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MEMORANDUM TO: VLA Optical Processor File

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SUBJECT: Basic Limitations of Encoding Methods

In this memo we compare various encoding methods as to their basic limitations and characterize the intrinsic noise in each. Emphasis is placed on three methods that are the most serious contenders. The section headings of this memo are as follows:

On-Axis Impulse Response
Negative or Phase Material
Film Noise
Interimage Film Noise
Lohmann Method
Other Encoding Methods
R+-I+- Method
Conclusions

The three most promising methods of encoding the complex visibility function

$$V(u,v) = |V(u,v)|e^{j\phi(u,v)} = R(u,v)+j\ I(u,v)$$

are the simple carrier method, using the transmittance

$$H(u,v) = B_o + 2\,|V(u,v)|\cos[\omega_o u + \phi(u,v)]$$

$$= B_o + V(u,v)e^{j\omega_o u} + V^*(u,v)e^{-j\omega_o u}$$

where $\omega_o u$ is a carrier frequency term required to separate the twin images and B_o is a bias term required to make the transmittance non-negative; the real-imaginary method, using two transparencies of transmittance

$$H_R(u,v) = B_o + R(u,v)$$

and

$$H_I(u,v) = B_o + I(u,v)$$

respectively; and the modified Lohmann method, using a transparency of binary transmittance with apertures of area proportional to $|V(u,v)|$ located at the peaks of the function $\cos[\omega_o u - \phi(u,v)]$. Here it is assumed that $H(u,v)$ is normalized to be ≤ 1.0 everywhere. The brightness distribution is $B(x,y) = \mathcal{F}\{V(u,v)\}$.

On-Axis Impulse Response

All three encoding methods under consideration have a bias term that produces an unwanted undiffracted wave. An example of this is the B_o term for the transmittance of a simple carrier transparency. The transmittance equations above should be replaced with $H(u,v)A(u,v)$, where $A(u,v)$ is a real, non-negative function describing the aperture in the u-v plane, including its finite width and the elliptical tracks. The corresponding image amplitude is

$$\mathcal{F}\left\{A(u,v)H(u,v)\right\} = B_o\ a(x,y) + a(x,y) * \left[B\left(x - \frac{\omega_o}{2\pi}\lambda f\right) + B\left(-x - \frac{\omega_o}{2\pi}\lambda f\right)\right]$$

where $a(x,y) = \mathcal{F}\left\{A(u,v)\right\}$, the impulse response of the system. Let B_{max} be the maximum value of $a(x,y) * B(x,y)$; $\overline{B^2}$ = (area of image)$^{-1}$ $\iint [a(x,y) * B(x,y)]^2\, dx\, dy$; and N be the number of independent picture elements in the image. Then $B_{max} = \left(\eta_c \overline{B^2}\right)^{\frac{1}{2}}$ where η_c is N times the fraction of the total energy going into the brightest picture element. η_c varies between 1 for an image of constant brightness everywhere to N for an image that consists of a single star. Note that $N^{\frac{1}{2}}$ is approximately 10^3, allowing B_{max} to have a wide range of values depending on the structure of the image. Since ideally the impulse response of the system is delta function, we have the maximum value of $a(x,y)$, $a_{max} = (N\ \overline{a^2})^{\frac{1}{2}}$, where $\overline{a^2} \equiv$ (area of image)$^{-1}$ $\iint |a(x,y)|^2\, dx\, dy$. Using Parseval's theorem, we have

$$\overline{a^2} = k \iint |A(u,v)|^2\, du\, dv \equiv k\ A_o$$

and

$$\overline{B^2} = k \iint |A(u,v)V(u,v)|^2 \, du\, dv \equiv k A_o \overline{|V|^2}$$

where k is a constant. Therefore, the ratio of the maximum amplitude of the image to the peak amplitude of the on-axis impulse response of the system is given by

$$\frac{B_{max}}{B_o a_{max}} = \left(\frac{\eta_c \overline{|V|^2}}{N B_o^2} \right)^{\frac{1}{2}} .$$

Since the transmittance of the transparency $H(u,v) \geq 0$ everywhere, then we have $B_o - 2|V|_{max} \geq 0$, or $2|V|_{max} = \sqrt{\eta_f}\, B_o$, where $\eta_f \leq 1$ is the diffraction efficiency factor due to the use of only a fraction of the dynamic range of the film. (For the real-imaginary method, $|V|_{max} = \sqrt{\eta_f}\, B_o$.) The equation above becomes

$$\frac{B_{max}}{B_o a_{max}} = \left(\frac{\eta_c \eta_f \eta_v}{4N} \right)^{\frac{1}{2}}$$

where $$\eta_v = \frac{\overline{|V|^2}}{|V|^2_{max}}$$

For example, if the scene consists of 100 equally bright point-like stars, and $N = 10^6$, making $\eta_c = 10^6/10^2 = 10^4$, and if $\eta_f = 0.5$ and $\eta_v = 0.2$, then $B_{max}/(B_o a_{max}) = (10^4 \cdot 0.5 \cdot 0.2/4 \cdot 10^6)^{1/2} \approx 0.02$. Then the image fails the 1% criterion wherever the amplitude of sidelobes of $B_o\ a(x,y)$ is greater than 2×10^{-4}, or the intensity is greater than 4×10^{-8}, times the peak value. For a filled rectangular aperture in the u-v plane, the sidelobes of the sinc function impulse response (which decrease with amplitude $1/(\pi x)$ along with x-axis) would be above this level out to the 1600th independent (resolution) picture element from the optical axis. For the real-imaginary and Lohmann methods the corresponding figures would be 800 and 1260, respectively. This example points out the importance of weighting $A(u,v)$ to minimize the sidelobe levels of $a(x,y)$ and also of using a carrier frequency ω_o greater than the minimum necessary to separate the twin images, in order to move the desired image away from the on-axis impulse response of the system. For a filled circular aperture in the example above, the sidelobes would be too great out to only the 80th picture element. In order to predict the effect of the actual sidelobes $a(x,y)$ on the image we would need to know the detailed structure of $a(x,y)$, and we presently do not have that information.

The effect of the term $B_o\ a(x,y)$ is similar for all three encoding methods, except that for the real-imaginary method, the ratio $B_{max}/B_o a_{max}$ is twice as great as the expression above, due to its greater diffraction efficiency. Furthermore, since for the real-imaginary method the image is on-axis in the center of the image, a careful weighting of $A(u,v)$ in order to reduce the sidelobes of $a(x,y)$ is absolutely essential.

From the equation for $B_{max}/B_o a_{max}$ above, we see that there are two important data-dependent quantities that will have a direct impact on the performance of the optical processor. One of these is the ratio $\eta_V = |V|^2/|V|^2_{max}$. The diffraction efficiency of the signal transparency is proportional to η_V for all encloding methods. This arises from the fact that the encoding of $|V|_{max}$ is accomplished by modulating the transmittance of the signal transparency the maximum allowable and the modulation required to encode all other values of $|V(u,v)|$ must be scaled down with respect to that required for $|V|_{max}$. Consequently, of the light transmitted at a given point in the signal transparency, only a fraction goes into the image, accoring to the local modulation, which is less than the maximum possible modulation since $|V(u,v)|$ is less than $|V|_{max}$. Thus, while the noise remains constant, the signal amplitude is proportional to $\sqrt{\eta_V}$, and so the signal-to-noise ratio in amplitude is proportional to $\sqrt{\eta_V}$. For an image consisting of a single point star, $\sqrt{\eta_V}$ is equal to 1; and for an image consisting of a uniformly bright extended object over the entire field-of-view, V(u,v) is a delta-function-like and η_V is equal to $(1/N)^{\frac{1}{2}} = 10^{-3}$ for $N = 10^6$. Thus, depending on this one quantity alone, the final signal-to-noise ratio can vary over three orders of magnitude.

The second important data-dependent quantity is $\eta_c = B^2_{max}/\overline{B^2}$. The reason that this quantity is important is that for a fixed ratio of total energy in the image to total energy in the noise, the signal-to-noise ratio depends on the degree to which the signal is either highly concentrated or smeared out over the field-of-view. The signal-to-noise ratio in amplitude is proportional to $\sqrt{\eta_c}$, which, as mentioned above, can vary from 1 to 10^3 for $N = 10^6$.

Also important is the fact that if η_c is large (or small) then η_V also tends to be large (or small), and vice versa. Thus, the ratio $B_{max}/B_o a_{max}$, which is proportional to the signal-to-noise ratio in amplitude, can be drastically different for different data. For this reason, for the 1% criterion to be truly meaningful, η_c and η_V must also be specified. For the purpose of this study it is important to know the range of values typically taken on by η_c and η_V. These numbers are also important for the determination of the required laser power and the integration time of the detectors.

Negative or Phase Material

If the transparency is made using a negative film, then an additional impulse-response-like term appears due to the fact that the film would be perfectly transmitting in the area between tracks. This is also true for a pure-phase transparency. Let $A'(u,v) = \{1 \text{ wherever } A(u,v) \neq 0;\ 0 \text{ wherever } A(u,v) = 0\}$. Then for negative materials an additional term $A_o(u,v)[1-A'(u,v)]$ is present, resulting in the additional terms $a_o(x,y)*[\delta(x,y) - a'(x,y)]$ in the image, where $A_o(u,v)$ is a physical aperture in the u-v plane and $a_o(x,y) = \mathcal{F}\{A_o(u,v)\}$ and $a'(x,y) = \mathcal{F}\{A'(u,v)\}$. If $A_o(u,v)$ is a circular aperture, then $a_o(x,y)$ is of the form $(2/\pi r)J_1(\pi r)$, where $r \equiv (x^2+y^2)^{1/2}$. Here r = 1 corresponds to one picture element. The diffraction efficiency of the image is given by

$$\eta = \eta_f \eta_m \eta_{uv} \eta_V$$

where η_m is the theoretical maximum diffraction efficiency of the encoding method and η_{uv} is the fractional area of the u-v plane within the large circular aperture that is encoded with V(u,v). Then the ratio of B_{max} to the peak amplitude of the $(2/\pi r)J_1(\pi r)$ term is approximately given by

$$\left(\eta_f \eta_m \eta_{uv} \cdot \frac{\eta_c}{N} \cdot \eta_V\right)^{1/2} .$$

Using the same numbers as in the previous example, with $\eta_m = 1/16$ for the simple carrier method and $\eta_{uv} = 0.25$, we get the ratio $4 \times 10^{-3}:1$. Then the image fails the 1% criterion wherever the amplitude of $(2/\pi r)J_1(\pi r)$ is greater than 4×10^{-5} in amplitude or 1.6×10^{-9} in intensity. Using the fact that the amplitude of the oscillations of $(2/\pi r)J_1(\pi r)$ is $2/(\pi^2 r^{3/2})$ for large values of r, we find that the sidelobes of this term are above that level out to the $r = \left[2/(\pi^2 4 \times 10^{-5})\right]^{2/3} = 234^{th}$ picture element from the optical axis. For the real-imaginary method, $\eta_m = 1/4$, and for the same example as above, the 1% criterion is met beyond the 92nd picture element from the optical axis. Thus, for the real-imaginary method it is highly desirable to use a positive film, irrespective of the type of weighting for the u-v plane aperture, A(u,v), lest a significant fraction of the image be degraded beyond the 1% criterion.

For a square aperture, $a_o(x,y)$ is of the form $\sin(\pi x) \cdot \sin(\pi y)/(\pi^2 xy)$ which drops to the 4×10^{-5} level at y = 0, x = 8,000 picture elements; however, at an angle of 45° with the edge of a square aperture, it drops to that level at x = y = 89 picture elements. Thus, if a phase modulating or negative amplitude modulating material is used for the simple carrier method, a square aperture should be used and the carrier should be $\omega_o(u + v)$ in order to move the image to the area in which the sidelobes of the impulse response $a_o(x,y)$ are at a minimum. (An alternative method would be to use a carrier of $\omega_o u$ but tilt the rectangular aperture at 45^o; however, this method would result in the strong sidelobes cutting through two corners of the image.) Depending on the aperture $A_o(u,v)$ used, the carrier frequency ω_o can be chosen to be large enough to move the image away from the region in which the sidelobes from the on-axis terms are too great, even when a phase or negative amplitude material is used.

Film Noise

Another source of noise will be film noise. (Other sources of scattered light in the optical system can be treated in a similar manner to the analysis in this section.) Biederman [1] found that the scattered flux spectrum for a number of holographic materials is given by the formula

$$\frac{\Phi(\nu)}{\Phi_o} = a \cdot \nu^{-b}$$

where Φ_o is the transmitted flux, equal to the incident flux times B_o^2. For Kodak 649F plates, $a = 2.6 \times 10^{-4}$, $b = 2.26$, and the equation above is valid only for spatial frequencies in the range $6 < \nu < 80$ cycles/mm. For lower spatial frequencies, the noise is less than that predicted by this equation. In the equation above, $\Phi_o = \iint I(x,y)\, dxdy$ where $I(x,y)$ is the intensity of the on-axis term and the area of the integral includes most of the intensity of the on-axis term. If the peak intensity of the on-axis germ is I_o (which is equal to $B_o^2 a^2_{max}$) and diffraction limited spot size is Δx_o, then we have $\Phi_o \approx I_o(\Delta x_o)^2$. The spatial frequency, ν, is related to a position in the brightness plane by the relation

$$\nu = x/\lambda f$$

The scattered flux $\Phi(\nu)$ is normalized for an area equivalent to $(1\ \text{cycle/mm})^2$. In the brightness plane, a distance corresponding to 1 cycle/mm is given by λf (1 cycle/mm). Then $\Phi(\nu)$ is given by

$$\Phi(\nu) = \iint_x^{x+\lambda f/1mm} I_n(x,y)\, dxdy = I_n(x,y) \cdot (\lambda f/1mm)^2$$

where $I_n(x,y)$ is the intensity of the scattered light, and the spatial frequency ν corresponds to the position (x,y). Then, using the relationship $\Delta x_o = \lambda f/D$, where D is the diameter of the signal transparency,

$$\frac{\Phi(\nu)}{\Phi_o} = \frac{I_n(x,y)(\lambda f/1\text{mm})^2}{I_o(\Delta x_o)^2} = \frac{I_n(x,y)}{I_o}(D/1\text{mm})^2$$

or, rewriting,

$$\frac{I_n(x,y)}{I_o} = \frac{\Phi(\nu)}{\Phi_o}\cdot\left(\frac{1\text{mm}}{D}\right)^2$$

$$= a\cdot\nu^{-b}\left(\frac{1\text{mm}}{D}\right)^2 \quad \text{for } 6 < \nu < 80 \text{ cyc./mm}$$

In terms of amplitude, the equation above becomes

$$\frac{\left(I_n(x,y)\right)^{\frac{1}{2}}}{B_o a_{max}} = \left(\frac{1\text{mm}}{D}\right)\sqrt{a}\ \nu^{-b/2}$$

Using the example described previously, in which $B_{max}/(B_o a_{max}) = 0.02$, and assuming a signal transparency aperture of diameter D = 65 mm, we find that the 1% criterion is satisfied using 649F plates only for spatial frequencies above

$$\nu = \left[\frac{(1\text{mm}/D)\sqrt{a}}{(.01)B_{max}/(B_o a_{max})} \right]^{2/b} = 1.1 \text{ cycles/mm}$$

which corresponds to a distance $x = \nu\lambda f = 1.5$ mm which corresponds to $x/x_o = \nu\lambda f \cdot (\lambda f/D)^{-1} = \nu \cdot D = 72$ picture elements from the optical axis for an optical system in which $\lambda = 0.488 \times 10^{-3}$ mm and the focal length $f = 2850$ mm. However, since Biedermann's formula overestimates the noise for spatial frequencies below 6 cycles/mm, the 1% criterion would be satisfied in this example for spatial frequencies somewhat below the 1.1 cycles/mm computed above. For the real-imaginary method, using the same parameters as above, the 1% criterion would be satisfied at spatial frequencies above 0.6 cycles/mm, corresponding to the 39th picture element.

Film noise would also be present in the binary transmittance Lohmann encoding method, but techniques are available for making it negligible [2].

The fraction of the image that is disturbed by film noise and by sidelobes of the undiffracted beam can be minimized in a number of ways. One way would be to demagnify the transmittance function H(u,v) on the transparency, giving it a higher spatial frequency content. Another way, for the simple carrier and

Lohmann methods, would be to use a higher carrier frequency ω_o in order to move the image further from the optical axis. Alternatively, two transparencies can be made; a first with $H(u,v) = B_o + 2|V(u,v)|\cos[\omega_o u + \phi(u,v)]$ and a second with $H(u,v) = B_o + 2|V(u,v)|\cos[\omega_o u - \phi(u,v)]$; looking to one side of the optical axis, the second image is a mirror image of the first, and in each case a different portion of the image is disturbed by the noise associated with the undiffracted beam. The full image is obtained from one half of the first image and the other half of the second image. Finally, the weighting of the u-v plane should be chosen to minimize sidelobes, a film with the lowest scattered noise should be used, and the diffraction efficiency of the signal transparency should be optimized.

Interimage Film Noise

Since film noise is a multiplicative error, the result is the convolution of the entire image with the noise term $[\phi(\nu)/\phi_{inc}]^{\frac{1}{2}}$. In the analysis above we considered only the convolution of the noise term with the undiffracted beam. In addition, we should consider the convolution of the noise term with $B(x,y)$ as well, which we shall call "interimage film noise". The amount of degradation caused by this convolution depends on the structure of the image as well as the shape of the noise term. As an example, consider an optical system with an input aperture of D = 65 mm, focal length f = 2850 mm and wavelength 0.488×10^{-3} mm. A single picture element then has a width of $\lambda f/D = 0.021$ mm, corresponding to a spatial frequency of 1/D = 0.015 cycles/mm. The formula $\sqrt{a}\,\nu^{-b/2}$ is valid only for spatial frequencies above 6 cycles/mm, which is equivalent to 400 picture elements, and hence is not useful for considering the film noise associated with $B(x,y)$. Unfortunately, we do not have a formula for $\Phi(\nu)$ for very low spatial frequencies; however, we took a quick qualitative look at the scattered flux spectrum of a sample of 649F in our laboratory, and it appeared to continue to increase below 6 cycles/mm (where, according to Biedermann, its value is 4.5×10^{-6} (cycles/mm)$^{-2}$). Suppose, for the sake of argument, that at 0.6 cycles/mm, corresponding to 40 picture elements, $\Phi(\nu)/\Phi_o = 10^{-4}$, and suppose that one star were surrounded by 20 other stars each 40 picture elements away from the one star, and all 21 stars being of the same brightness. Then, since the noise from each of the 20 stars will add incoherently, we have the noise intensity due to the 20 stars at the position of the one star being $I_n = 20\ I_o \times 10^{-4}(1\text{mm}/D)^2 = 4.7 \times 10^{-7}\ I_o$. Then the percent error at the one point, due to the scattered film noise associated with the other twenty points would be $\sqrt{4.7 \times 10^{-7}} \simeq 7 \times 10^{-4}$, which is well below the 1% criterion. Except to say that this interimage film noise term will probably not be significant, it would be difficult to make any general statements about the effect of this noise term since it depends on the detailed structure of the brightness distribution and on the scattered flux spectrum at very low spatial frequencies, neither of which are known.

Lohmann Method

Aside from the inherent sources of noise mentioned above, both the simple carrier method and the real-imaginary method are theoretically capable of producing perfect imagery. The Lohmann binary method, on the other hand, which can be implemented in such a way as to eliminate film noise [2], has certain errors that are inherent in the method. The nature of the errors involved is discussed in the literature [3,4,5] and will not be repeated here.

The proper position of the center of a given aperture is given by the position (u_p, v_p) determined by the equation [4]

$$2\pi(\omega_o u_p - \ell_p) = \phi(u_p, v_p) + 2\pi L_p$$

where L_p is an integer and $\ell_p = [\omega_o u_p + \frac{1}{2}]$ (greatest integer). This corresponds to the locations for which $\cos[\omega_o u_p - \phi(u_p, v_p)] = 1$. Given four adjacent samples of $\phi(u,v)$ along a track, a cubic interpolating polynomial is defined that can be used for $\phi(u_p, v_p)$ in the equation above. The resulting cubic equation can be solved exactly [6] or numerically by Newton's method for the position (u_p, v_p), which defines the center of the aperture. The area of the aperture is made proportional to $|V(u_p, v_p)|$ which is determined by a cubic interpolation of $V|(u,v)|$ using the four nearest samples of $|V(u,v)|$ on the track. A problem with this procedure for the VLA Optical Processor application arises for the "vertical" part of a track (the part of a track parallel to the v-axis). Since the fringes of $\cos[\omega_o u - \phi(u,v)]$ tend to be parallel to the v-axis, a vertical track is not likely to cross a fringe and no solution would be found for the equation above. Thus, the vertical parts of a track would not contribute as desired to the encoded transmittance. One possible solution would be to interpolate $V(u,v)$ between neighboring tracks; this might involve a significantly greater amount of computation, and might be inaccurate as well. A second solution would be to approximate $\phi(u_p, v_p)$ by a constant equal to a specific value of $\phi_o = \phi(u_t, v_t)$ on the track. Then the equation above reduces to the linear equation $2\pi(\omega_o u_p - \ell_p) = \phi_o + 2\pi L_p$. However, the use of this simpler equation introduces a phase error roughly equal to $|(u_t - u_p)\frac{\partial\phi}{\partial u}|$.

Another potential problem of the Lohmann method for this application is the quantization of the encoded amplitude $|V(u,v)|$. Aside from the value of zero, the minimum encodable value of $|V(u,v)|$ corresponds to the area of the writing spot of the recording device. Consequently, low values of $|V(u,v)|$ might be quantized to the zero level. The effects of this quantization of amplitude would deserve further study in the event that a binary transmittance encoding method were required.

Other Encoding Methods

A number of other encoding methods [7-9] can be considered, but all are unsuitable for the VLA Optical Processor for one reason or another, except for a recently suggested method that will be described in the next section.

The ROACH method [8,10] consists of modulating the amplitude $|V(u,v)|$ by absorption and the phase $\phi(u,v)$ by variation in surface relief height in an on-axis manner using a multi-emulsion film. For the ROACH, η_m = 100% and all the transmitted light goes into the desired on-axis image and none into an undiffracted term. Consequently, except for interimage film noise, the ROACH would not suffer from any of the inherent sources of noise discussed above. Unfortunately, presently available multiemulsion films suitable for the ROACH are plagued with severe cross-talk and other problems that prevent them from being useful for the present application [11].

Another method for which η_m = 100% and is on-axis and without an undiffracted term is Chu's parity-sequence method [12,13]. A "parity sequence" image is added to the desired image such that the sum of the two images have a Fourier transform with constant amplitude allowing the transparency to modulate only the phase of the wavefront. The image has a dot-like structure with rows of parity elements interlaced with, but not overlapping, rows of the desired image elements. This method is useful only where discrete samples of the image at the Nyquist rate is required, as in the case of the computer memory application, and not where the image is to be oversampled, as in the present application. Furthermore, the concept is based upon a regular grid of points in the u-v plane, rather than the quasi-random sampling of interest for the visibility function. A related method is Chu's synthetic coefficient method [8,13] which suppreses the parity elements near the optical axis, allowing an oversampling detection of the desired image. However, the parity elements are suppressed less well for image points away from the optical axis, causing a background of noise that would be far too great for the purposes of the present application.

Other binary detour-phase methods besides Lohmann's have been developed by Haskell [14-16], but all of them produce only approximations to the desired image not within the 1% criterion.

Lee [17] and Burckhardt [18] developed a continuous-tone detour-phase method that avoids some of the intrinsic errors of the Lohmann method. This method can be shown to be a sampled half-wave-rectified version of the simple carrier method, but without the bias B_o. Davenport and Root [19] analyze the case of a signal having a spectral density (the modulus squared of the Fourier transform) of width W and offset x_o, and find that the output after a half-wave linear detection of that signal results in additional terms in the spectral density, including a zero-frequency delta-function (as though the signal had a bias), and self-convolution terms of width 2W at frequencies zero and $2x_o$. Thus, in order to avoid overlap of the extra terms with the desired image, the offset would have to be W·3/2. Thus, three times the bandwidth is required as compared to the simple carrier method, for which the offset need be only slightly larger than W/2. In addition, there are weak terms of width 4W centered at 0 and $2x_o$ that would overlap the desired term. The degree of this degradation is not presently known and would require a considerable amount of analysis to be determined. Furthermore, the Lee method requires a regular gridding of the sample points in the u-v plane, which would introduce aliasing. This aliasing can be avoided by altering Lee's method by allowing samples of arbitrary location of the function

$$H(u,v) = \begin{cases} |V(u,v)|\cos[\omega_o u + \phi(u,v)] & \text{, where } > 0 \\ 0 & \text{, otherwise} \end{cases}$$

However, this method does not seem to have any advantages over using the simple carrier, except for a minor increase in diffraction efficiency, but has the disadvantage of introducing additional undesired noise terms.

Another method similar to the simple carrier method is the method analogous to a hologram:

$$H(u,v) = \frac{1}{B_o} \left| B_o e^{j\omega_o u} + V(u,v) \right|^2$$

$$= B_o + \frac{1}{B_o} |V(u,v)|^2 + 2|V(u,v)| \cos[\omega_o u + \phi(u,v)]$$

which produces an image

$$h(x,y) = \mathcal{F}\{H(u,v)\}$$

$$= B_o \delta(x,y) + \frac{1}{B_o} B(x,y) \star B(x,y) +$$

$$B(x - \frac{\omega_o}{2\pi} \lambda f, y) + B(-x - \frac{\omega_o}{2\pi} \lambda f, y)$$

where $\star$ indicates the autocorrelation operation. This method has the advantage not requiring B_o to be greater than twice the maximum of $|V(u,v)|$, but at the expense of the on-axis autocorrelation term of width 2W requiring a spatial offset of the image by W·3/2 in order to avoid overlap. Thus, three times the bandwidth is required as compared to the simple carrier method. For this method, as well as for the simple carrier method, $\eta_m = 1/16$.

Still another similar method not requiring a strict bias term is that of Huang and Prasada [20], using

$$H(u,v) = 2|V(u,v)| + 2|V(u,v)|\cos[\omega_o u + \phi(u,v)]$$

This method replaces a bias term with an on-axis term of lesser energy, but of width approximately W. To avoid overlap with this term, a spatial offset of the image of W is required, which is twice that required by the simple carrier method. Furthermore, the transmittance of a Huang-Prasada transparency would tend to be near zero over much of the u-v plane, requiring a more accurate compensation for film nonlinearities than would be required for the simple carrier method. For this method $\eta_m = 1/16$.

R+-I+-Method

A method recently suggested is to make four transparencies:

$$R_+(u,v) = \begin{cases} R(u,v) & \text{, where } R(u,v) > 0 \\ 0 & \text{, otherwise} \end{cases}$$

$$R_-(u,v) = \left\{\begin{array}{ll} -R(u,v) & \text{, where } R(u,v) < 0 \\ 0 & \text{, otherwise} \end{array}\right\} = R_+(u,v) - R(u,v)$$

$$I_+(u,v) = \begin{cases} I(u,v) & \text{, where } I(u,v) > 0 \\ 0 & \text{, otherwise} \end{cases}$$

and

$$I_-(u,v) = \left\{\begin{array}{ll} -I(u,v) & \text{, where } I(u,v) < 0 \\ 0 & \text{, otherwise} \end{array}\right\} = I_+(u,v) - I(u,v)$$

where

$$V(u,v) = R_+(u,v) - R_-(u,v) + j\, I_+(u,v) - j\, I_-(u,v) .$$

The desired complex visibility function $V(u,v)$ could be recovered exactly on-axis with no other terms present by the simultaneous illumination of all four transparencies and the interferometric combination of the four resulting wavefronts with the appropriate constant relative phase shifts between them. Alternatively, the four wavefronts can be processed serially and the results added digitally. Since $V(u,v)$ is Hermitian, $R(-u,-v) = R(u,v)$, and so $R_+(-u,-v) = R_+(u,v)$ and $R_-(-u,-v) = R_-(u,v)$; thus $R_+(u,v)$ and $R_-(u,v)$ are both Hermetian with their corresponding images $B_{R+}(x,y)$ and $B_{R-}(x,y)$ both being real and even. Thus, $R_+(u,v)$ and $R_-(u,v)$ can be processed each by itself, each component image being detectable by the method of reference beam with zero and pi radians phase shift. Also, since $V(u,v)$ is Hermetian, $I(-u,-v) = I(u,v)$, that is, $jI(u,v)$ is Hermetian, and so the component image produced by $jI(u,v)$ is real and odd, and therefore is detectable by the method of reference beam with zero and pi radians phase shift. However, $I_+(u,v)$ and $I_-(u,v)$ individually have no such symmetry, since if one of them is non-zero at (u,v) then it is equal to zero at $(-u,-v)$. Thus, the component brightness terms B_{I+} and B_{I-}, produced by I_+ and I_-, respectively, do not have constant phase, and cannot be detected simply by the method of reference beam with zero and pi radians phase shift. However, since the sum $jB_{I+}-jB_{I-}$ is real-valued, only the real components of B_{I+} and B_{I-} are of interest. Thus, we can use the method of reference beam with zero and pi radians phase shift to extract just the real parts of B_{I+} and B_{I-}, which are all that is needed. However, the fact that B_{I+} and B_{I-}, are complex-valued implies a loss of a factor of two in signal-to-noise ratio [21].

The advantage of the R+-I+- method over the real-imaginary method is the elimination of the on-axis impulse response term in the image. This is done at the cost of requiring twice as many transparencies and twice the number of readout operations.

A major problem with R+-I+- method is that near the optical axis the brightness map will be arrived at by taking the difference of large numbers to yield a relatively small number. This can be seen from the analysis of Davenport and Root [19] in connection with the Lee hologram mentioned in the previous section. Both R_+ and R_- are half-wave rectified versions of R(u,v) as are I_+ and I_- of I(u,v). Consequently, included in each component image is an on-axis delta-funtion (the on-axis impulse response) term of total energy approximately the same as the total energy in the rest of the component image. Thus, although the on-axis impulse response term is eliminated from the sum of all four component images, it is present in each component image. Although the on-axis impulse response term is subtracted out, it does cause a loss of accuracy where it is large compared to the desired image. One would expect, though, that the effect of the on-axis term would be considerably less for the R+-I+-method than for the real-imaginary method. Nevertheless, the existence of the on-axis impulse response term does detract from the only advantage of the R+-I+-method over the real-imaginary method.

Another on-axis impulse response term worth considering is that due to an effective bias term arising from the fact that the film will have a certain maximum optical density and a corresponding minimum non-zero amplitude transmittance. It is also possible that a certain minimum exposure and corresponding minimum amplitude transmittance will be used in order to avoid the low-transmittance region of the film characteristic curve if accurate control is difficult in that region. This will be particularly important for small values of $\eta_v = |V|^2 / |V|^2_{max}$, for which most values of R_+, R_-, I_+, and I_- are small compared to the maximum value. This effective bias term is unimportant for the other methods since they have large bias terms that overshadow this one. If, for example, the maximum optical density is 2.0, then this bias term would have approximatley 0.01 times the energy as the additional term discussed previously in connection with negative film. Thus, the image plane on-axis impulse response term would have a maximum amplitude of one-tenth that in the negative film case. Using the same example as before, assuming a circular aperture, the sidelobes of this term would be above the 1% criterion level out to the $[\ 2/\pi^2 \times 10 \times 4 \times 10^{-5})]^{2/3} = 50^{th}$ picture element. At this point we should distinguish between two parts of this bias term: the part in areas covered by tracks in the u-v plane and the part in areas not covered by tracks. It is convenient to consider what happens in the interferometric combination of the four components in the u-v plane. The part in areas not covered by tracks is the same for all four components, and they are added with appropriate phase factors in such a way as to cancel. The part in areas covered by tracks does not cancel, since two of the components (either R_+ or R_- and either I_+ or I_-) will have greater than the minimum transmittance and the other two that do have the minimum transmittance will have a relative phase shift of $\pi/2$ radians and not cancel. The bias term of all four components will cancel everywhere only if the minimum transmittance value is purposely added to the non-zero transmittance values everywhere. The problem of subtracting two large numbers to arrive at a relatively small result will only be slightly aggravated by this procedure. Assuming that this correction procedure is performed, the one percent criterion would be violated only within a radius of a couple of tens of picture elements from the optical axis due to the problem of subtracting large numbers to obtain a relatively small result.

In summary, the R+-I+-method loses far fewer picture elements near the optical axis than does the real-imaginary method, but at the expense of producing twice as many transparencies and requiring either a much more complicated interferometric optical system or twice the number of read-out operations and additional digital operations. In addition, the R+-I+-method is at a disadvantage if it is difficult to accurately control the film transmittance at small values of transmittance.

Still another possibility is the use of three transparencies, with half-wave rectified versions of Real $\{ e^{-jk\pi/3} \cdot V(u,v) \}$, k = 0, 1 and 2, respectively, in a manner similar to the R+-I+-method above. Since neither the three transparencies nor I_+ and I_- are Hermetian, these methods cannot take advantage of recording and processing only 1/2 the u-v plane.

CONCLUSIONS

The encoding method most likely to succeed is the simple carrier method. Of the major contending methods capable of producing an image that satisfied the 1% criterion everywhere, the real-imaginary method fails near the center of its image, and so does the R+-I+-method, but to a lesser degree. The Lohmann method tends to fail near the edges of the image field. The simple carrier method places a greater accuracy requirement on the recorder, but the real-imaginary and R+-I+-methods place a greater accuracy requirement on the optical processor and the output detector.

Since I_+ and I_- are not Hermetian, the R+-I+- encoding method does not allow us to record and process only half of the u-v plane, although it is possible for all the other methods.

JRF:sd

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