6 October 1976

MEMORANDUM TO: VLA File

FROM: C. C. Aleksoff

SUBJECT: Film and Film-Gate Considerations

In this memo we consider the magnitude of the aberrations associated with the film and the film-gate windows (included are the effects of beam splitters).

The conclusions are that ideal (flat) windows or beam splitters of a 1 cm thickness can be inclined up to about 10^{o} before significant error is introduced for our f#/45 system. The film should be an cellulose triacetate base film and not an estar base film. The index matching between fluid and film base should be better than 0.0024.

Film Gates

The basic processor configuration that we are considering is illustrated in Figure 1. A converging spherical wave passes through window #1, illuminates the film, and then the first order diffracted beam passes through window #2 at an inclined angle. By evaluating the effects of a tilted plate on a converging spherical wave, as indicated in Figure 2, we can obtain results for the most general case which includes windows normal to the spherical wave.

Referring to Figure 2, the transverse spherical aberration is given by[1]

1. W.J. Smith, "Modern Optical Engineering," McGraw Hill, 1966, pg. 84.

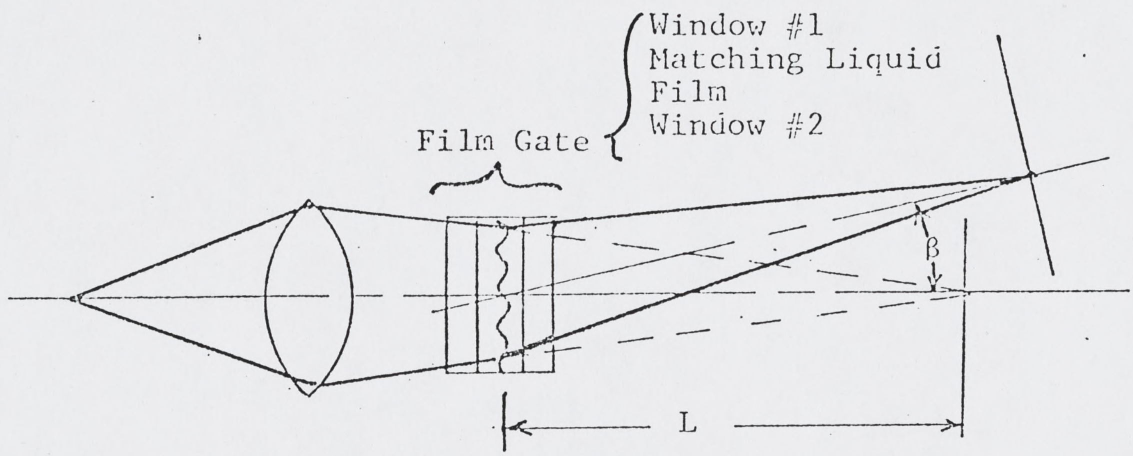

Figure 1

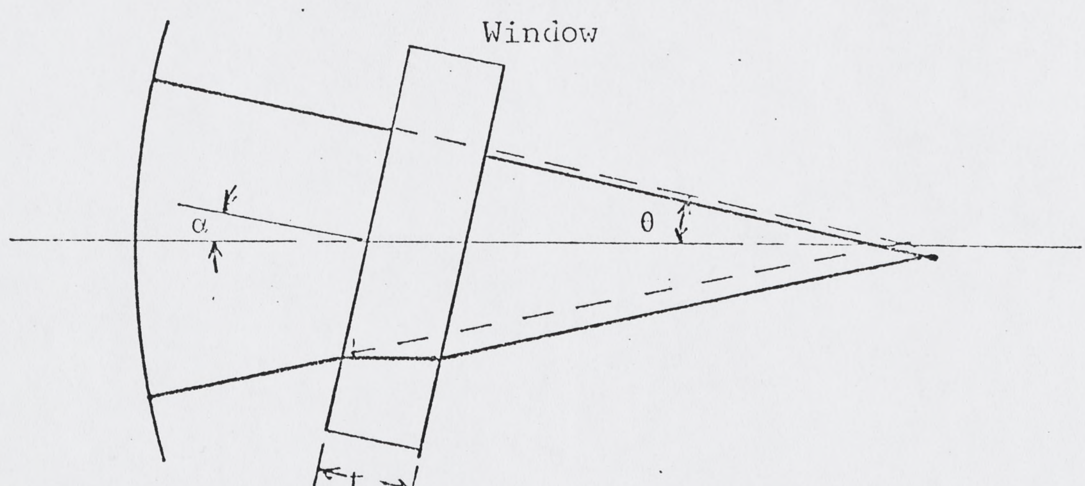

Figure 2

$$S = \frac{t}{N}\left[1 - \frac{N\cos\theta}{\sqrt{N^2 - \sin^2\theta}}\right]\tan\theta \qquad (1)$$

$$\approx \frac{t\,\theta^3\,(N^2 - 1)}{2N^2} \qquad \text{for } \theta \ll 1$$

The transverse error due to astigmatism is

$$A = \frac{t\tan\theta}{\sqrt{N^2 - \sin^2\alpha}}\left[\frac{N^2\cos^2\alpha}{N^2 - \sin^2\alpha} - 1\right]$$

$$\approx \frac{t\alpha^2\theta(N^2 - 1)}{N^3} \qquad \text{for } \theta,\ \alpha \ll 1$$

and the sagittal coma is

$$C \approx \frac{t\theta^2\alpha(N^2 - 1)}{2N^3} \qquad \text{for } \theta,\ \alpha \ll 1$$

We take;

$N = 1.5$ (refractive index)

$t = 1$ cm

$\theta = .011$ (corresponds to a clear aperture of 66 mm and a convergence length of L = 3m)

The spot size (between 3 dB points) is

$$\rho \approx \frac{1.22\lambda}{2\theta}$$

Using ρ = .6328 μm,

$$\rho = 35.1 \ \mu m$$

Then the spot size divided by the transverse aberration is as given in Table I, as a function of the tilt angle α.

Table 1

α =	0^o	1^o	10^o
ρ/s	406	406	406
ρ/A	∞	85	.85
ρ/C	∞	263	26.3

As indicated in a previous memo[(2)] a spot size to aberration ratio of greater than 25 is sufficient (but not necessary) to guarantee satisfying the 1% criteria. Obviously the 0^o and 1^o tilt cases satisfy this criterion.

2. Aleksoff, C.C., Memo to VLA File, Relating Phase and Aberration Error, 7 September 1976.

To further test the 10^o tilt case a ray trace program was run for this tilt. This program calculates the phase error about an ideal reference wave using an 8^{th} degree polynomial. From these calculations the phase error given in Table II were found.

Table II
$\alpha = 10^o$

phase error order	magnitude in terms of wavelength
2nd (astigmatism)	1.3×10^{-2}
3rd (coma)	1.6×10^{-3}
4th (spherical)	1.3×10^{-5}

It is seen that none of the phase errors exceeded $\lambda/50$. Further, the maximum peak-to-peak phase error for all terms in combination did not exceed .014 wavelengths. The RMS error was less than 3.5×10^{-3} wavelengths. The most aberrated ray deviated by 1/12 of spot size from the ideal focus.

From these results it is clear that even a 10^o tilted plate would not exceed the 1% criteria.

The overall conclusion is that ideal windows or beam-splitters of less than 10^o tilt, of about 1 cm thickness, and 1.5 refractive index will not degrade the output signal beyond the 1% criteria.

Film Thickness Variations

In house experiments[3] have shown that cellulose tri-acetate base film can have as much as 4 μm of peak-to-peak thickness variation. This is a slowly varying signal with a maximum frequency of 0.1 cycles/mm. The refractive index[4] can vary as much as ± .01 about a nominal value of N = 1.48. Using the equation

$$\Delta\phi = \frac{2\pi}{\lambda} \; \Delta N \; \Delta T$$

where ΔT is the thickness variation and ΔN the refractive index difference between the film and matching fluid we have that ΔN must satisfy

$$\Delta N < \frac{\Delta\phi\lambda}{2\pi\Delta T}$$

where Δϕ now is the maximum allowed peak-to-peak phase error. A λ/50 criteria corresponds to about 7° of phase error. Using λ = 0.5 μm, ΔT = 4 μm, and the λ/50 criteria gives the requirement that

$$\Delta N < \frac{1}{400} = .0025$$

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3. Experiments performed by J.C.Dwyer on 5460 microfilm.

4. Properties of Kodak Materials for Aerial Photographic Systems, Vol. III, 1974, Book #087985-082-5.

Thus, to satisfy this criteria the liquid will have to be matched to each roll of film since the film-to-film refractive index change can be as large as 8 times this value.

Estar Film

Estar based film is birefringent with a possible index difference of .02 in the plane of the film sheet. Even using an ultra thin base of 38 μm we find a possible phase shift of

$$\frac{\Delta\phi}{2\pi} = \Delta N \cdot \frac{T}{\lambda} \; .$$

$$= 1.52 \text{ wavelengths}$$

which is obviously excessive.

Trying to match the direction of polarization for a linear polarized input wave to that of a principal index axes in the film is not practical since the orientation of the index ellipsoid can vary in a film and from film-to-film.

For a comparison, birefringence in cellulose estar base is about .00001, which is not significant for our purposes.

CCA/pw

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