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MEMORANDUM TO: VLA File

FROM: C. Aleksoff

SUBJECT: Antihermitian Input Possibilities

In this memo we present the concept that the VLA input (visibility) signal could be written as an antihermitian signal as easily as it can be written as an hermitian signal. We have previously considered the case of a hermitian input and the case of a half-plane input. However, the input signal could be written as an antihermitian function and hence the desired part of the output signal would now be the imaginary part rather than the real part.

Let us consider the problem mathematically. An input signal $S(u,v)$ can always be broken up into its hermitian part S_h and its antihermitian part S_a;

$$S = S_h + S_a \tag{1}$$

where

$$S_h = \frac{S+S^\dagger}{2} \tag{2a}$$

$$S_a = \frac{S-S^\dagger}{2} \tag{2b}$$

$$S^\dagger(u,v) = S^*(-u,-v) \tag{2c}$$

If its FT is s(x,y) then we can write

$$s = s' + i\,s'' \tag{3}$$

where s' is its real part and s" its imaginary part such that S_h is the FT of s' and S_a is the FT of i s".

Hence, the output intensity of the processor is given by

$$I(\phi) = R^2 + s^2 + 2Rs' \cos\phi - 2Rs'' \sin\phi \tag{4}$$

where $Re^{i\phi}$ is the reference wave, and s the signal wave. We note that

$$I(\pi) - I(0) = 4Rs' \tag{5}$$

and that

$$I(-\tfrac{\pi}{2}) - I(\tfrac{\pi}{2}) = 4Rs'' \tag{6}$$

Hence, either the real or imaginary part of s can be isolated.

If the aberrations were of proper form then it might be more advantageous to use the antihermitian form rather than the hermitian form on input. For example, if the phase errors were of odd order, i.e., $\phi \propto X^{2n+1}$ then all these aberration errors would contribute to the real part of the output (while every even order phase error contributes to both the imaginary and real part). However, a check of the spherical errors associated with the spherical wave processor configuration indicates that the real part of the second order phase error (focusing error) is large and would need to be locally corrected in the output plane by focusing. But, if it were corrected by focusing, the next significant error is the 4th order, which we could expect to be small for the VLA aperture. A linear positional correction is entirely avoided.

The main advantage to using the antihermitian part appears to be in its removal of the bias effect. Consider that if the signal is recorded on film it has the form

$$S = A(b + V) \tag{7}$$

where b is the bias level, A(u,v) the aperture, and V(u,v) the visibility function Thus

$$s = ba + a * B \tag{8}$$

where brightness B is the FT of V and a(u,v) is the FT of A. Typically A is hermitian. If V is also made hermitian then s would be entirely real. Notice that the bias term could overlap into the desired term. However if V were made antihermitian then a * B would be pure imaginary and the bias effects would be removed by using the subtraction process indicated by eq (6).

Now if the desired output signal is entirely in the real or imaginary part then the other-part can be used for calibration signals. For example, a grid and some reference stars of different brightnesses could be incorporated in the other-part to test the system for errors. Notice, that aberration errors could couple energy from one part to the other and hence low level calibration signals might be necessary.

We should also point out that a time varying signal, could be used to do real time phase stabilization by making only "one part" of the output insensitive to the time variations. For example, if

$$\phi = M \cos (\omega t) + \phi_o$$

is used in (4) it is seen that $I(\phi)$ has time varying components that could be monitored in real time to make sure the phase ϕ_o is proper. However for long integrations (with respect to $1/\omega$) the output given by (5) for ϕ_o changed from 0 to π is

$$4Rs'J_o(M)$$

where J_o is the zero order Bessel function. For small amplitude M we have $J_o(M) \approx 1$. Thus, the output would be the same as before except for the addition of continuous phase monitoring capabilities.

CA/pw

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Appendix A
Detailed Equations for VLA Input

Let $V_n = M_n \exp(i\phi_n)$, where $M_n = |V_n|$, be the complex input value for the position (u_n, v_n). We form the signal S from cosine or sine functions as

$$S = \sum_n M_n H(u - u_n, v - v_n) {}^{\sin}_{\cos}(2\pi x_o u + \phi_n)$$

$$+ M_n H(u + u_n, v + v_n) {}^{\sin}_{\cos}(2\pi x_o u - \phi_n) \qquad (1)$$

where H describes the envelope of the recorded spot and x_o is the offset frequency. The FT of S is

$$s = \left({}^{-i}_{+}\right) h(x,y) \{ \delta(x - x_o) *$$

$$\Sigma M_n \cos [2\pi(u_n x + v_n y) + \phi_n]$$

$$\mp \delta(x + x_o) *$$

$$\Sigma M_n \cos [2\pi(u_n x + v_n y) - \phi_n] \} \qquad (2)$$

Thus, as long as H(u,v) is hermitian, i.e., h(x,y) real, then s will be pure real if a cosine is used and pure imaginary if a sine carrier is used.