7 September 1976

MEMORANDUM TO: VLA File

FROM: C. C. Aleksoff

SUBJECT: Relating Phase and Aberration Errors

In a previous memo[(1)] we gave some results for the spread function errors in terms of the input plane phase errors. It is often more convenient to interpret the above results in terms of ray aberrations. This memo develops the connection between input phase errors and output ray aberrations.

Let $\phi(u,v)$ describe the input phase error. The local surface wave vector is then given by

$$\overline{K} = \nabla_s \phi \tag{1}$$

where

$$\nabla_s = \hat{u} \frac{\partial}{\partial u} + \hat{v} \frac{\partial}{\partial v} \tag{2}$$

is the surface gradient operator. Let the illumination wave vector be given by $\overline{k}_c$ and the resulting transmitted ray vector by $\overline{k}_T$ (see Fig. 1). We note that for free space

---

(1) C.C. Aleksoff, Phase Error Plots, ERIM memo to VLA File, SA-761060-123401, August 16, 1976.

$$k_c = |\bar{k}_c| = \frac{2\pi}{\lambda}$$

$$k_t = |\bar{k}_t| = \frac{2\pi}{\lambda} \quad (3)$$

where λ is the optical wavelength. The basic equation governing the three wave vectors is[2]

$$(\bar{k}_t - \bar{k}_c - \bar{K}) \times \hat{z} = 0 \quad (4)$$

where $\hat{z}$ is normal to the input plane, as indicated in Figure 1. Alternatively we can write (4), using the angles shown in Figure 1, as

$$k_t \sin \theta_t - k_c \sin \theta_c = K \quad (5)$$

or using (3)

$$\sin \theta_t - \sin \theta_c = \frac{K\lambda}{2\pi} \quad (6)$$

If we now define a local grating frequency F defined by

---

[2] G. Toraldo di Francio, "Parageometric Optics," JOSA, 40, September 1950.

$$\overline{F} = \frac{\overline{K}}{2\pi} = \frac{1}{2\pi} \nabla_s \phi \qquad (7)$$

then (5) becomes

$$\sin \theta_t - \sin \theta_c = \lambda F \qquad (8)$$

which is in the form of the classical grating relationship defining incident and refracted rays (waves) for a grating of frequency F. We note that $\overline{k}$, $\overline{k}_t$, $\overline{k}_c$ define a plane in which the θ_i angles are measured.

For small angles (8) becomes

$$\Delta\theta \triangleq \theta_t - \theta_c \approx \lambda F \qquad (9)$$

or

$$\Delta\theta = \frac{\lambda}{2\pi} |\nabla_s \phi| \qquad (10)$$

where $\Delta\theta$ is the angular aberration. The ray aberration D, which is the distance between the actual ray intercept and the ideal ray intercept measured on the output plane, is thus given by

$$D = \ell\Delta\theta$$

$$= \ell\lambda F$$

$$= \frac{\ell\lambda}{2\pi} |\nabla_s \phi| \qquad (11)$$

where ℓ is the distance from the input to output planes. Thus, the ray aberration is proportional to the gradient of the phase error.

Given an input aperture width W the diffraction limited output spread function width ρ is approximately

$$\rho = \frac{\lambda\ell}{W} \qquad (12)$$

Thus, in terms of ρ, the aberration is

$$D/\rho = WF \qquad (13)$$

Consider one-dimensional phase errors of the form

$$\phi = 2\pi h \left(\frac{u}{W/2}\right)^n \qquad (14)$$

the spread function error for some of these phase errors were plotted in a previous memo[1]. From (13) and (7) the ray aberration due to phase errors of the form (14) is

$$D/\rho = 2hn \left(\frac{u}{W/2}\right)^{n-1} \qquad (15)$$

For example, if n = 1 and A = 2h is the peak-to-peak amplitude of this linear phase shift then the peak-to-peak aberration is

$$(D/\rho)_{pp} = 2A \tag{16}$$

Thus, ½ wavelength of peak-to-peak phase error translates the spread function by one spread function width. This is the expected result since the first null for a sinc spread function occurs when the rays from the aperture edges differ by $\lambda/2$ in optical path length.

Next consider quadratic error (first order defocus), i.e., n = 2. Then

$$D/\rho = \frac{8hu}{W} \tag{17}$$

or

$$(D/\rho)_{pp} = 4A \tag{18}$$

where A = h is now the peak-to-peak phase error. Thus, a ¼ peak-to-peak phase error produces a peak-to-peak ray aberration of one spread function width.

It is useful to have a relationship for the focus change necessary to obtain the ray aberration given in (18). From Figure 2 it is seen that

$$\Delta \ell = 2 \frac{\ell}{W} D_{pp}$$

$$= \frac{\ell}{W} 8A\rho$$

$$= \frac{8A\rho^2}{\lambda} \qquad (19)$$

Thus, if ℓ = 3m, W = 50 mm, and λ = 0.5 μm, then

$$\rho = 30 \ \mu m$$

and

$$\Delta \ell = 14.4 \ A \qquad (\Delta \ell \text{ in mm})$$

Thus, a peak-to-peak phase error of 1λ corresponds to a defocus of ±14.4 mm. From the phase plots in reference 1 it was seen that a λ/20 peak-to-peak phase error was good enough to satisfy the 1% criteria if only the real part of the spread function were used. This corresponds to a defocus of ± 0.70 mm.

CCA/pw
cc: I. Cindrich M. Hidayet
A. Klooster J. Fienup

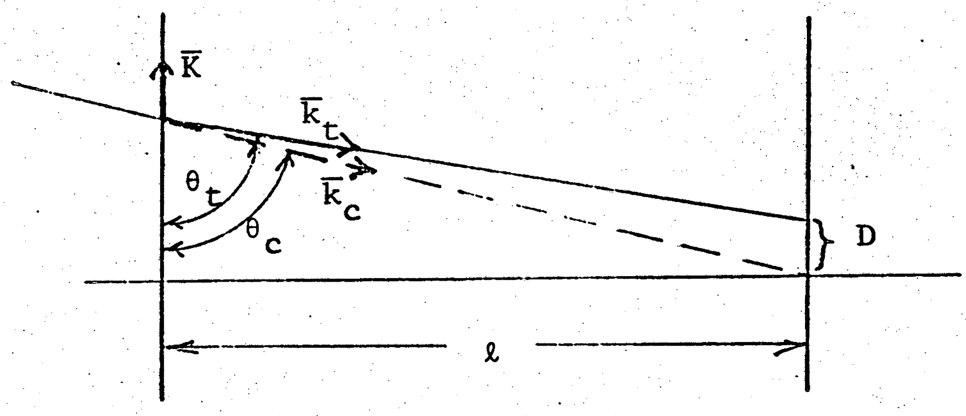

Figure 1

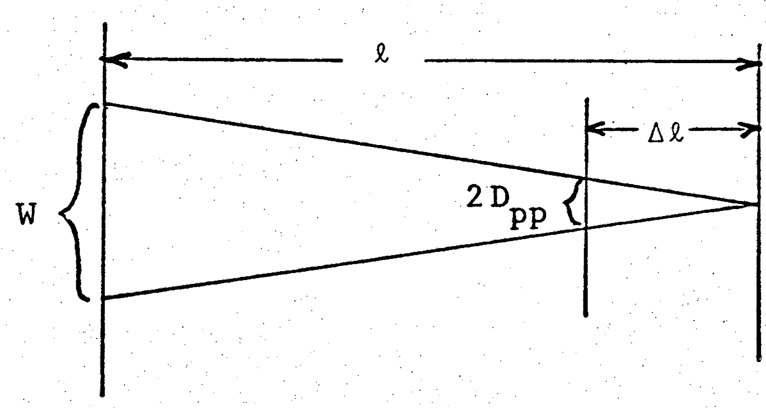

Figure 2