ERRORS IN AN OPTICAL FOURIER TRANSFORM PROCESSOR DUE TO CERTAIN PHASE AND AMPLITUDE DEFECTS

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I. INTRODUCTION

Defects (amplitude and phase) in an optical processor cause errors in the output map. These effects have been studied analytically and numerically. Here we present a tabular summary of our efforts to date. Our goal is to share current understanding for the purpose of identifying phase (and to a lesser extent, amplitude) defect specifications which will result in VLA maps meeting an astronomically acceptable "1%" map error criteria.

The following pages are a tabular summary of

1. Simulation Performed
2. Calibration and Compensation Techniques
3. Error Criteria
4. Numerical Results
5. Some Design Considerations and Conclusions

II. SIMULATION

Our simulations are essentially numerical experiments performed in a digital computer. We have concentrated on two principal kinds of phase defects. They are (1) the spherical wave defect and (2) polynomial defects. Both are so-called "low frequency defects" because they contain less than one full cycle of a sin wave (in the input aperture of the processor) if expanded in a Fourier series. Highfrequency defects, i.e., more than one cycle of a Fourier component in the input aperture are not considered here. They must, however, be kept to less than 1/100 the minimum visibility amplitude we desire to map.

The spherical wave defect has its origin in the Huggens spherical wave fronts which diverge from every point of the input plane to every point in the output plane. These wave fronts can only be approximated by plane waves (i.e., the Fourier kernel) over a finite region. This leads to the concept of a finite space-bandwidth product. The magnitude of this product depends on the magnitude and type of error acceptable in the processor output. Since this defect (the difference between the perfect Fourier transform and the optical Fourier transform) has its origin, the propagation of spherical wave fronts, the concept of a finite space-bandwidth product for optical transform instruments is fundamental.

Polynomial defects originate in the usual wave aberrations of an optical system, as well as in the positioning errors inherent in laser and electron-beam recorders (the most attractive types for this processor). More correctly, the positioning errors can be modeled conveniently and accurately with a power series in the input plane.

As will be seen, we are most concerned with the low-order, odd-power (1, 3, 5) terms in the power series, the even powers (2, 4, 6) contributing significantly less error to the output map. We have simulated the low-order even and odd terms and calculated the resulting map error for both the VLA aperture and a clear circular aperture. By the VLA aperture we mean an aperture of unity transmission at the locus of points in the V-v plane sampled by the telescope. A clear aperture, in contrast, has unity transmission everywhere. Uniform aperture weighting was usually simulated.

Since it is possible to remove some of the resulting map errors by gain and position adjustment of the diode sensors array (in the output plane), we have simulated these error-correction techniques also. The term "error compensation" refers to error correction of stationary defects in the processor. The term "error calibration" refers to error correction of map-to-map dependent errors.

This simulation has been performed with both FFT and DFT (direct Fourier transforms) techniques in a large digital computer. There are interesting differences in our results which depend on the size of the FFT array used. These are important differences and I (LES) do not fully understand their significance. It seems, however, that the larger the FFT array (1024 vs 512, for example) the closer tne simulated result is to the true Fourier transform.

We turn now to calibration and compensation techniques and means of simulating them. This is followed by some numerical results.

III. CALIBRATION AND COMPENSATION

Calibration and compensation remove output-plane dependent geometric and radiometric (brightness) defects only. Four principal defects can be partially removed this way. They are (1) linear phase shift in the input plane, (2) cubic phase shift in the input plane, (3) quadratic phase shift in the input plane and (4) scale change in the input plane. Also, many amplitude and phase defects reduce the brightness in the output plane by a predictable amount. This effect too can be partially removed.

The spherical wave defect is represented by terms of the form (in one dimension for ease)

$$D_{\substack{uv\\xy}} = \frac{e^{ik\left[Z\left(1 - \frac{4U^3x}{8Z^4} + \frac{6U^2x^2}{8Z^4} - \frac{4Ux^3}{8Z^4} + H.O.\right)\right]}}{Z^3\left[1 + \left(\frac{U-x}{Z}\right)^2\right]\sqrt{1 + U^2/Z^2}\ \sqrt{1 + x^2/Z^2}}$$

For large Z, $D_{\substack{uv\\xy}}$ is almost equal to $1/Z^3$ being slightly less in magnitude and containing a real, as well as an imaginary component. Z is a constant fixed distance in the processor, U is the input plane coordinate and x is the output plane coordinate. Details of the derivation of $D_{\substack{uv\\xy}}$ are found in VLA memorandum .

It is possible to compensate for the linear phase shift term $\exp\left[-ikZ(4Ux^3/8Z^4)\right]$ in the output plane by positioning the sensor array at the corresponding displacement in the output plane. Similarily, the focal shift term $\exp\left[ikZ(6U^2x^2/8Z^4)\right]$ can be compensated by a shift of focus being applied to the sensor array. The cubic phase shift can be partially compensated by a displacement in the output plane corresponding to a linear approximation to the cubic.

In the numerical results (following), we have simulated these compensations by removing the corresponding defect from $D_{\substack{uv\\xy}}$. The linear phase shift compensation is referred to as "linear"; the quadratic as "focus".

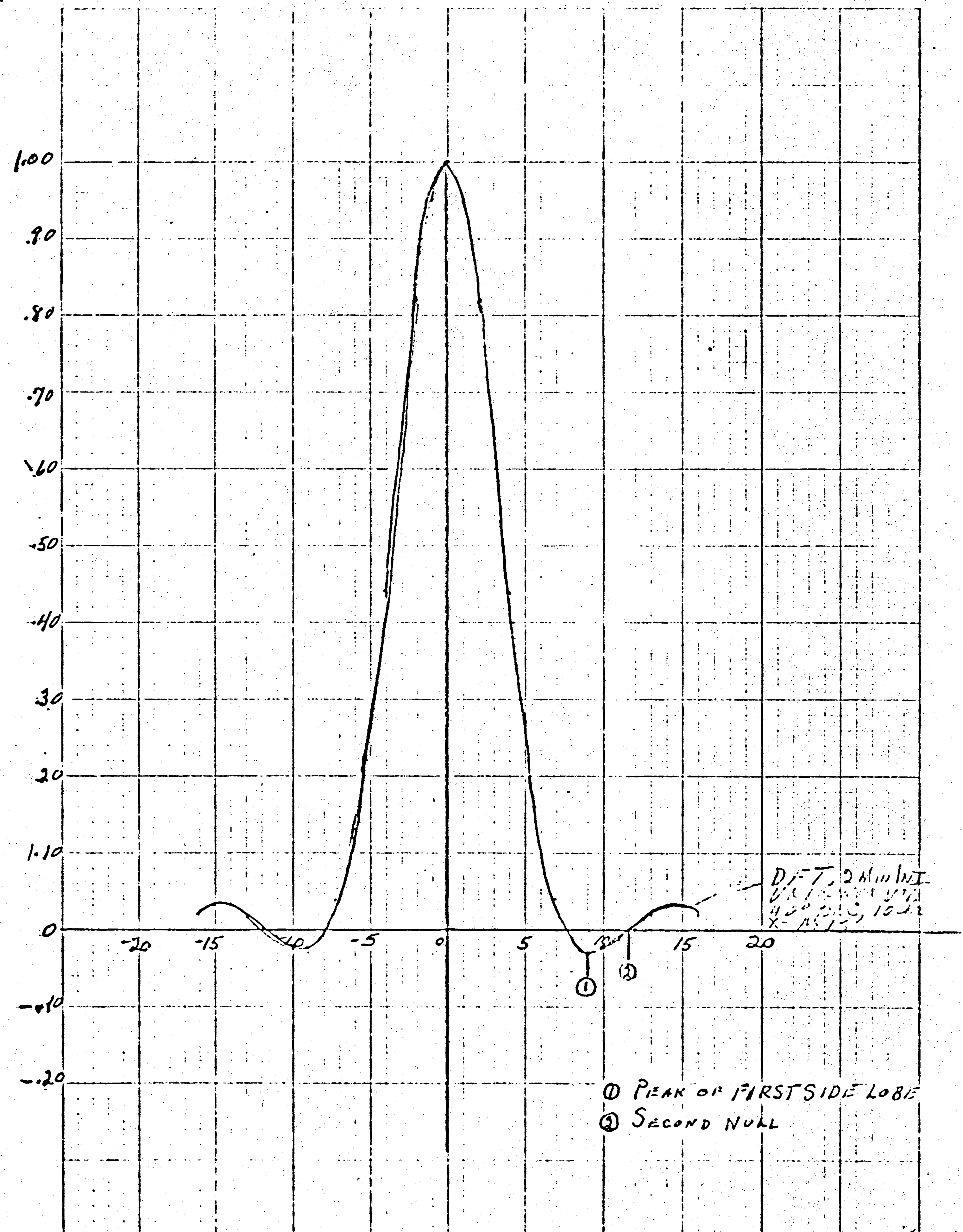
Turning to the polynomial defect simulations, these two can be partially compensated as described for the spherical wave defect terms of power 1, 2 and 3 (in U). We (NRAO) have not performed any compensated polynomial simulations at this writing. Aleksoff (ERIM) has simulated the removal of the cubic by means of a linear phase shift with meaningful improvement (a map error reduction of 2.5).

IV. ERROR CRITERIA

We are concerned with the difference between the true Fourier transform and the output of the optical processor. We wish this difference or error to be less than 1% of the peak value of the map (image) of a point source. This error is to be less than 1% of the peak radio brightness at the center of the point source image and everywhere beyond (at values of x and y greater than) the peak of the first side lobe of the point source image. We have also included in this study the errors beyond the second null of the point source image.

In terms of a synthesized beam, the above regions of interest are shown in Figure 1.

The errors we have studied with FFT and DFT techniques have always been in the region of the point image; e.g., four to six side lobes in all directions. We have found that the errors (error maps) are well behaved with peaks (peak error) near the 3 dB point of the main lobe and falling nanotomically to small values within a few beam diameters. We have not studied map errors in many hundreds of beam diameters from the point object. Should this be done?


1.00
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.80
.70
.60
.50
.40
.30
.20
.10
0
-.10
-.20
-20
-15
-10
-5
0
5
10
15
20
① Peak of First Side Lobe
② Second Null

FIGURE 1