25 October 1976

MEMORANDUM TO: I. Cindrich

FROM: L. Somers, F. Schwab and C. Aleksoff

SUBJECT: Spherical Wave Defects for a Particular Optical Fourier Transform Configuration

Introduction

The optical Fourier transform (FT) configuration considered is shown in Figure 1. It consists of an input plane (uv) and an output plane (xy) the latter contains a sensor array which is operated in a mode such that the output signal is proportional to

$$R\ (E_r^* \ E_{xy}) \tag{1}$$

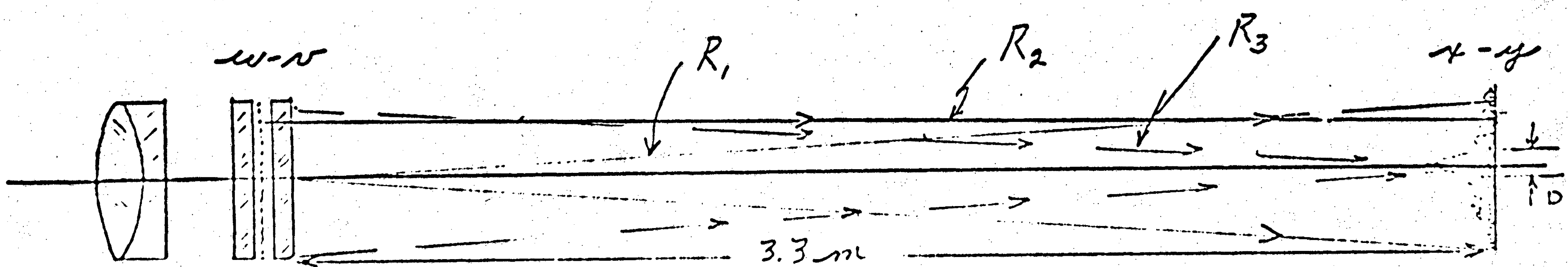
E_r is a coherent reference wave, here taken for simplicity to be

$$E_r = |E_r| \frac{e^{-ikR_1}}{iR_1} \tag{2}$$

That is, a diverging spherical wavefront with origin at $u = v = z = 0$. This reference wave is introduced to linearize the processor output.

E_{xy}, the wave containing the FT of the data placed in the u-v plane, is the result of diffraction which takes place between the u-v and x-y planes. The Huggens-Fresnel integral (solved by Sommerfeld) represents the physics;

$R_1 + R_3 - R$



(Z)

LENS
{ CONV. WAVE-FRONT

MODULATOR
{ U-V FILM IN INDEX MATCHING LIQUID GATE

LINEAR DIODE ARRAY
{ x-SCAN ELECTRONIC
y-SCAN MECHANICAL

50 X 50 mm

$f_c = 30$ C/mm

$N_v = 1500 - 2000$ Cycles

$$E_s = |E_s| \frac{e^{ikR_3}}{R_3} \qquad E_r = |E_r| \frac{e^{-ikR_1}}{i R_1}$$

100 X 100 mm

100 μm = D

4 DIODES/D

4000 DIODES

DUE TO DIFFRACTION:

$$E_{xy} = \frac{1}{i\lambda} \iint_{-\infty}^{\infty} E_{uv} \frac{e^{-ikR_2}}{R_2} \frac{Z}{R_2} \, du\, dv$$

$$R_1 = Z\sqrt{1 + x^2/Z^2 + y^2/Z^2} \qquad E_{uv} = T_{uv} E_s$$

$$R_2 = Z\sqrt{1 + \left(\frac{u-x}{Z}\right)^2 + \left(\frac{v-y}{Z}\right)^2} + \omega x \qquad T_{uv} = V_{uv} \cos(2\pi f_c u) + B$$

$$R_3 = Z\sqrt{1 + u^2/Z^2 + v^2/Z^2} \qquad V_{uv} = \text{Sampled Visibility Functio[n]}$$

Can go to $u \to (u - x_0)$ and $v \to (v - y_0)$ for shifted FT (convergent beam)

FIG 1

$$E_{xy} = \frac{1}{i\lambda} \iint_{-\infty}^{\infty} (E_{uv}) \left(\frac{e^{-ikR_2}}{R_2}\right) \left(\frac{Z}{R_2}\right) \quad dudv \tag{3}$$

(E_{uv}) is the complex amplitude of the optical wave in the u-v plane

$(\frac{Z}{R_2})$ is an "obliquity" term

$\left(\frac{e^{-ikR_2}}{R_2}\right)$ is the diverging Huggens spherical wavefront from every u-v point

$R_2 = Z\sqrt{1 + \left(\frac{u - x}{z}\right)^2 + \left(\frac{u - y}{z}\right)^2}$ the distance between points (u,v,o) and (x,y,z)

E_{uv}, the optical wave exiting the u-v plane, is the result of an illuminating wave E_s and the transmission (amplitude of the w-v plane modulator. (We call the combination, liquid gate and input signal film, a modulator because it introduces the desired input signal onto a uniform wavefront). That is

$$E_{uv} = E_s \; T_{uv}$$

$$E_{uv} = |E_s| \left(\frac{e^{ikR_3}}{R_3}\right) \left[B + V_{uv} \cos (W_c U + \phi_{uv})\right] \tag{4}$$

where the first term is a uniform converging illumination wave and the second term is the amplitude transmission of the film.

Substituting 2, 3 and 4 into 1 results in

$$R\left\{ |E_r| \frac{e^{ikR_1}}{R_1\lambda} \iint_{-\infty}^{\infty} \left[|E_s| \frac{e^{ikR_3}}{R_3} \right] \left[B + V_{uv} \operatorname{Cos} \left(W_c U + \phi_{uv} \right) \right] \left[\frac{Z}{R_2} \right] \left[\frac{e^{-ikR_2}}{R_2} \right] dudv \right\} \tag{5}$$

which is proportional to the voltage output from the sensor array. A slightly different form of this equation is shown in Figures 2 and 3 where the usual optical aberrations $W_{\substack{uv\\xy}}$ (referred to the u-v plane) are also included.

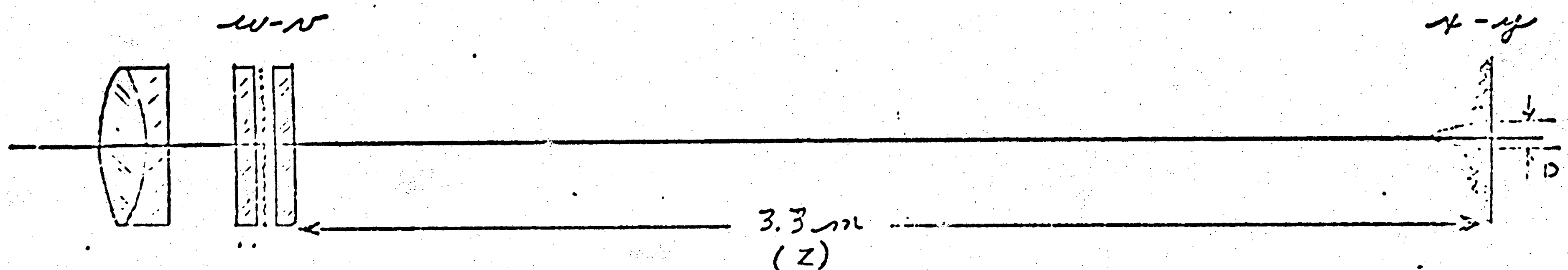
Equation 5 can be rewritten as

$$R\left\{ \frac{|E_r|\,|E_s|}{\lambda} \iint_{-\infty}^{\infty} \left[B + V_{uv} \operatorname{Cos} \left(W_c U + \phi_{uv} \right) \right] \frac{Z}{R_1 R_2^2 R_3} e^{ik(R_1+R_3-R_2)} \times \right.$$
$$\left. \times\, e^{i\frac{k}{z}(ux+vy)}\, e^{-i\frac{k}{z}(ux+vy)} \, dudv \right\}$$

or as

$$R\left\{ \frac{|E_r|\,|E_s|}{\lambda} \iint \left[B + V_{uv} \operatorname{Cos} \left(W_c U + \phi_{uv} \right) \right] e^{i\frac{k}{z}(ux+vy)} D_{\binom{uv}{xy}} \, dudv \right\} \tag{6}$$

where



THE PHYSICS

$$\Re_e\left\{\frac{i|E_r|e^{+ikR_1}}{R_1}\frac{1}{i\lambda}\iint_{-\infty}^{\infty}\left[V_{uv}\cos(2\pi f_c u)+B\right]\left[|E_s|\frac{e^{+ikR_3}}{R_3}\right]\left[\frac{e^{-ikR_2}}{R_2}\right]\left[\frac{z}{R_2}\right]\left[e^{ikW_{uv,xy}}\right]du\,dv\right\}$$

REF. W. MODULATOR SIGNAL W. H-F OB. ABERR

THE FOURIER TRANSFORM

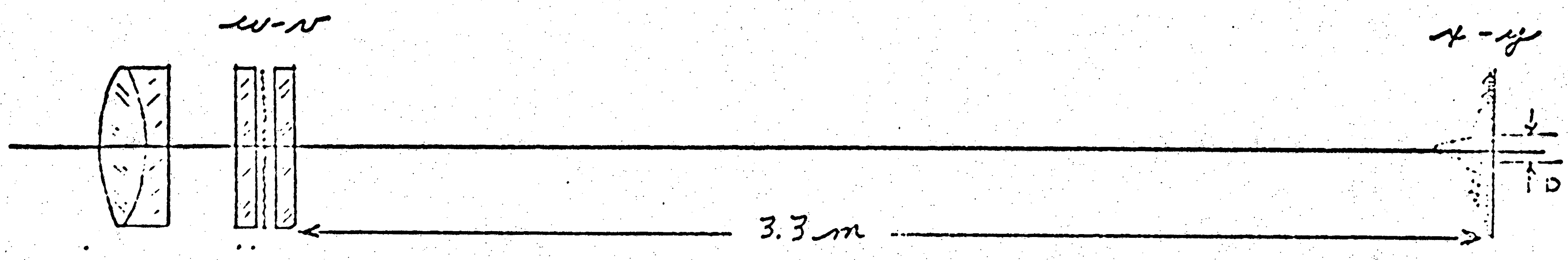
$$\Re_e\left\{\frac{|E_r||E_s|}{\lambda}\iint_{-\infty}^{\infty}\left[B+V_{uv}\cos(2\pi f_c u)\right]e^{\frac{ik}{z}(ux+vy)}D_{uv,xy}\,du\,dv\right\}$$

WHERE

$$D_{uv,xy}=\frac{z\,e^{ikW_{uv,xy}}\,e^{-ik\left(\frac{ux}{z}+\frac{vy}{z}\right)}\,e^{ik(R_1+R_3-R_2)}}{R_1R_3R_2^2}$$

FIG 2

$$\approx\frac{1}{z^3}\left[1+ik\left\{W_{uv,xy}-\left(\frac{ux}{z}+\frac{vy}{z}\right)+(R_1+R_3-R_2)\right\}\right]$$



$$P_{\substack{uv\\xy}} = \frac{Z}{R_1 R_3 R_2^2}\, e^{ikW_{\substack{uv\\xy}}}\, e^{ikZ\left(2 - \frac{-4u^3x + 6u^2x^2 - 4ux^3}{8Z^4} + H.O.\right)}$$

| ACCEPTABLY SMALL | | CAN BE UNACCEPTABLY LARGE |

FIG 2'

$$D_{\binom{uv}{xy}} = \frac{Z}{R_1 R_2^2 R_3} \; e^{ik(R_1+R_3-R_2)} \; e^{-i\frac{k}{z}(ux+vy)} \qquad (7)$$

$D_{\substack{uv\\xy}}$ can be thought of as the defect in the optical FT instrumentation. For the case of small values of u,v and x,y, $D_{uv} \approx \frac{1}{Z^3} e^{ikZ}$ which is a constant and can be "normalized" out. As u,v or x,y become larger, R_1, R_2 and R_3 are no longer constants (equal to z) and the FT operation of the instrument becomes degraded. The result is the introduction of errors into the output of the FT instrument.

A Look at the Defect and Resulting Errors

There are at least two ways to examine this defect, numerically and with series expansions. The former has been done for the VLA aperture, A-array at 40^{o} Dec and 12-hour coverage. A 512 × 512 FFT was used. About 10 points per synthesized beam (to null diameter) were used. The results are shown on the following pages.

The first series of printouts is the calculated beam (equation 5 or 6) normalized to 100 at the peak. The parameter in the top right corner of each plot is the beam location in the output plane (in mm) and the second term is the absolute amplitude of the peak in the beam. Thus 10/0.9999 is the calculated beam for 40^{o} Dec, 12-hour coverage, A-array, 10 mm from the optical axis. The peak beam amplitude (absolute) is 0.9999.

The second series of printout is the difference between the previously calculated beams (above) and a defect-free beam (N.B.) This difference was calculated after a coarse calibration for shift of position and change of gain had been made. The heading information (mm/peak absolute amplitude) is the same as the first printout and there is a 1:1 correspondence between pages in each series.

The numerical calculation performed for the error maps was

$$E = R\left\{\iint e^{i\frac{k}{z}(ux+vy)} S_{uv}\left[\frac{1}{\iint S_{uv}\,dudv} - \frac{D_{uv}^{xy}}{e^{ikz}\iint \frac{S_{uv}\,dudv}{\left(z^2+u^2+v^2\right)^{3/2}}}\right]dudv\right\}$$

(N.B.)

We must still check the output plane scale factors. The input plane scale factors are:

$$72000 \text{ m (uv)} \; \alpha \; \underset{(\pm 25)}{50 \text{ mm}} \text{ or } \underset{(\pm 30)}{60 \text{ mm}}$$

The output plane scale factors should be about

$$100 \text{ mm} \; \alpha \; 61' \text{ arc}$$

Are they?

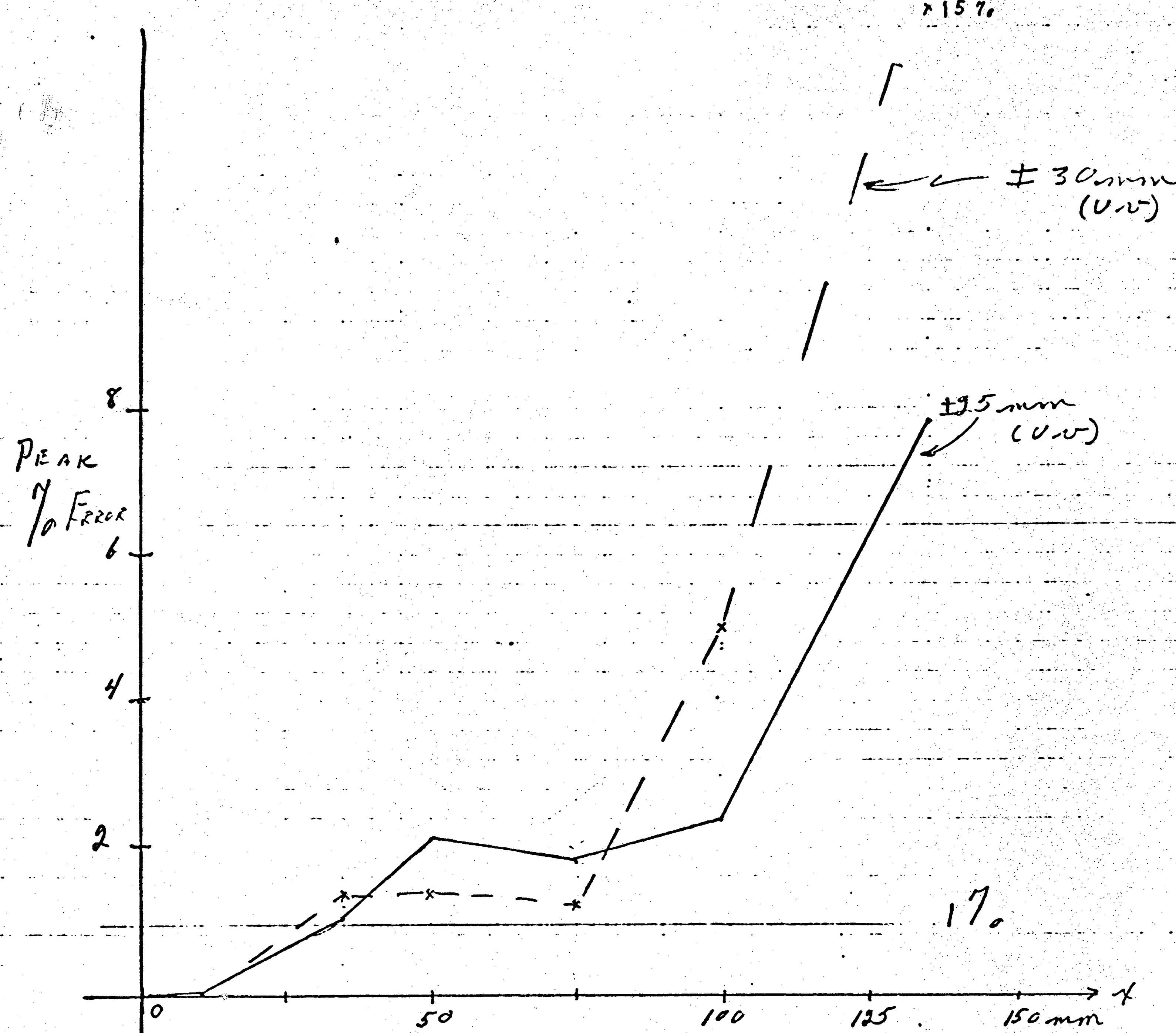
Areas of Interest

We are primarily concerned with the processor error at:

1. the central point in the beam (It should be within 1% of its correct value.)

2. the region outside the peak of the first side lobe (Here the error should be less than 1% of the value at the peak of the beam.)

In these calculations of the error maps, the gain was adjusted so condition (1) is always 0%. The resulting peak error outside the ring of the first side lobe has been manually searched and evaluated. The results are shown on the next page.



"MANUAL PEAK ERROR SEARCH RESULTS"
(FOR COARSE CALIBRATION (ΔX) SIMULATIONS)

VARIABILITY BELIEVED DUE TO QUANTIZING EFFECTS, I.E. ΔX IS TOO LARGE.

We are in the process of simulating an improved calibration for both beam position shift as well as beam focus shift. This will be done for both VLA and clear (fully sampled) apertures (circular).

The second point of view, series expansions, are as follows:

$$D_{\substack{uv\\xy}} = \frac{Z}{R_1 R_2^2 R_3} \left\{ 1 + ik\left(R_1 + R_3 - R_2 - \frac{ux}{z} - \frac{uv}{z}\right) - \frac{k^2}{2}\left(R_1 + R_3 - R_2 - \frac{ux}{z} - \frac{vy}{z}\right)^2 + H.O \right\} \quad (8)$$

$$\approx \frac{1}{Z^3}\left\{ 1 + ik\bar{R} - \frac{k^2}{2}\bar{R}^2 \right\}$$

The resulting map or Fourier transform is (from 6)

$$R\left\{ \frac{|E_r|\,|E_s|}{\lambda}\left[B\,\delta_{(xy)} + b_{xy} * \delta_{\left(x \pm \frac{wc}{2\pi}\right)} \right] * \left[\frac{\delta_{(xy)}}{Z^3} + \frac{ik}{Z^3}\,\bar{r} - \frac{k^2}{2Z^3}\,\bar{r} * \bar{r} \right] \right\} \quad (9)$$

from which we see the defect generated errors to be principally of the form

$$R\left\{ b_{xy} * \frac{k^2}{2Z^3}\,\bar{r} * \bar{r} \right\}$$

where b_{xy} is the map (beam) and $\bar{r}$ is the FT of $\bar{R}$. (N.B.) This result is not strictly correct because $\bar{R}$ is a function of uv and xy. It, thus, must be evaluated for each value of xy. The only value I find in the preceding is the physical understanding intrinsic in equation 9.

It is also possible to write the defect term

$$D_{\substack{uv\\xy}} = \frac{Z}{R_1 R_2^2 R_3}\, e^{ik\left(R_1 + R_3 - R_2 - \frac{ux}{z} - \frac{vy}{z}\right)} \tag{7}$$

as (in one-dimension for ease)

$$\frac{ze^{ik\left(-\frac{ux}{z} + z\sqrt{1+x^2/z^2} + z\sqrt{1+u^2/z^2} + \sqrt{1+(u-x/z)^2}\right)}}{z^4\sqrt{1+x^2/z^2}\;\sqrt{1+u^2/z^2}\left(\sqrt{1+\frac{u-x}{z}}\right)^2}$$

Expanding the square roots

$$\frac{R_1}{Z} = \sqrt{1 + x^2/z^2} = 1 + \frac{x^2}{2Z^2} - \frac{x^4}{8Z^4} + \text{H.O.}$$

$$\frac{R_3}{Z} = \sqrt{1 + u^2/z^2} = 1 + \frac{u^2}{2Z^2} - \frac{u^4}{8Z^4} + \text{H.O.}$$

$$\frac{R_2}{Z} = \sqrt{1 + \frac{u-x}{z}^2} = 1 + \frac{u^2 - 2ux + x^2}{2Z^2} - \frac{\left(u^2 - 2uv + x^2\right)^2}{8Z^4} + \text{H.O.}$$

$$u^4 - 4u^3x + 6u^2x^2 - 4ux^3 + x^4$$

and forming $\left(R_1 + R_3 - R_2 - \frac{ux}{Z}\right)$ we get

$$D_{\substack{uv\\xy}} = \frac{e^{ik\left\{z\left(1 - \frac{4u^3x}{8z^4} + \frac{6u^2x^2}{8z^4} - \frac{4ux^3}{8z^4} + \text{H.O.}\right)\right\}}}{z^3\left[1 + \left(\frac{u-x}{z}\right)^2\right]\sqrt{1 + \frac{u^2}{z^2}}\;\sqrt{1 + \frac{x^3}{z^2}}} \tag{10}$$

Equation 10 shows the form and source of defects and resulting map errors, to wit,

	Function	Error	Calibration
PHASE DEFECTS	$e^{-ik\frac{u^3x}{2Z^3}}$	"Comalike"	Difficult
	$e^{ik\frac{2v^2x^2}{3Z^3}}$	Focus (Field Curvature)	Focal Shift of Sensors
	$e^{-ik\frac{ux^3}{2Z^3}}$	Displacement (Field Dependent)	Δx or Δy Shift of Sensors
AMPLITUDE DEFECTS	$\frac{1}{\left[\left(1+\frac{x-u}{Z}\right)^2\right]}$		Difficult
	$\frac{1}{\sqrt{1+u^2/Z^2}}$	Input Plane Weighting	Input Plane Illumination
	$\frac{1}{\sqrt{1+x^2/Z^2}}$	Output Plane Weighting	Output Plane Gain Adjustment