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MEMORANDUM TO: VLA Processor File

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SUBJECT: Optical Processor System Model

The processor system receives visibility function data as digital I and Q signals which are a function of u, v position. Its output is an electronic signal representative of the sky map obtained by detector array scan of the optical Fourier transform as shown in functional form in Figure 1. The additional functions of map recording and viewing are also included in the diagram. We write expressions for the model without error and noise sources. Work is continuing on incorporation of amplitude and phase error sources.

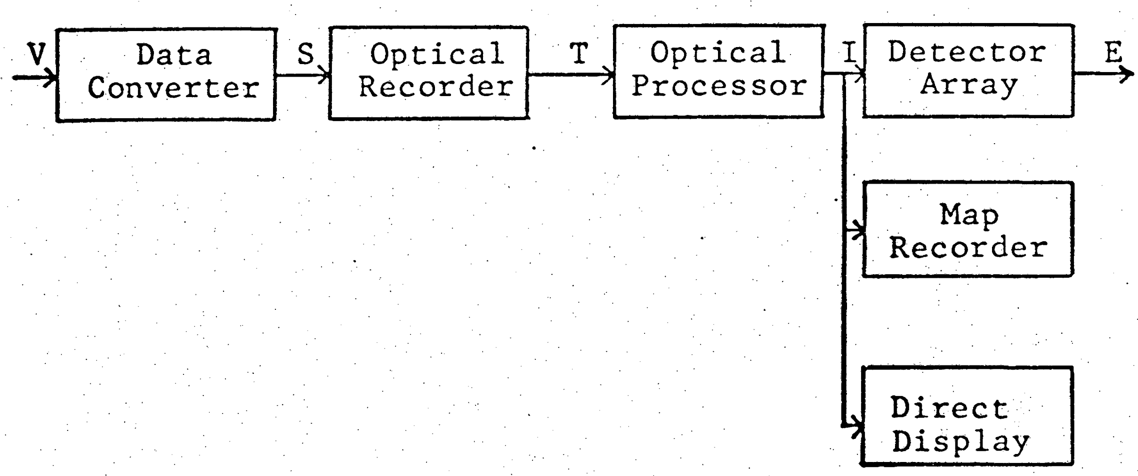

FIGURE 1. PROCESSOR SYSTEM

Data Converter

The data converter serves to generate a real voltage signal S on a suitable carrier from the in-phase and quadrature digital signals V_i and V_q. The signal S which is to be recorded has the form

$$S(u,v) = A(u,v) \left\{ A_T(u,v) \left[b_T + |V| \cos(\omega_1 u + \phi(u,v) \right] + b(1 - A_T(u,v) \right\}$$

Here $|V(u, v)|$ is the magnitude of the visibility function, $\phi(u, v)$ is its phase and ω_1 is the prescribed carrier frequency inserted by the data converter. The aperture of the elliptical tracks is denoted as $A_T(u, v)$. The outer boundary of the total frame of data which may be a rectangular, circle, or Gaussian function, etc., is given by A. The real form of the visibility function has a bias level b_T. The region within the frame aperture A, exclusive of the data tracks A_T, is $(1 - A_T)$ and it has a bias level b.

Optical Recorder

The optical recorder takes the time varying signal S, shown herein in its u,v co-ordinates, and converts it to a spatial recording taken as a spatial variation of optical amplitude transmission t(u, v). Using an analog coding we represent the recorder functions as follows:

$$t(u, v) = S(u, v) * h_1(u, v) * h_2(u, v)$$

Here $h_1(u, v)$ is the impulse response of the writing beam at the surface of the recording material. It characterizes the beam spot distribution and the gain (beam intensity versus input voltage) and can be expressed as

$$h_1 = K_1 e^{-\frac{u^2+v^2}{2\sigma_1^2}}$$

K_1 is the beam gain and has dimensions of beam intensity per unit input voltage. For linear operation K_1 is a constant and for non-linear operation K_1 will be a function of the input voltage S. The Gaussian function is the beam spatial distribution. We note that $S * h_1$ gives the exposure energy density $\mathcal{E}(u, v)$ caused by the writing beam. Beam position errors ε_u, ε_v can occur in u and v, (i.e., $u \to u + \varepsilon_u$ and $v \to v + \varepsilon_v$) taken in either h_1 or A_T but not both.

The recording film is characterized by its impulse response $h_2(u,v)$ and can be approximated as

$$h_2 = K_2 e^{-\frac{u^2+v^2}{2\sigma_2^2}}$$

K_2 is the recording film gain (sensitivity) and has dimensions of film amplitude transmission per unit input exposure E (energy/area). For linear operation K_2 is a constant while for non-linear operation it is a function of the input exposure $e(u, v)$. The Gaussian function characterizes the spatial distribution of the film impulse response.

The spatial Gaussian distributions for h_1 and h_2 when Fourier transformed give the corresponding MTF's. The product of the individual MTF's is the MTF of the complete optical recorder. We include in the beam response h_1 the response of the amplifier used to feed the beam intensity modulator.

The recorded data t(u, v) can be expanded to show its input signal dependences as follows:

$$t(u,v) = T_0(u,v) + T_*(u,v) + T(u,v)$$

$$T_0(u,v) = \{A(u,v)A_T(u,v)(b_T-b) + bA(u,v)\} * h_1(u,v) * h_2(u,v)$$

$$T_*(u,v) = \{A(u,v)A_T(u,v)|V| \; e^{-j[\omega_1 u+\phi(u,v)]}\} * h_1(u,v) * h_2(u,v)$$

$$T(u,v) = \{A(u,v)A_T(u,v)|V| \; e^{+j[\omega_1 u+\phi(u,v)]}\} * h_1(u,v) * h_2(u,v)$$

Optical Processing Channel

The optical processing channel, which will be taken here as the beyond the lense input plane type with an output reference wave, provides as its output a light intensity distribution I(x,y) at the processor output plane. This output has the form

$$I(x, y) = |U(x, y) + U_r(x, y)|^2$$

$$= \left| A_i \, e^{j\theta_i} F[t] + A_r \, e^{j\theta_r} \right|^2$$

Here we have U as the optical Fourier transform field and U_r the reference wave. $A_i(u, v)$ is the amplitude of the convergent optical Fourier transform illumination beam at the input plane and A_r is the amplitude of the reference wave at the output Fourier transform plane. θ_i and θ_r are light field phase terms which are a function of x and y. t(u, v) is the recorded data that is Fourier transformed as expressed below.

$$F[t] = \iint \left[T_0 + T_* + T\right] e^{i2\pi \left[\left(\frac{x}{\lambda d}\right) u + \left(\frac{y}{\lambda d}\right) v\right.} \; du \, dv$$

Expansion of $|U + U_r|^2$ for I(x, y) gives the output light intensity of the processor which can be written as

$$I(x, y) = \left\{ A_i e^{j\theta_i} F \right\} \left\{ A_i e^{j\theta_i} F \right\}^*$$

$$+ \left\{ A_r e^{j\theta_r} \right\} \left\{ A_r e^{j\theta_r} \right\}^*$$

$$+ \left\{ A_i A_r^* \, e^{-j(\theta_r - \theta_i)} F \right\} + \left\{ A_i^* A_r e^{j(\theta_r - \theta_i)} F^* \right\}$$

$$= A_r^2 + A_i^2|F|^2$$

$$+ 2A_iA_rF' \cos(\theta_r - \theta_i)$$

$$+ 2A_iA_rF'' \sin(\theta_r - \theta_i)$$

Here we have written F for the transform $F[t(u, v)]$ with F' and F'' being real and imaginary parts of F. Note that F is normally real (F'' = 0) however certain instrument phase errors can give rise to F''.

Finally, we denote the Fourier transform $F[t]$ as

$$F[t(u, v)] = B_o(x,y) + B_*(x,y) + B(x,y)$$

Here B_* and B are simply the conjugate pairs of output plane sky map data, while B_o is the zero-order diffracted light at the output. They are written as

$$B_o(x,y) = F[T_o(u, v)]$$

$$B_*(x,y) = F[T_*(u, v)]$$

$$B(x,y) = F[T(u, v)]$$

Recall that

$$T_o = \left\{A(u,v)\ A_T(u,v)(b_T - b) + bA(u,v)\right\} * h_1(u,v) * h2(u,v)$$

$$T_* = \left\{A(u,v)\ A_T(u,v)\ |V| e^{j[\omega_1 u + \phi]}\right\} * h_1(u,v) * h_2(u,v)$$

$$T = \left\{A(u,v)\ A_T(u,v)\ |V| e^{-j[\omega_1 u + \phi]}\right\} * h_1(u,v) * h_2(u,v)$$

We can now write the output light intensity distribution of the optical channel as

$$I(x,y) = A_r^{\ 2} + A_i^{\ 2}\left(|B_o|^2 + |B_*|^2 + |B|^2\right)$$

$$+ A_i^{\ 2}\left(B_o B_*^* + B_o^* B_* + B_o B^* + B_o^* B + B_* B^* + B_*^* B\right)$$

$$+ 2A_i A_r\ (B_o + B_* + B)'\ \cos(\theta_r - \theta_i)$$

$$+ 2A_i A_r\ (B_o + B_* + B)''\ \sin(\theta_r - \theta_i)$$

Note that A_r, B_o, B_*, B, θ_r, θ_i are in general x, y dependent. The desired sky map portion of the output is taken here as B and its corresponding light intensity distribution I_B is

$$I_B = 2A_iA_r B' \cos(\theta_r - \theta_i)$$

To obtain an output term of this form and to be rid of the other terms in I, we must provide for the following:

(1) B should be suitably isolated from B_o and B_*. This will be provided by proper choice of the carrier ω_1 on the input S(u,v) and by proper control of the zero order beam B_o through selection of the input bias $b_T - b = 0$ and with proper dynamic orientation of A(u,v) and therefore B_o relative to B.

(2) The term containing the imaginary part ( )" $\sin(\theta_r - \theta_i)$ should be removed. This will be accomplished by use of an on-axis reference beam such that $(\theta_r - \theta_i) = 0$ or π. In addition ~~or in one~~ by taking the difference between two separate scans of the output plane for the cases where $\theta_r - \theta_i = 0$ and $\theta_r - \theta_i = \pi$ ~~we~~ cancel terms not containing the factor $\cos(\theta_r - \theta_i)$.

(3) We note that the term $B_o B_*^* + B_o^* B_* + B_o B^* + B_o^* B + B_* B^* + B_*^* B$ is known and must be made small and/or cancelled. This will be accomplished by (1) and (2) above for the first four parts of this term. The terms $B_* B^*$ and $B_*^* B$ are essentially non-overlapping and therefore zero.

If these provisions are made with no error in the process, we will have, when interpreted at the processor output plane,

$$I_B(x,y) = I_o(x,y) - I_\pi(x,y)$$

$$= 4A_i A_r \, B(x,y) \cos(\theta_r - \theta_i)$$

At present, the subtraction will be accomplished electronically at some point after the output has been scanned with a detector array. This procedure may be done in various ways. Using the conjugate sky map data B_* at the processor output, along with the data for B may allow implementing the I_o and I_π concurrently. In this case the subtraction $I_o - I_\pi$ might be done on a line-by-line basis in order to minimize buffer storage demands. Alternatively, only the B data (not B_*) can be used and the detector can dwell at each line position in the output plane while reading out the data first for I_o and then I_π. Still another possibility is the scan of the complete frame for I_o and then the I_π frame, and then subtraction of these data frames.

Output Map Detector Array

The photo-detector array used to convert the desired portion of the processor light distribution I_B to an electronic signal, is a set of discrete detector elements. Each detector element has a local spatial detection response $h_3(x,y)$ where we will write h_3 as

$$h_3 = K_3 A_d(x,y)$$

Here K_3 is a sensitivity or gain constant of the detector element which has units of volts per Joule/mm^2 of light energy incident. For linear operation K_3 would be independent of the magnitude of the input $I_{(x,y)}$. A_d defines the detector element spatial response and its finite aperture size.

In general, we use the detector array to detect over the area of interest, that is, the detector provides a two-dimensional sampling of the output. The sampling is defined by the comb function with spacing d_x and d_y which is written as

$$\text{Ш}\left(\frac{x}{d_x}\right)\text{Ш}\left(\frac{y}{d_y}\right)$$

Errors in detector array position will be identified as ε_x and ε_y and if present require that we let $x \rightarrow x + \varepsilon_x$ and $y \rightarrow y + \varepsilon_y$ in either Ш or $I(x,y)$.

Operation of the detector array provides the following voltage output

$$E(x, y) = \left\{ I(x, y) * K_3 A_d(x, y) \right\} \Pi\Pi \left(\frac{x}{d_x}\right) \Pi\Pi \left(\frac{y}{d_y}\right)$$

Directly Viewed Output

The light intensity distribution at the optical processor sky map output may be viewed directly by projecting this output onto a viewing screen. Such a viewing system would make use of a suitable projection lense and probably a back illuminated small screen. The combined response of the provisions used for such viewing can be characterized by a spatial impulse response function denoted here as $h_4(x, y)$. This response can be approximated as

$$h_4 = K_4 \, e^{-\frac{x^2+y^2}{2\sigma_4^2}}$$

K_4 is the gain constant with units of output (viewing) watt per watt per unit area and unit solid angle ~~from~~ for a given light ~~distribution~~ power density at the processor output plane.

Sky Map Recording

The sky map data can be recorded in at least three possible ways:

(1) A direct snapshot of the sky map output plane with or without the reference wave.

(2) Recording the detector array output onto film using a line scanned optical recorder.

(3) Detector array output conversion to digital tapes which are then played back.

(i) into a line scanned optical recorder

(ii) onto a TV screen which is then photographed

We will not treat sky map data recording for the present time in our system model.

Model Analyses

The model will be used to estimate system performance theoretically and to assess in particular the affects of:

(1) Recorder scan uncertainties (phase errors), amplitude noise sources, film phase error contributions and film shrink distortions.

(2) Phase errors and additive noise in the optical components of the optical processing channel (including laser noise.

(3) Detector array scan errors and amplitude noise sources.

(4) Non-linear gain and scale factor properties of the recorder, optical channel and detector array.

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