ERIM
FORMERLY WILLOW RUN LABORATORIES, THE UNIVERSITY OF MICHIGAN

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MEMORANDUM TO: VLA Optical Processor File

FROM: James R. Fienup J.F.

SUBJECT: Computation of $B^2_{max}/\overline{B^2}$ for Three Hypothetical Brightness Maps.

In an effort to characterize the extremes between which the forms of typical brightness maps might lie, we consider here two examples described by Dr. Frazier Owen of NRAO and one example described by Dr. Lewis Somers of NRAO. For each we compute a parameter that is of importance to the signal-to-noise ratio in the output of the optical processor:

$$\eta_c = \frac{B^2_{max}}{\overline{B^2}}$$

where B(x,y) is the brightness map and an overbar indicates the average value.

CASE I

B(x,y) consists of 100 bright point-like stars. The k^{th} star has a brightness of $10^{-(k-1)/10}$, for stars numbered k = 1 to 12, and brightness $10^{-1-.01k}$ for k = 13 to 100; that is, the respective brightnesses are 1.0, .79, .63, .50, .40, .32, .25, .20, .16, .126, .10, .079, .074, .072, .071,..., .010. Using these numbers, we find that B^2_{max} = 1.0 and

$$\sum_{k=1}^{100} B_k^2 = 2.8188$$

If there are $N = 10^6$ independent picture elements, then $\overline{B^2} = 2.82/N = 2.82 \times 10^{-6}$. Thus, for this case

$$\eta_c = \frac{B^2_{max}}{\overline{B^2}} = \frac{N}{2.82} = 3.55 \times 10^5$$

CASE II

B(x,y) consists of an anulus of outer diameter 0.5 W and inner diameter 0.4 W , where W is the width of the field-of-view. The area of that anulus is then $\pi(.25W)^2 - \pi(.20W)^2 = 0.0707W^2$. For $N = 10^6$ picture elements, only $0.071 \times N = 7.1 \times 10^4$ are non-zero. Of those 7.1×10^4 picture elements 10%, or 7.1×10^3, have a brightness value of 5.0 and the rest have a brightness value of 1.0. Then $B^2_{max} = 25.0$ and

$$\overline{B^2} = \frac{1}{N} \sum B^2 = \frac{1}{N} (25 \times 7.1 \times 10^3 +$$

$$1.0 \times 0.9 \times 7.1 \times 10^4)$$

$$= 2.4 \times 10^5/N = 0.24$$

Thus, for this case,

$$\eta_c = \frac{B^2_{max}}{\overline{B^2}} \simeq N \cdot 10^{-4} = 100$$

The value of 100 for the ratio above is independent of N in this case.

Comparing these two cases, we see that η_c is likely to vary from 100 to 4×10^5, or by a factor of 4×10^3. Thus, the signal-to-noise ratio in amplitude can vary by a factor of $\sqrt{4 \times 10^3} = 63$.

<u>CASE III</u>

$B(x,y)$ consists of 100 equally bright point-like stars, each of brightness value 1.0. Then $B^2_{max} = 1.0$, $\sum B^2 = 100$, $\overline{B^2} = 100/N = 10^{-4}$ and

$$\eta_c = \frac{\overline{B^2}_{max}}{\overline{B^2}} = \frac{N}{100} = 10^4,$$

which is a result that lies between the extreme Cases I and II.

The similarly important parameter $\eta_v = \overline{|V|^2} / |V|^2_{max}$ cannot be readily computed from the information given above without knowing the relative positions of the stars.

JRF:sd

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