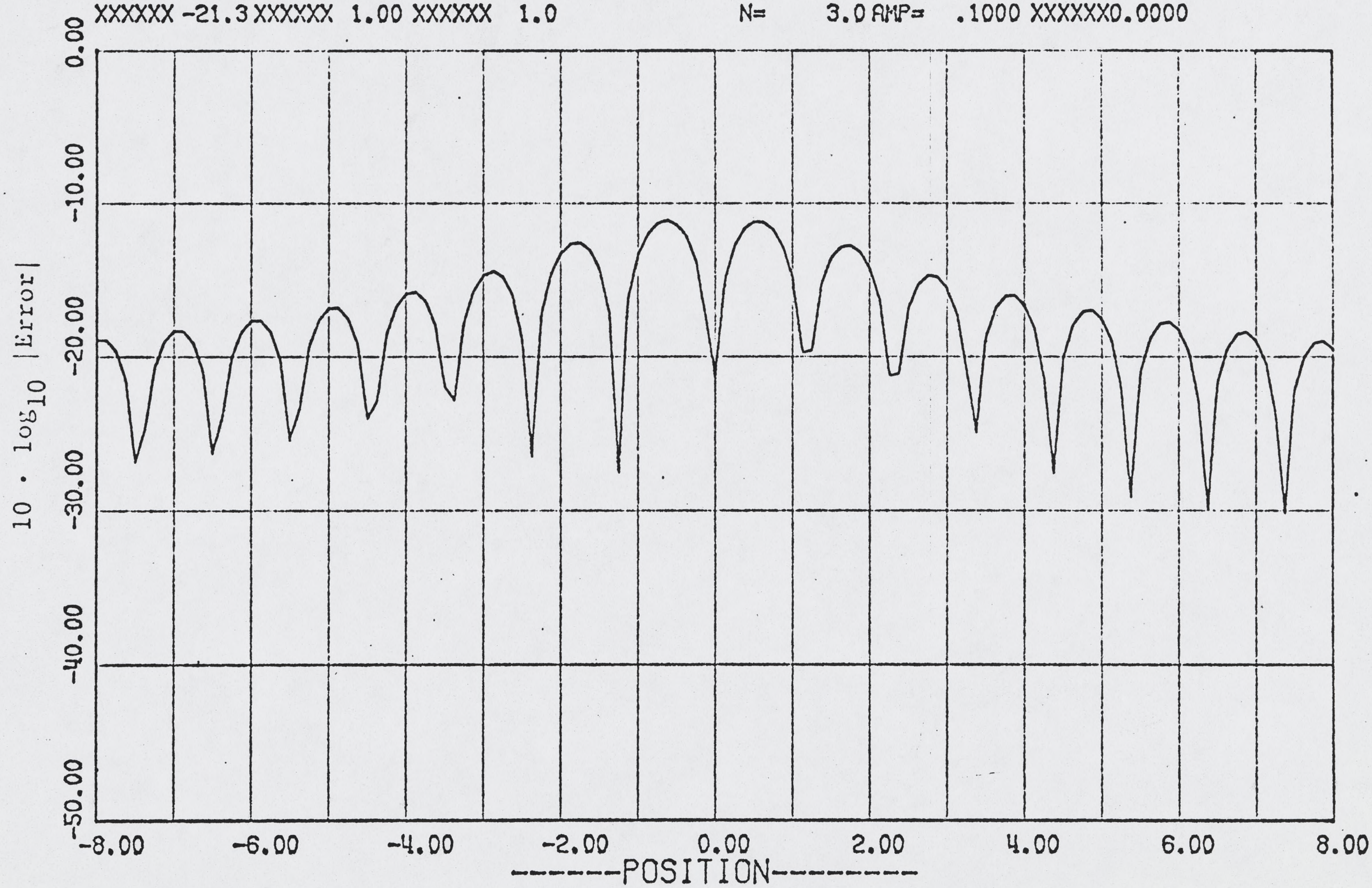

FOURIER (RECTANGULAR) WINDOW
XXXXXX -7.8 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 3.0 AMP= .5000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
$10 \cdot \log_{10} |Error|$
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------
PLOT 29

FOURIER (RECTANGULAR) WINDOW

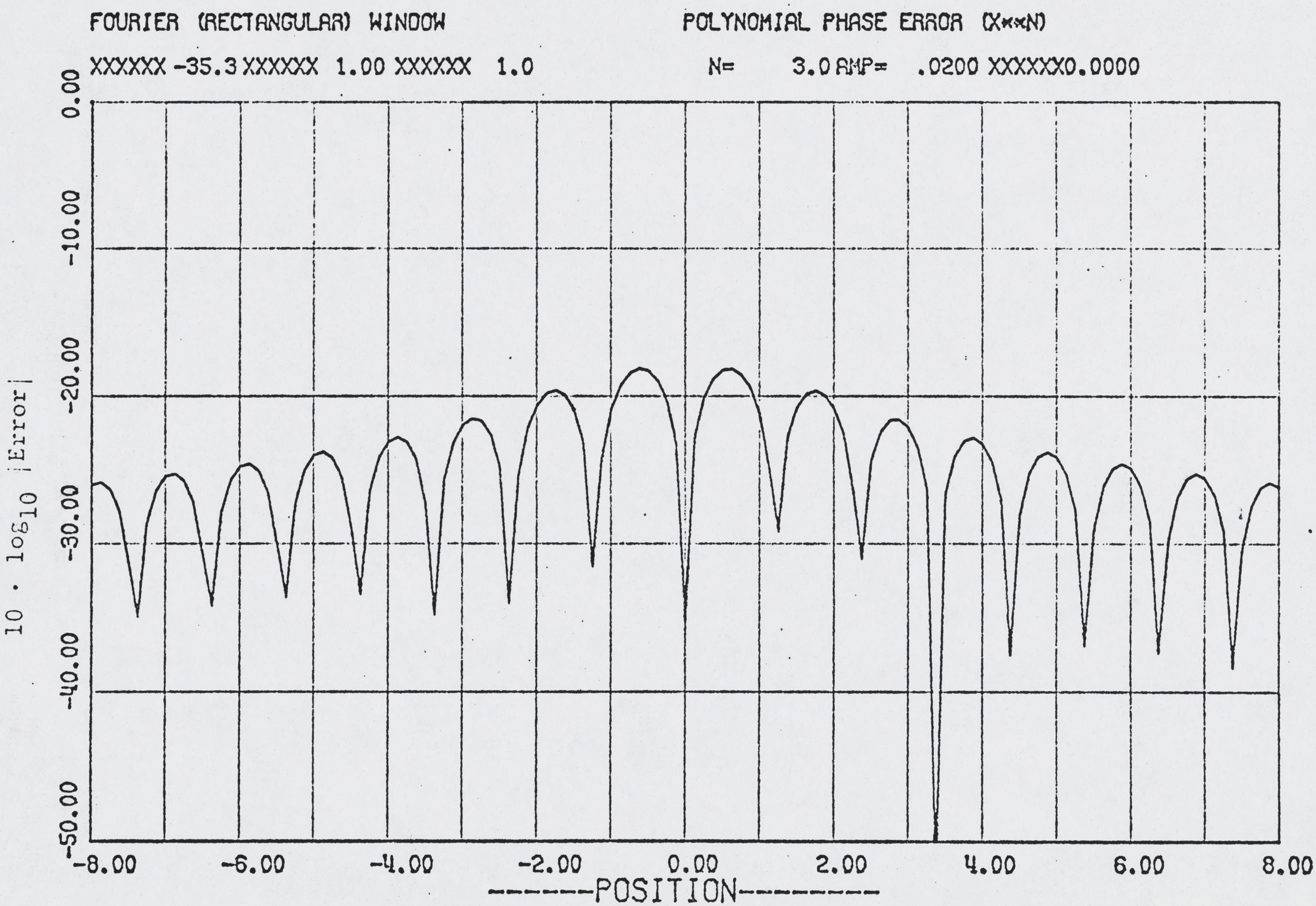
XXXXXX -21.3 XXXXXX 1.00 XXXXXX 1.0

POLYNOMIAL PHASE ERROR (X**N)

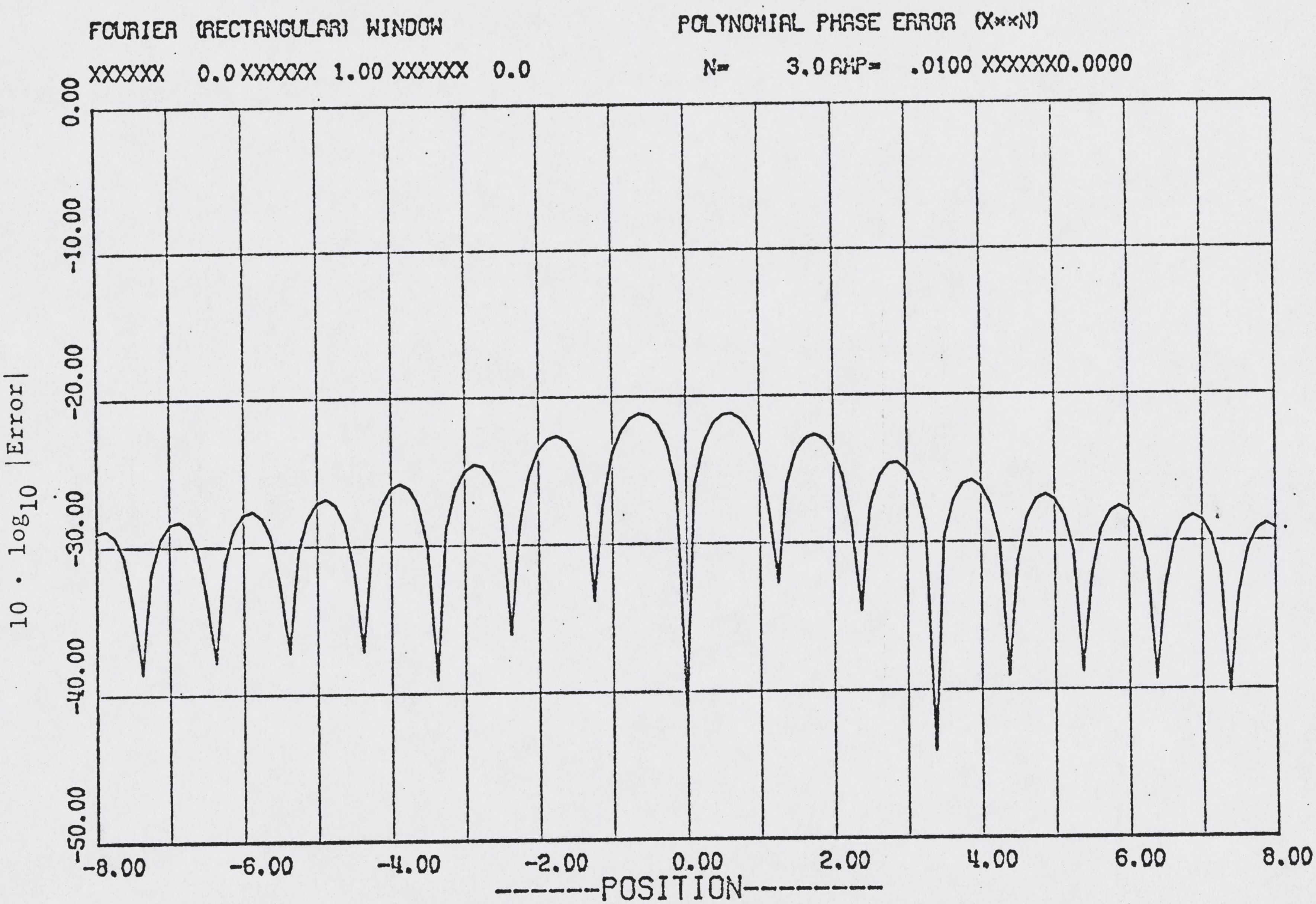
N= 3.0 AMP= .1000 XXXXXX0.0000

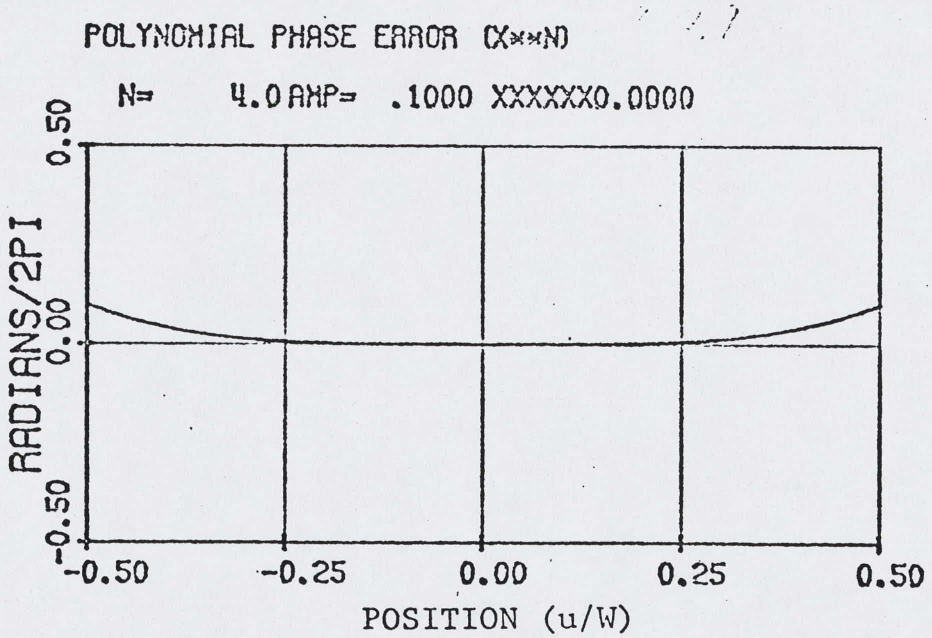



PLOT 30

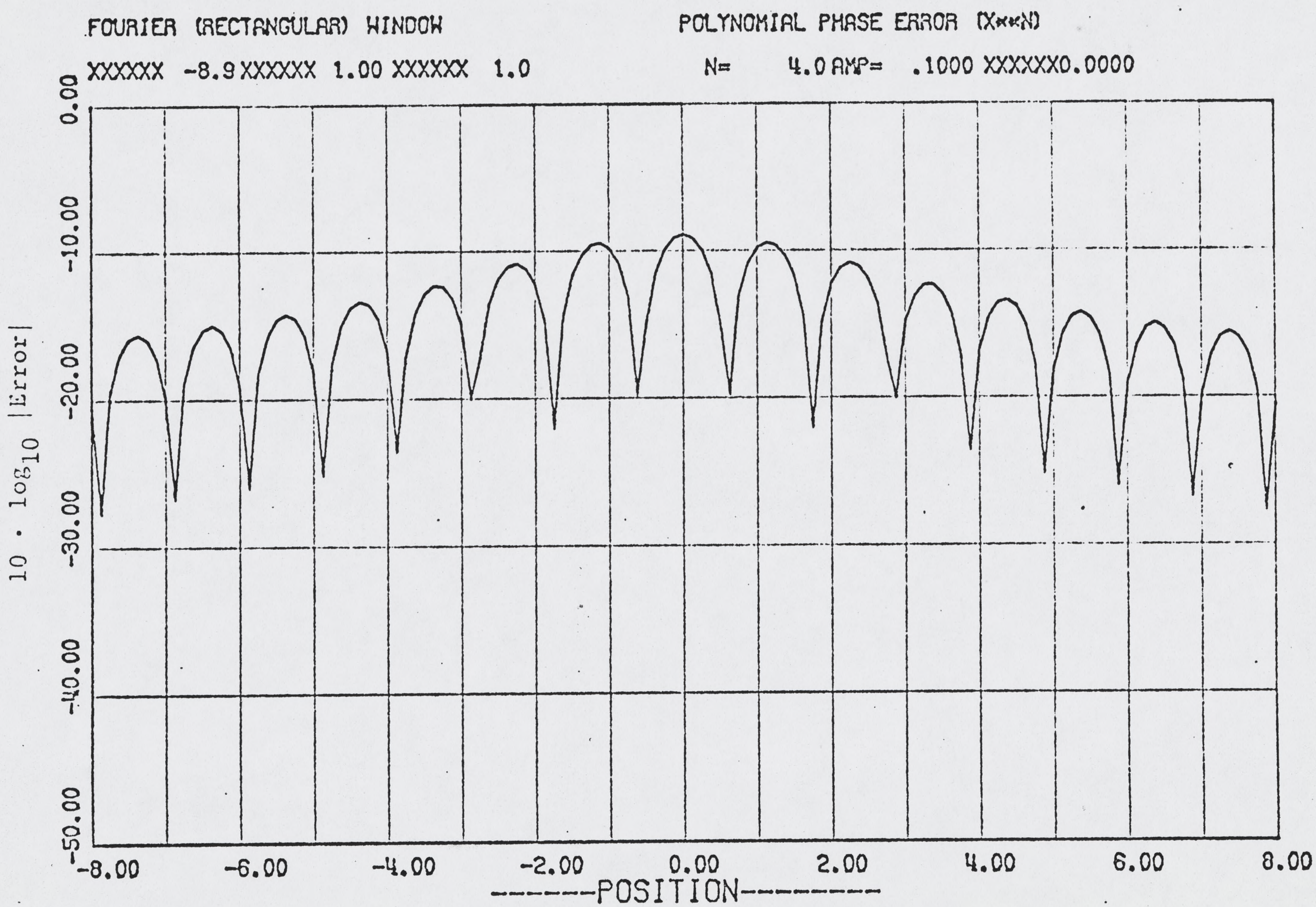

FOURIER (RECTANGULAR) WINDOW
XXXXXX -35.3 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 3.0 AMP= .0200 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 31

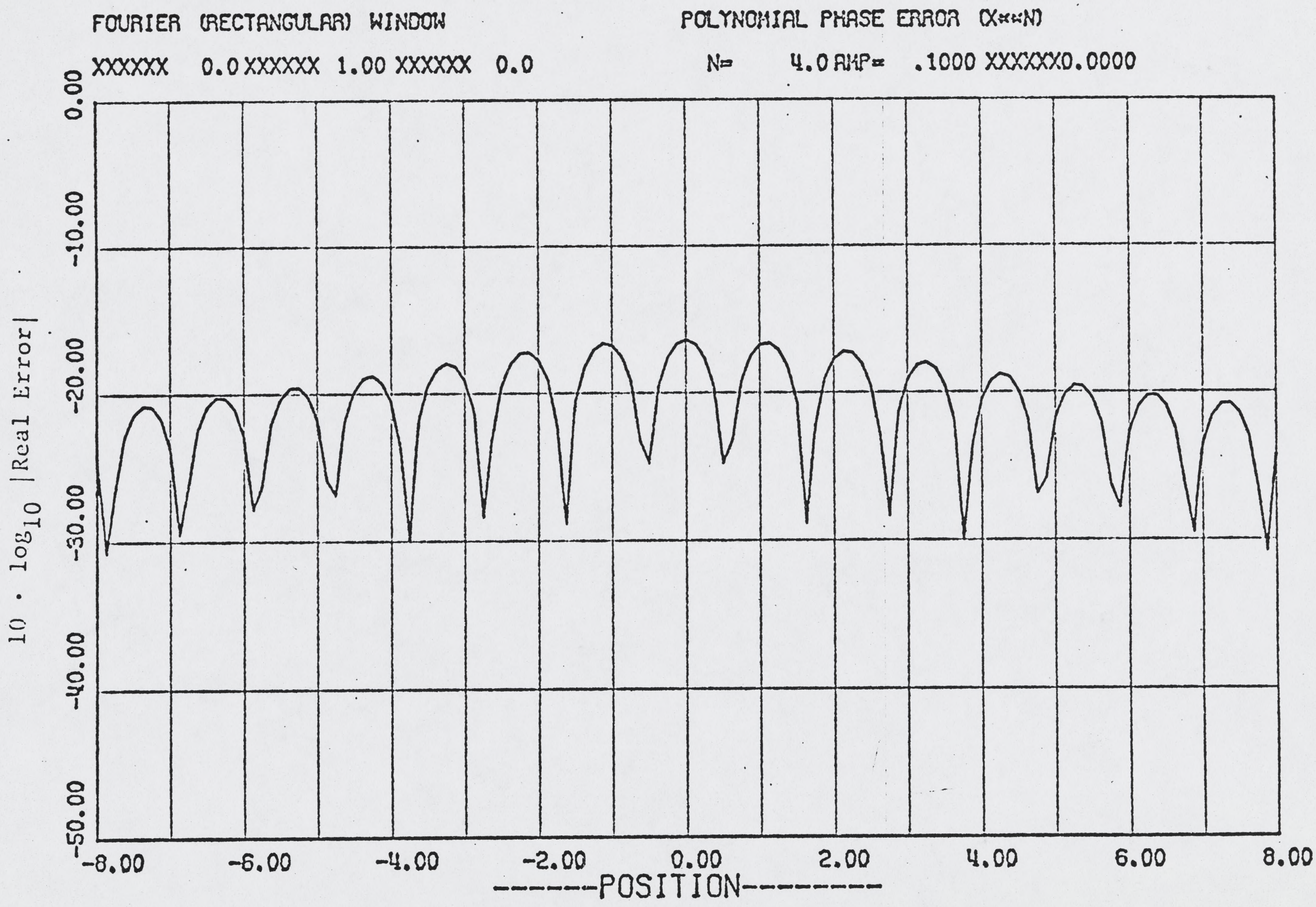

FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 3.0 AMP= .0100 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------
PLOT 32


POLYNOMIAL PHASE ERROR (X**N)
N= 4.0 AMP= .1000 XXXXXX0.0000
RADIANS/2PI
0.50
0.00
-0.50
-0.50
-0.25
0.00
0.25
0.50
POSITION (u/W)

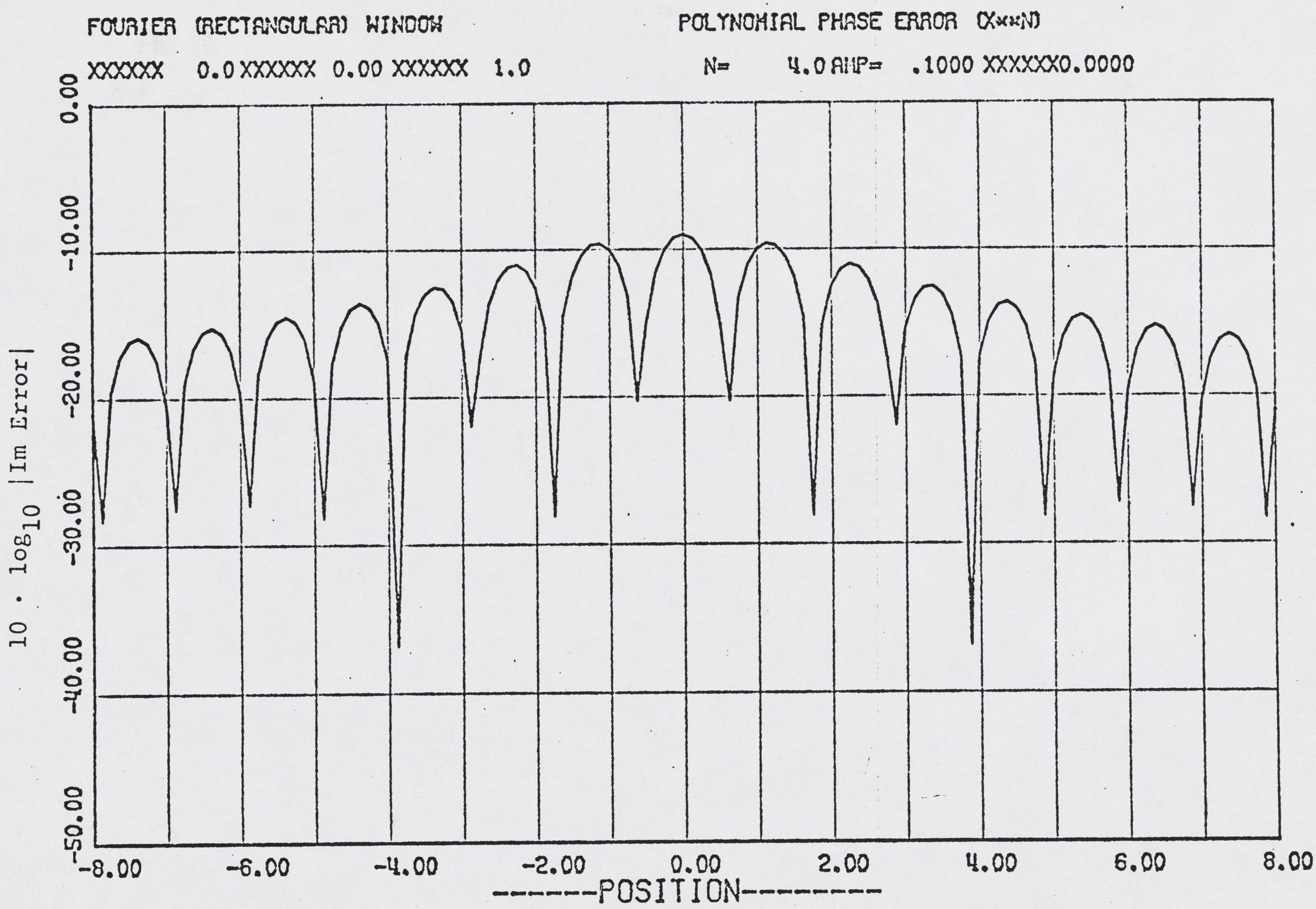
PLOT 33

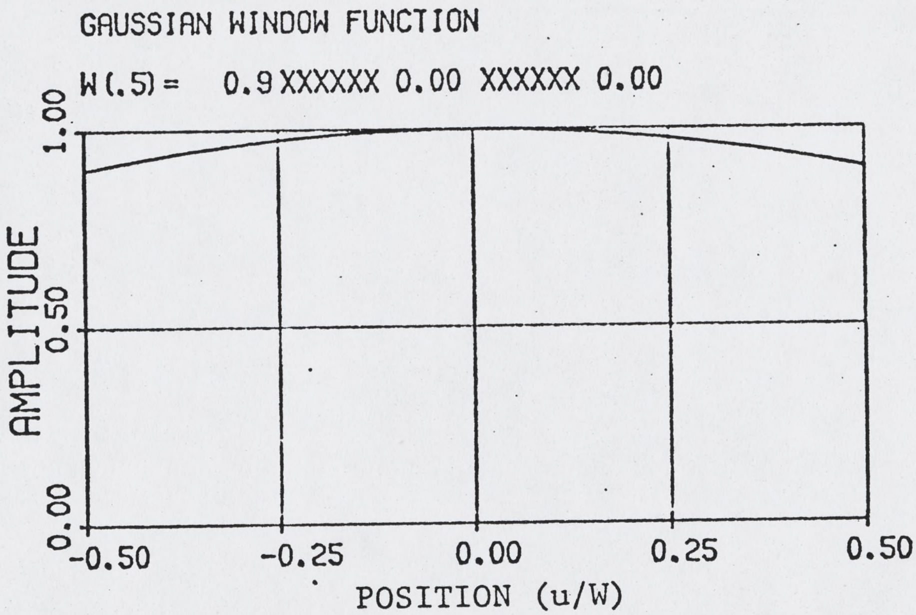

FOURIER (RECTANGULAR) WINDOW
XXXXXX -8.9 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 4.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION-------

PLOT 34


FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 4.0 AMP= .1000 XXXXXX0.0000
10 · log10 |Real Error|
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

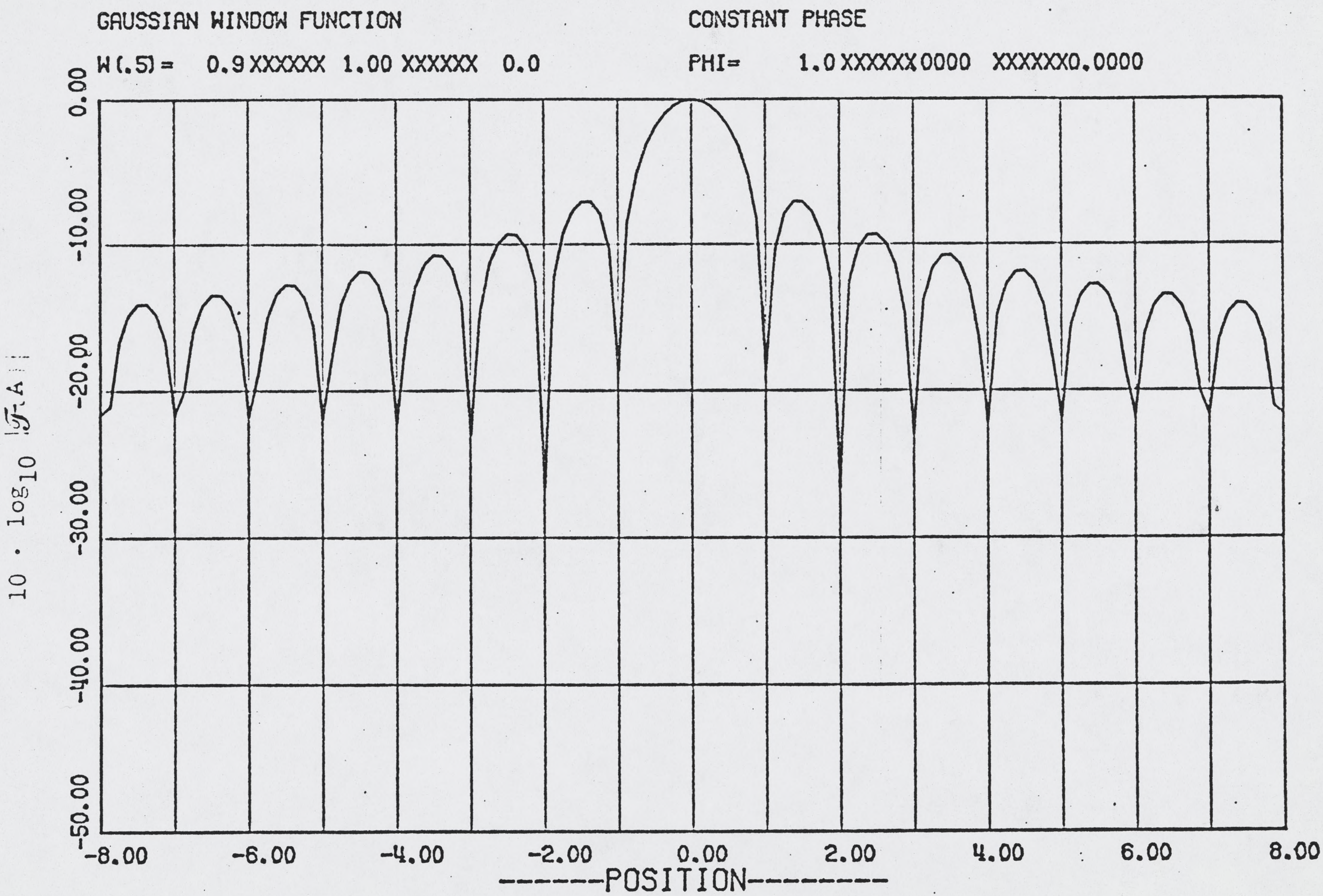
PLOT 35


FOURIER (RECTANGULAR) WINDOW
XXXXXX 0.0 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 4.0 AMP= .1000 XXXXXX0.0000
10 · $\log_{10}$ |Im Error|
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------
PLOT 36

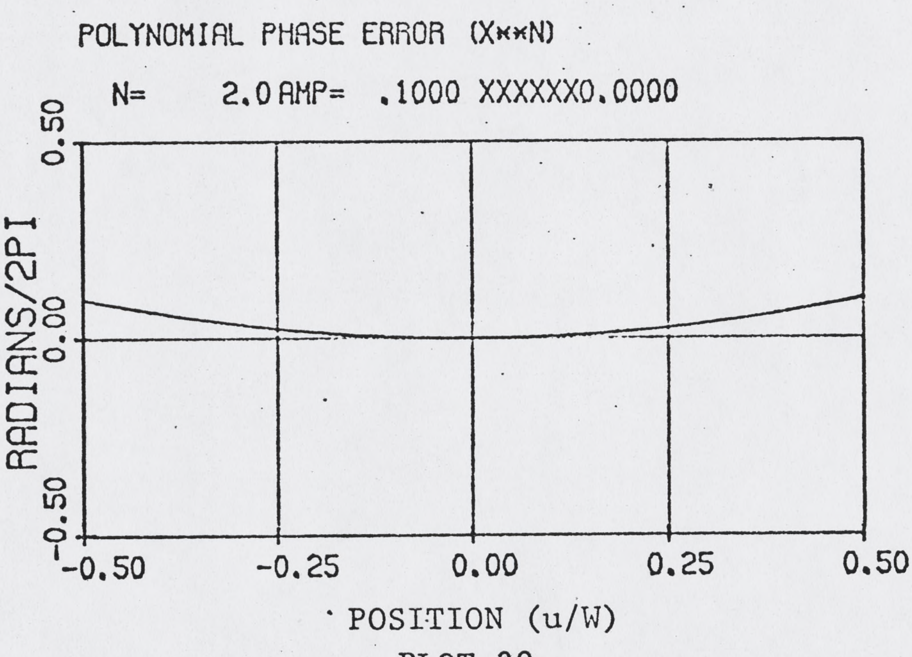

GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 0.00 XXXXXX 0.00
AMPLITUDE
1.00
0.50
0.00
-0.50
-0.25
0.00
0.25
0.50
POSITION (u/W)

PLOT 37

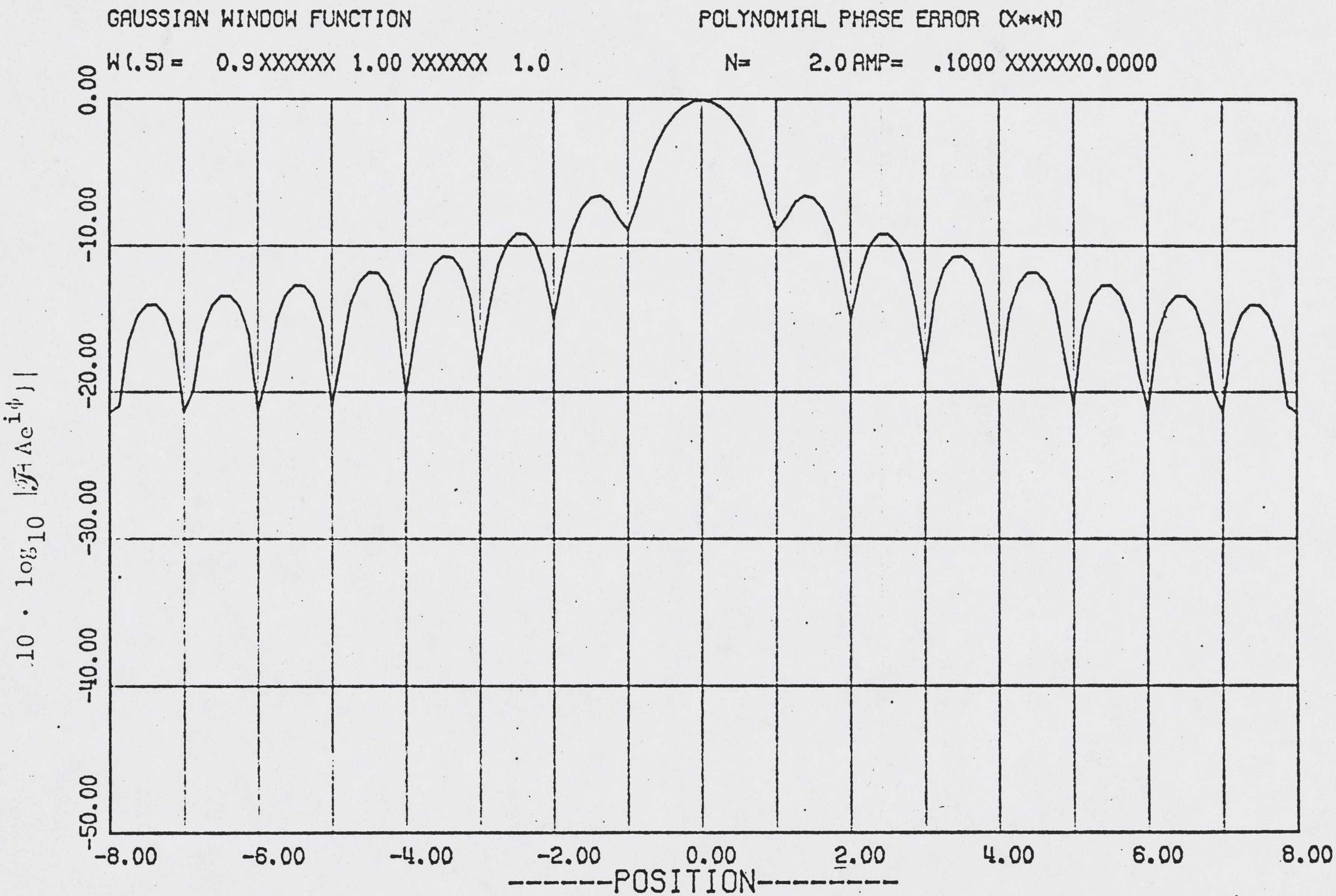
8.36.7


GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 1.00 XXXXXX 0.0
CONSTANT PHASE
PHI= 1.0 XXXXXX 0000 XXXXXX 0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |𝓕 A|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 38

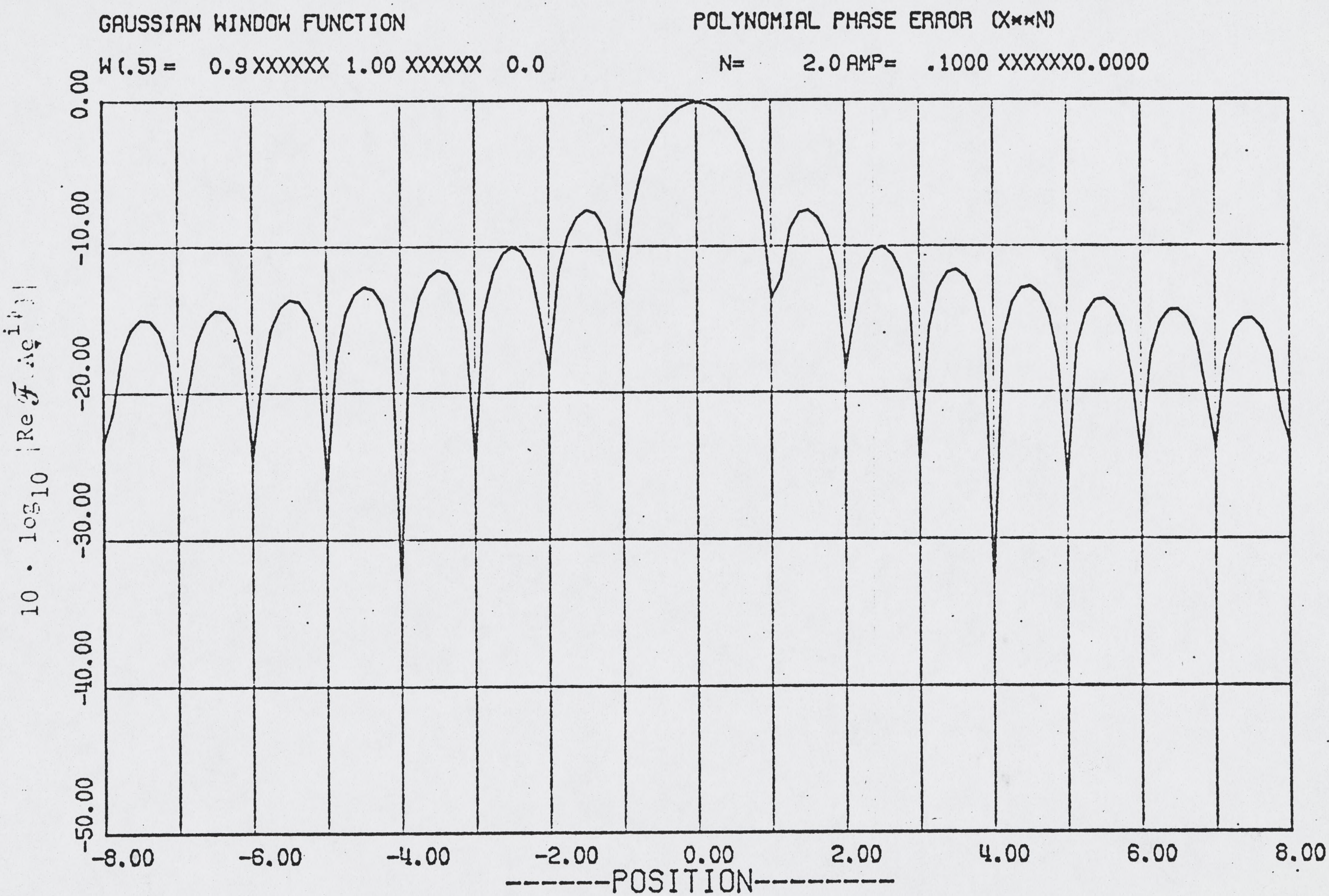

POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.50
RADIANS/2PI
0.00
-0.50
-0.50
-0.25
0.00
0.25
0.50
POSITION (u/W)

PLOT 39



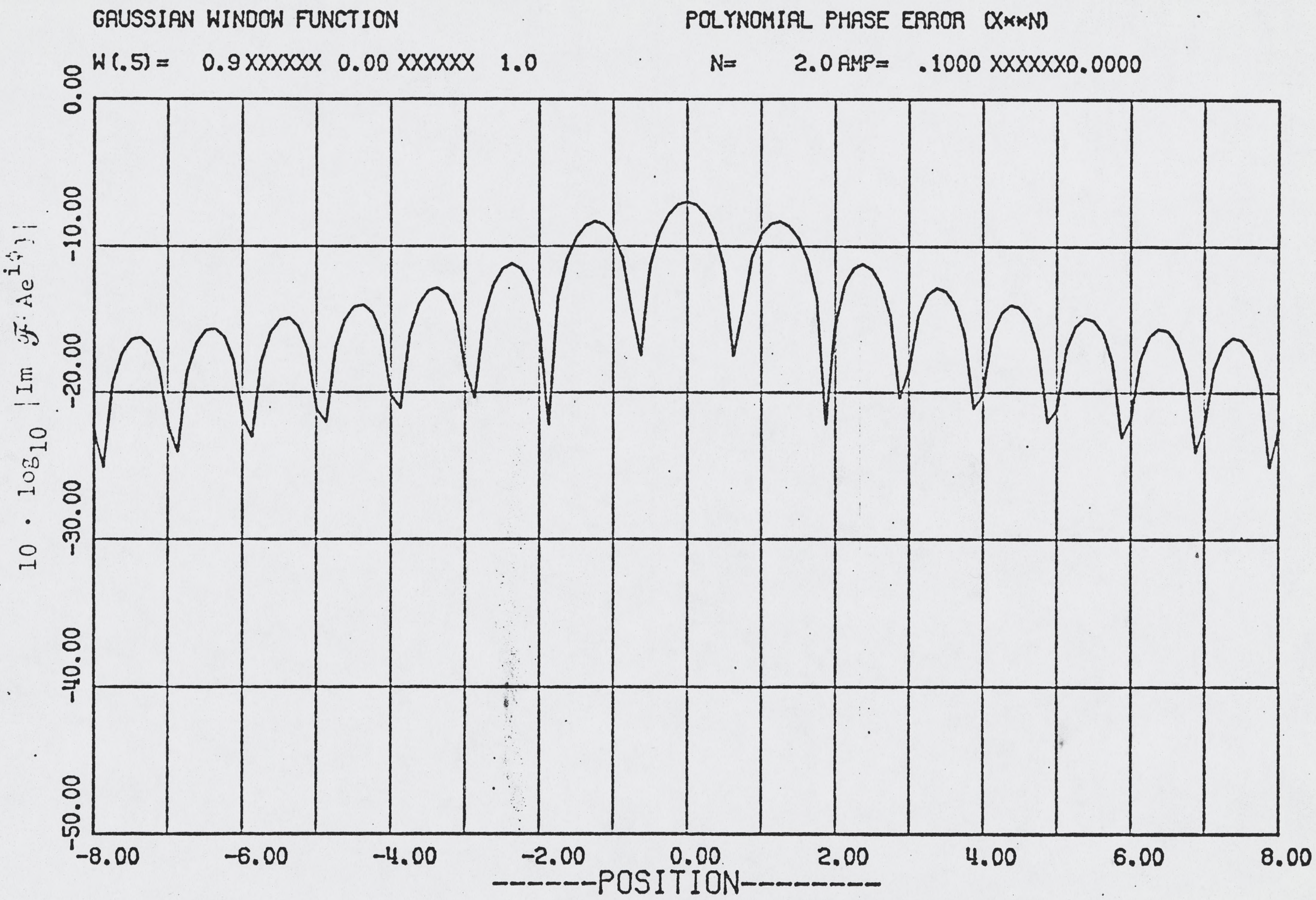
GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |ℱ Ae^{iφ}||
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------


PLOT 40


GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

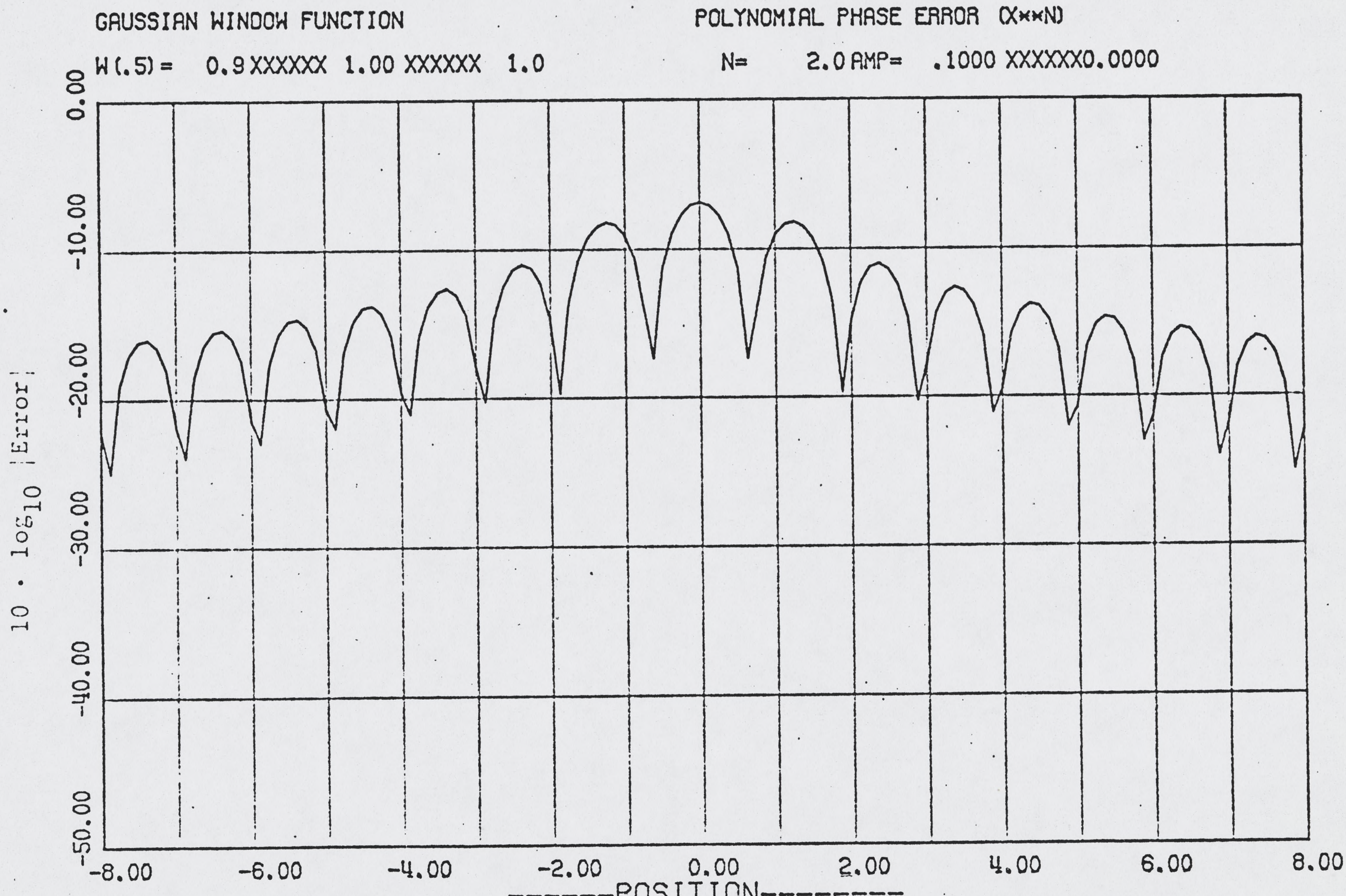
PLOT 41

5-36.5

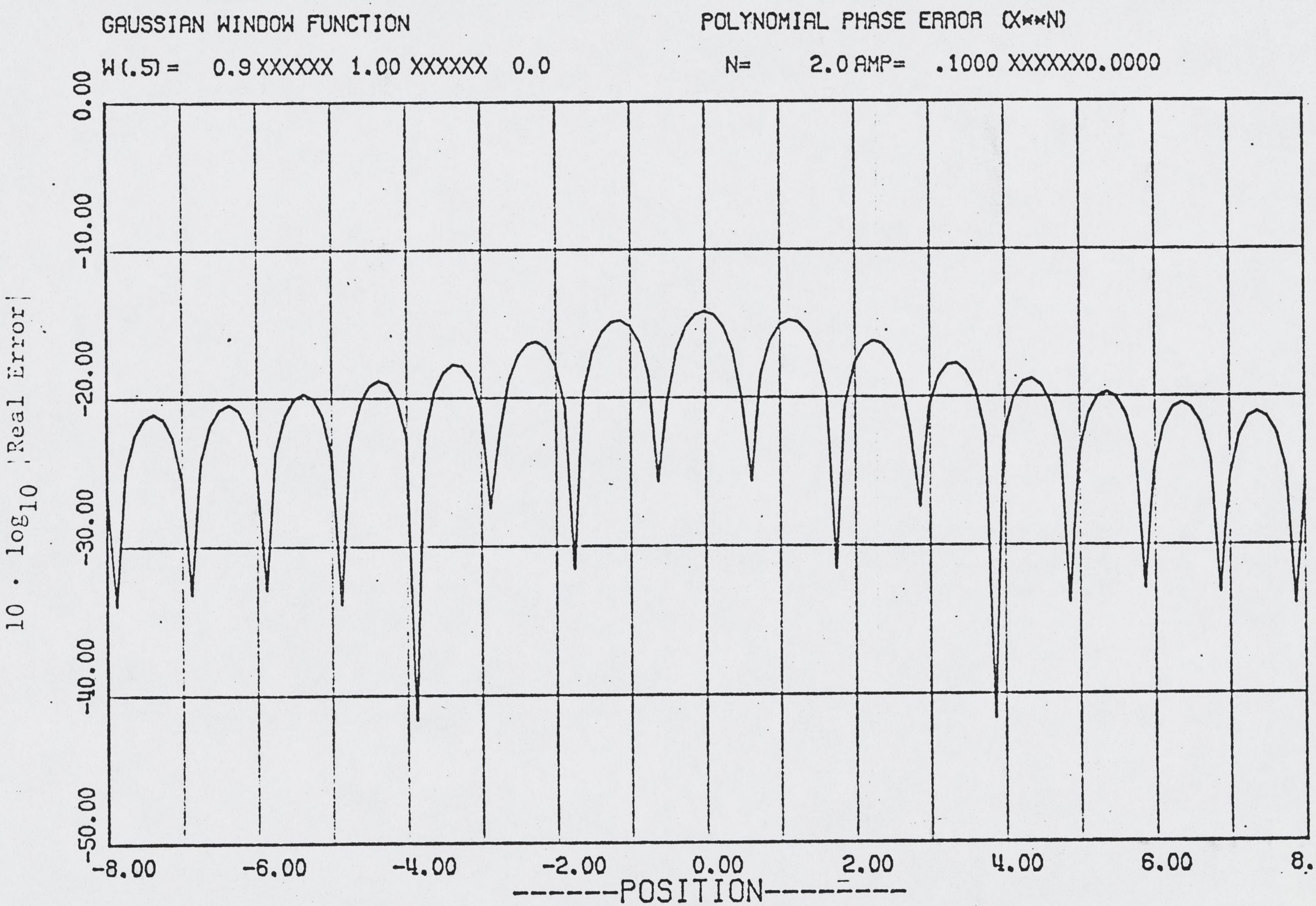


GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Im 𝓕 · Ae^{iφ}|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

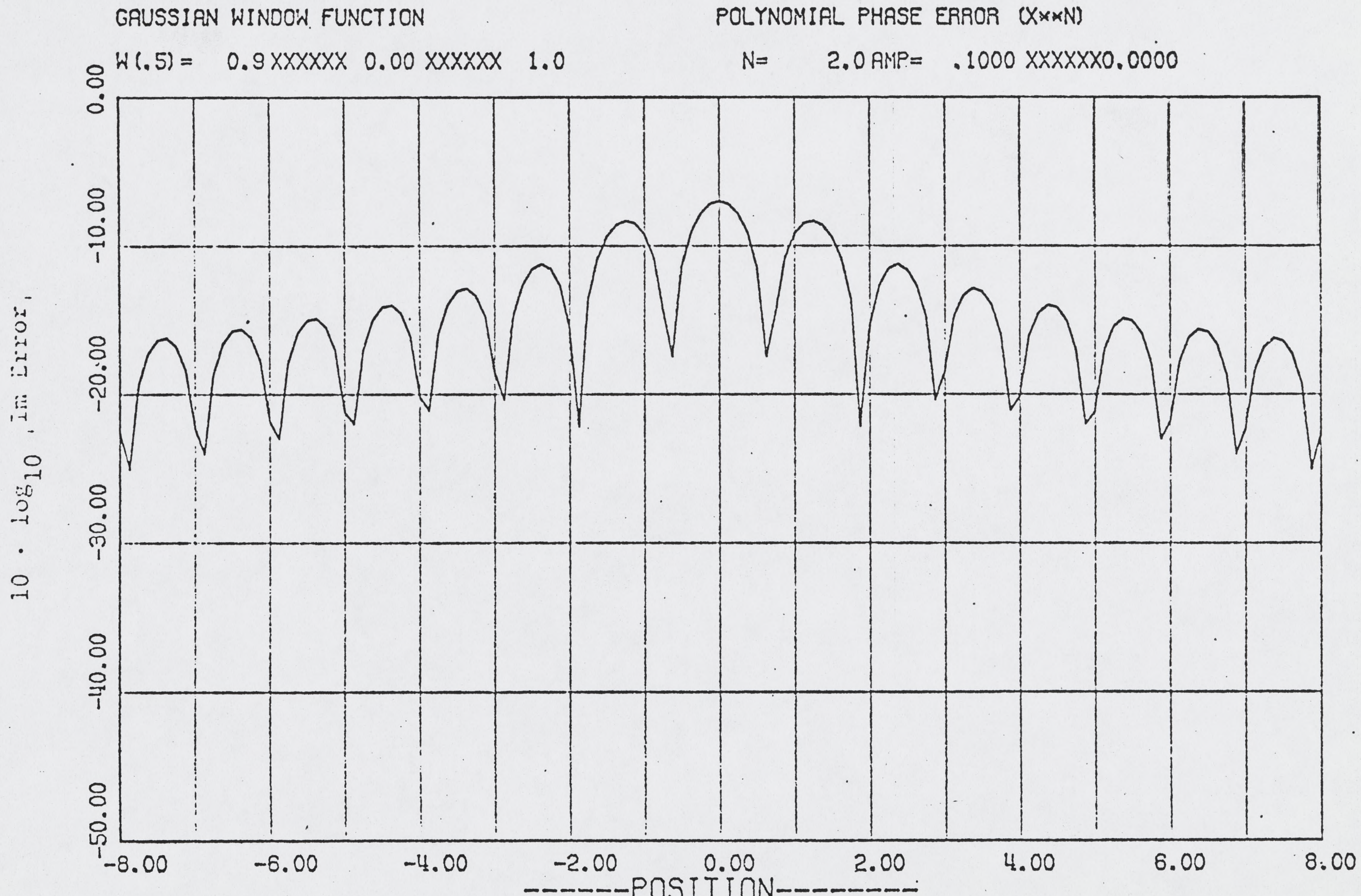

PLOT 42


GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 1.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
-------POSITION--------

PLOT 43


GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 1.00 XXXXXX 0.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
10 · log10 |Real Error|
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION--------

PLOT 44


GAUSSIAN WINDOW FUNCTION
W(.5) = 0.9 XXXXXX 0.00 XXXXXX 1.0
POLYNOMIAL PHASE ERROR (X**N)
N= 2.0 AMP= .1000 XXXXXX0.0000
10 · log10 ,Im Error,
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00
------POSITION-------

PLOT 45