D. Andrich

ERIM EO-77-1002

FORMERLY WILLOW RUN LABORATORIES, THE UNIVERSITY OF MICHIGAN

7 January 1977

MEMORANDUM TO: VLA Optical Processor File

FROM: James R. Fienup

SUBJECT: On-Axis Impulse Response

In this memo we expand on the analysis of the on-axis impulse response terms discussed in a previous memo (J.R. Fienup, "Basic Limitations of Encoding Methods", 21 September 1976), including the effect of non-zero bias in the non-track areas.

CONTENTS

1. Introduction
2. Reduction of the On-Axis Dirty Beam
    2.1 Subtract it
    2.2 Make the brightness distribution imaginary
    2.3 Equalize the bias levels
    2.4 Clip $|V|_{max}$ to boost η_V
    2.5 Complementary weighting
    2.6 Summary; combination of methods
3. Large-Aperture Impulse Response
4. Scattered Light
    4.1 Film noise
    4.2 Optical system noise
5. Summary and Conclusions

1. INTRODUCTION

In a Fourier transform optical processor using a lens of focal length f and wavelenth λ, the complex amplitude of the output wavefront, f(x,y), is related to the input wavefront, F(u,v) by the Fourier transform relation

$$f(x,y) = \frac{1}{\lambda f} \iint F(u,v) \exp\left[-\frac{j2\pi}{\lambda f}(ux + vy)\right] du\, dv \qquad (1)$$

where constant phase factors have been dropped. Rayleigh's (or Parseval's) theorem (the conservation of flux for an optical system) gives

$$\iint |f(x,y)|^2\, dx\, dy = \iint |F(u,v)|^2\, du\, dv \qquad (2)$$

In this memo we will consider the following Fourier transform pairs: the visibility function $V(u,v) = |V(u,v)| \exp[j\phi(u,v)]$ and the brightness distribution, B(x,y); the VLA aperture $A'(u,v) = \{1$ in track areas, 0 in non-track areas$\}$ and its impulse response (the "dirty beam") $a'(x,y)$; and, in the case of non-zero bias in the non-track areas, an overall aperture $A_o(u,v)$ and its impulse response $a_o(x,y)$. Note that $A'(u,v)$ is not exactly the VLA aperture: it assumes compensation for track overlap.

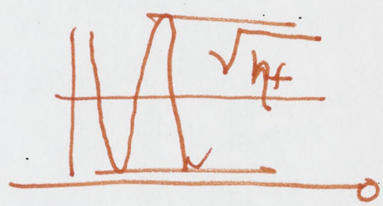
We assume that the transmittance of the input transparency, which is illuminated by a unit plane (or converging spherical) wave is given by

$$H(u,v) = A_o(u,v) \left\{ A'(u,v) \left[B_o + B_1 |V(u,v)| \cos(\omega_o\ u + \phi(u,v)) \right] + \beta \left[1 - A'(u,v) \right] \right\}$$

$$= A_o(u,v) \left\{ \beta + A'(u,v) \left[(B_o - \beta) + B_1 |V(u,v)| \cos(\omega_o\ u + \phi(u,v)) \right] \right\} \quad (3)$$

$B_1 = \frac{\sqrt{\eta_f}}{2|V|_{max}}$ ← Prevents clipping

where β is the bias transmittance in the non-track areas, B_o is the bias transmittance in the track areas, $\omega_o u$ is the carrier term, $B_1 = \sqrt{\eta_f}/(2|V|_{max})$, and $\sqrt{\eta_f}$ is the maximum peak-to-peak amplitude transmittance of the input transparency.



The resulting image is given by

$$h(x,y) = \beta\ a_o(x,y) + (B_o - \beta)\ a_o(x,y) * a'(x,y)$$

$$+ (B_1/2) a_o(x,y) * a'(x,y) * B(x + x_o, y) + \text{conjugate image} \quad (4)$$

where $x_o = \omega_o \lambda f$. If $A_o(u,v)\ A'(u,v) = A'(u,v)$, then $a_o(x,y) * a'(x,y) = a'(x,y)$.

Let $W_B{}^2$ be the area of the brightness distribution B(x,y) in the output plane of the processor; let $W_T{}^2$ be the area in the u-v plane covered by tracks:

$$W_T{}^2 = \iint A'(u,v)\,du\,dv \; ; \tag{5}$$

and let

$$W_o{}^2 = \iint A_o(u,v)\,du\,dv. \tag{6}$$

Also, make the following definitions:

$$B_{max} = \mathrm{Max}\left\{a'(x,y) * B(x,y)\right\} \tag{7}$$

$$\overline{B^2} = W_B{}^{-2} \iint |a'(x,y) * B(x,y)|^2\,dx\,dy \tag{8}$$

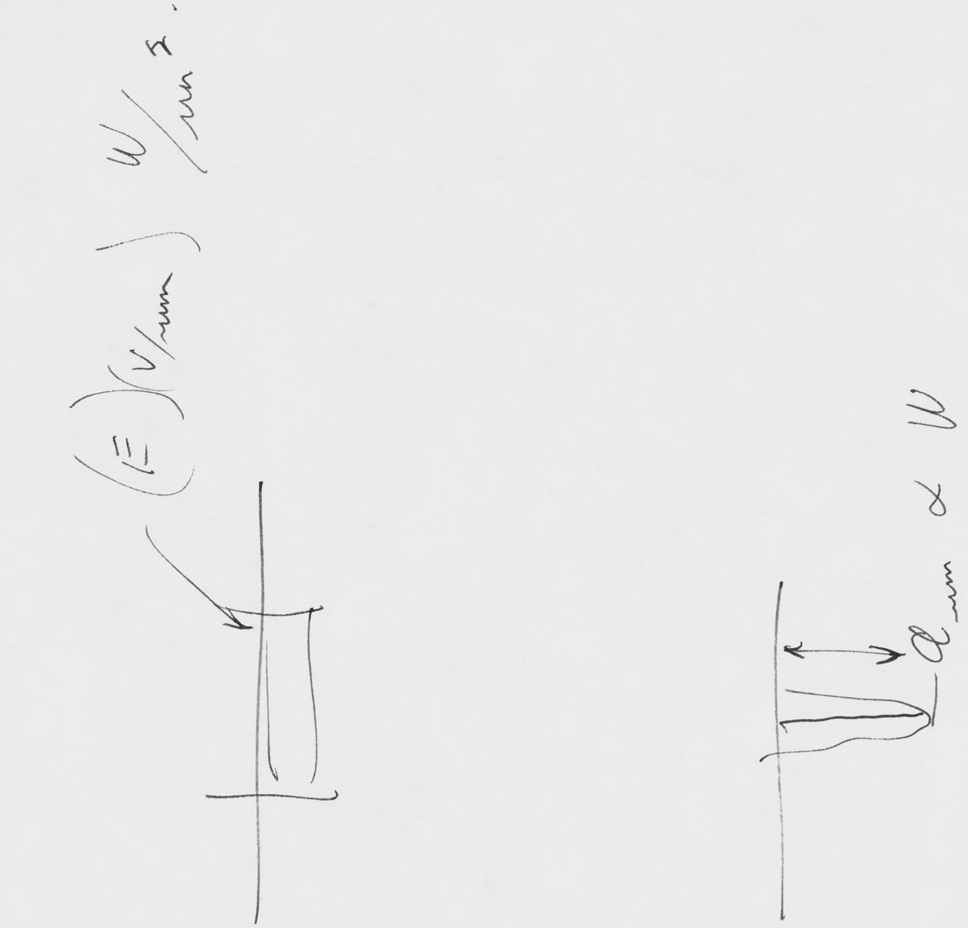
$$\eta_c = \frac{B^2{}_{max}}{\overline{B^2}} \tag{9}$$

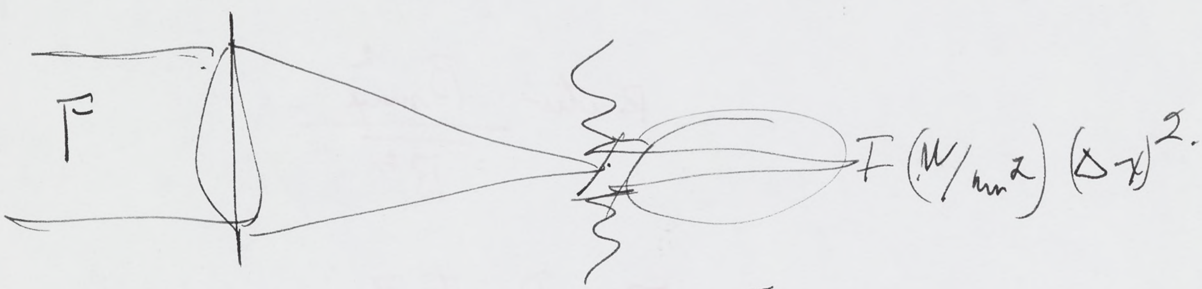
Ratio $\frac{B_{max}^2}{\overline{B^2}}$

$$a'_{max} = \mathrm{Max}\left\{a'(x,y)\right\} \tag{10}$$

← Purity Beam

$$\overline{a'^2} = W_B{}^{-2} \iint |a'(x,y)|^2\,dx\,dy \tag{11}$$





$$\frac{F}{(\Delta x)^2}$$

$$F = I (\Delta x)^2$$

$$|V|_{max} = \text{Max}\left\{A'(u,v)\ V(u,v)\right\} \tag{12}$$

max Vinish (?)

$$\overline{|V|^2} = W_T^{-2} \iint |A'(u,v)V(u,v)|^2\, du\, dv \tag{13}$$

and

$$\eta_V = \frac{\overline{|V|^2}}{|V|^2_{max}} \tag{14}$$

In the case of a square aperture of width D in the input plane, the output is equal to a_{max} sinc($x/\Delta x$)sinc($y/\Delta x$), where sinc(ζ) = sin($\pi\zeta$)/($\pi\zeta$) and $\Delta x = \lambda f/D$. Since $\int_{-\infty}^{\infty} \text{sinc}^2(\zeta)d\zeta = 1$, the total flux in that output would be given by $a_{max}{}^2(\Delta x)^2$. Thus, $a_{max}{}^2$ = (total flux)/$(\Delta x)^2$. Assuming that this relation also holds for the VLA aperture, we have

$$\overline{a'^2} = W_B^{-2} \iint |a'(x,y)|^2 dx\, dy = W_B^{-2}(\Delta x)^2 a'_{max}{}^2 \tag{15}$$

If we define a resolution element as having width Δx, which is the peak-to-null width (and is approximately the half-power width), then the number of resolution elements in the image is

$$N = W_B{}^2/(\Delta x)^2. \tag{16}$$

Then Equation (15) gives us

$$a'^2_{max} = NW_B^{-2} \iint |a'(x,y)|^2 \, dx \, dy \qquad (17)$$

Using Equation (2) (Rayleigh's theorem) and Equation (15), making use of the fact that $|A'(u,v)|^2 = A'(u,v)$, this becomes

$$a'^2_{max} = NW_B^{-2} W_T^2 \qquad (18)$$

Similarly, using Equations (9), (8), (2), and (13), we have

$$B^2_{max} = \eta_c \overline{B^2} = \eta_c W_B^{-2} W_T^2 \overline{|V|^2} \qquad (19)$$

From Equations (7) and (4), we see that the peak amplitude, S_p, of the brightness distribution output is given by

$$S_p = B_{max} B_1/2 = B_{max} \sqrt{\eta_f}/(4|V|_{max}) \qquad (20)$$

Combining Equations (19) and (14) with Equation (20) gives

$$S_p = (\eta_c \eta_V \eta_f W_B^{-2} W_T^2/16)^{\frac{1}{2}} \qquad (21)$$

Equation (21) makes sense from a physical viewpoint. The factor of 1/16 is the diffraction efficiency of the simple carrier method of encoding; W_T^2/W_B^2 is the ratio of the areas of the two domains in which the flux is concentrated; η_f is the efficiency at which the dynamic range of the film is used; η_V is the efficiency factor due to the scaling of $|V(u,v)|$ to be no greater than unity everywhere; and η_c is the compression gain of the Fourier transform process.

From Equation (4), we see that the peak amplitude, N'_p, of the on-axis "dirty beam" term is given by

$$N'_p = |B_o - \beta| a'_{max} \tag{22}$$

Inserting Equation (18) gives

$$N'_p = \left[(B_o - \beta)^2 N\, W_B^{-2} W_T^2\right]^{\frac{1}{2}} \tag{23}$$

Comparing this expression for "noise" with Equation (21) for the "signal" we see that the factor of 1/16 is replaced by unity, η_f is replaced by $|B_o - \beta|$, η_V is replaced by unity, and η_c is replaced by N. Combining Equations (21) and (23), we have the ratio of the peak amplitude of the signal to the peak of the noise:

$$\frac{S_p}{N'_p} = \left(\frac{\eta_c \eta_V \eta_f}{16N(B_o - \beta)^2}\right)^{\frac{1}{2}} \tag{24}$$

The peak value of the a'(x,y) noise term is on the optical axis; its sidelobes die down very quickly to about the 3×10^{-3} (-25 dB) level, but then the sidelobes stay at that level even for large distances from the peak. Thus, the ratio of the dirty beam noise at a given point in the image to the peak of the dirty beam is given by (on the average)

$$N'/N'_p \simeq 3 \times 10^{-3} \simeq 1/300 \tag{25}$$

Thus, the signal to noise ratio is

$$\frac{S_p}{N'} = \frac{N'_p}{N'} \frac{S_p}{N'_p} \simeq 300 \left(\frac{\eta_c \eta_V \eta_f}{16N(B_o - \beta)^2} \right)^{\frac{1}{2}}. \tag{26}$$

In order to satisfy the 1% criterion, this ratio must be equal to 100 or greater. Thus, it is required that

$$\frac{S_p}{N'_p} = 1/3 = (\eta_f \eta_V \eta_c / N)^{\frac{1}{2}} (4B_o |1 - \beta/B_o|)^{-1} \tag{27}$$

in order to just meet the 1% criterion. Table 1 shows values of $|1 - \beta/B_o| S_p/N'_p$ for various astronomical objects. As can be seen from Table 1, only a scene consisting of a single star can be processed to meet the 1% criterion unless the on-axis dirty beam is substantially reduced (or the signal increased).

Table 1
Signal to noise ratios for various astronomical objects. Here it is assumed that $\eta_f = 0.5$ and $B_o = 0.5$. The final row shows the degree to which the on-axis dirty beam term must be reduced in order to just satisfy the 1% criterion.

OBJECT	SINGLE STAR	100 WEIGHTED STARS	100 EQUAL STARS	Cas-A
η_c/N	1.0	.355	10^{-2}	1.44×10^{-4}
η_V	1.0	10^{-1} to 10^{-2}	10^{-1} to 10^{-2}	4.09×10^{-3}
$(\eta_f \eta_V \eta_c/N)^{\frac{1}{2}}/4B_o$	.35	$(.67 \text{ to } .21) \times 10^{-1}$	$(1.1 \text{ to } .35) \times 10^{-2}$	$.27 \times 10^{-3}$
same in dB (signal level)	5 dB	12 to 17 dB	20 to 25 dB	36 dB
1% of signal = "noise" allowed	25 dB	32 to 37 dB	40 to 45 dB	56 dB
reduction of 25 dB dirty beam required	0 dB	7 to 12 dB	15 to 20 dB	31 dB

η_c = Processor Gain
N = # Resolution cells/map.

η_f "

$\eta_v = \frac{|V|^2}{|V|^2_{max}}$ Source Dep

$\eta_c = \frac{B^2_{max}}{B^2} \sim 10^6$

η_f = max PP film amp Xmittance ≈ 1

2. REDUCTION OF THE ON-AXIS DIRTY BEAM

A relative reducing of the on-axis dirty beam can be accomplished by any of the following methods: (1) Subtract it, since its form and position are known; (2) make the brightness distribution and the reference beam imaginary; (3) transform it into a well-behaved impulse response by setting the bias β in the non-track areas equal to the bias B_o in the track areas; (4) boost the signal by increasing η_V by clipping $|V|_{max}$; (5) complementary weighting of the visibility data. These methods of increasing the signal to noise ratio will be discussed in detail below.

2.1 SUBTRACT IT

Since the exact form and position of a'(x,y) will be known, it can be digitally subtracted from the brightness data after detection. This is exactly analogous to subtracting the sidelobes of an extremely bright star that is just outside of the field of view. For the 100 equal stars case, the dirty beam sidelobes would be comparable to the star brightness, and the subtraction process would cause only a moderate loss in accuracy. However, for Cas-A, the dirty beam sidelobes would be an order of magnitude brighter than the brightest stars, and the subtraction process would cause a great loss in accuracy.

2.2 MAKE THE BRIGHTNESS DISTRIBUTION IMAGINARY

Replace the transmittance of Equation (3) by

$$A_o(u,v)\{\beta + A'(u,v)[(B_o - \beta) + B_1|V(u,v)|\sin(\omega_o u + \phi(u,v))]\} \tag{28}$$

Then the corresponding image becomes, replacing Equation (4),

$$\beta a_o(x,y) + (B_o - \beta) a_o(x,y) * a'(x,y)$$

$$+ j(B_1/2) a_o(x,y) * a'(x,y) * B(x - x_o, y) + \text{conjugate image} \tag{29}$$

Then the detection process would consist of the subtraction of the intensities $I_1 - I_2$, which are obtained by mixing with the brightness distribution reference waves that are of relative phase $\pi/2$ and $-\pi/2$, respectively. From a previous memo (J.R. Fienup, "Detected Signal," ERIM memo, 27 July, 1976, p. 2-3), for the exact Fourier transform case we have, assuming an on-axis reference beam $r_o = r_o' + jr_o''$ and adding the real-valued dirty beam term $a'(x,y)$,

$$I_1 = |r_o + a'(x,y) + jE_s(x,y)|^2 \tag{30}$$

$$= |r_o|^2 + |a'|^2 + |E_s|^2 + r_o a' + r_o^* a' +$$

$$+ jr_o^* E_s - jr_o E_s^* + ja'E_s = ja'E_s^*$$

$$= |r_o|^2 + a'^2 + |E_s|^2 + 2r_o' a'$$

$$+ 2r_o'' E_s' - 2r_o' E_s'' - 2a'E_s' \tag{31}$$

where $E_s(x,y) = E_s'(x,y) + jE_s''(x,y)$ is the signal wavefront, including aberrations. Shifting the reference beam by pi radians yields

$$I_2 = |r_o|^2 + a'^2 + |E_s|^2 - 2r_o'a'$$

$$- 2r_o''E_s' + 2r_o'E_s'' - 2a'E_s'. \qquad (32)$$

The difference, then, is

$$\Delta I(x,y) = I_1 - I_2 = 4(r_o''E_s' - r_o'E_s'') + 4r_o'a' \qquad (33)$$

By choosing $r_o = 0 + jr_o'' = r_o'' \exp(j\pi/2)$, we have

$$\Delta I(x,y) = 4r_o''E_s'(x,y) \qquad (34)$$

and the dirty beam term is eliminated.

Now consider the effect of the reference beam having a phase slightly different from $\pi/2$. Let

$$r_o = jr_o''e^{j2\pi\varepsilon_r} \simeq \underbrace{-2\pi\varepsilon_r r_o''}_{r_o'} - jr_o'' \qquad (35)$$

where ε_r is the absolute phase error, in wavelengths, of the reference beam. Then Equation (33) becomes

$$\Delta I(x,y) = 4r_o''E_s' + 4r_o'(a' - E_s'')$$

$$= 4r_o''E_s' + 8\pi\varepsilon_r r_o''(E_s'' - a') \qquad (36)$$

where the first term is the desired "signal" and the second term is "noise". Then in order to satisfy the 1% criterion, it is required that

$$\frac{8\pi\varepsilon_r r_o''|E_s'' - a'|}{4r_o''E_s'} \leq 0.01 \qquad (37)$$

For full-plane processing with low aberrations, $E_s'' << E_s'$ and can be neglected. This leaves

$$\varepsilon_r \leq \frac{0.01\ E_s'(x,y)}{2\pi a'(x,y)} \qquad (38)$$

Identifying the peak value of $E_s'(x,y)$ with S_p and $a'(x,y)$ with N' of Equations (21) to (26), we find that the residual error is

$$2\pi\varepsilon_r N'/S_p' \simeq 2 \times 10^{-2}\varepsilon_r(S_p/N_p')^{-1} \qquad (39a)$$

making the requirement

$$\varepsilon_r \lesssim \frac{1}{2} \frac{S_p}{N'_p} \tag{39b}$$

Thus, compared to the usual fractional error, N'/S_p, the error due to the sidelobes of the on-axis dirty beam is reduced by a factor of $2\pi\varepsilon_r$. From Table 1 we see that for the 1% criterion to be satisfied for the 100 equal stars case, the absolute phase of the reference beam would have to be accurate to within $\varepsilon_r = 1.7$ to 5.5×10^{-3} wavelengths (cycles), and for Cas-A, 1.3×10^{-4} wavelengths, assuming $\beta = 0$. Phase reading meters are capable of measuring phase with an accuracy of 0.1^o, or 2.8×10^{-4} wavelengths; however, it is not certain that the phase could be controlled with that accuracy even though it could be measured with that accuracy. Thus, although it will be adequate for the 100 equal stars case, it is doubtful that the phase of the reference beam could be controlled with sufficient accuracy to satisfy the 1% criterion for the worst case (Cas-A), and additional measures would have to be taken to diminish the residual on-axis dirty beam.

Another problem with making the image and reference beams imaginary is that, in the case of Cas-A, the stronger sidelobes from the on-axis dirty beam, although real-valued, use up most of the dynamic range of the detector. If, in Equation (31), $a' = 10\ r_o = 10\ E_s$, then the intensity would be scaled according to a'^2 rather than to $4r_o^2$, resulting in a loss of $10 \log_{10} (100/4)$ dB = 14 dB of the dynamic range of the detector. Thus, it is unlikely that the effect of the on-axis dirty beam can be eliminated by this method alone for the case of Cas-A.

2.3 EQUALIZE THE BIAS LEVELS (SET $\beta = B_o$)

If the transmittance β of the non-track areas is made equal to B_o, then the on-axis dirty beam term is eliminated, as can be seen from any of Equations (3), (4) or (24). However practical considerations prevent this from being done perfectly. We will consider two possible recording schemes for achieving $\beta = B_o$.

First is a two-step recording process. A mask is made with transmittance zero in the track areas and one in the non-track areas, i.e., with transmittance $1 - A'(u,v)$. The mask is placed in front of the film and a uniform exposure is made through the mask to expose the non-track areas to a uniform transmittance β without exposing the track areas. In a second exposure the film is exposed to the visibilty data in the track areas (without exposing the non-track areas). There would be a number of possible ways to insure that the first mask exposure would be in proper registration with the second exposure, but we will not explore them at this time. We will also note that the two-step exposure process could be replaced by a single exposure in the track areas on a negative film, having the non-track areas perfectly transparent. Then when the transparency is placed in the optical processor it is sandwiched with a mask of transmittance $A'(u,v) + \beta[1 - A'(u,v)]$. However, the two-exposure method would probably be preferred over the two transparency method.

A second recording scheme would be to use a gridded raster-scan recording and simply expose the non-track areas along with the track areas.

The sensitometry problem would be straightforward for the gridded raster-scan method, but more difficult for the two-exposure method. As can be seen from Equation (4) or (27), if $\beta/B_o = 1 \pm \varepsilon_\beta$, then the on-axis dirty beam is reduced by a factor of ε_β rather than being completely eliminated. In terms of density, let $\beta = 10^{-D/2}$, where D is the optical density in the non-track areas. Rewriting and differentiating,

$$D = -2 \log_{10}\beta$$

$$dD = -2 \log_{10} e \frac{dB}{\beta} = -2 \log_{10} e \cdot \varepsilon_\beta/\beta. \qquad (40)$$

Assuming $\beta \simeq 0.5$, an error ε_β in transmittance corresponds to an error in optical density given by

$$\Delta D = 1.7\, \varepsilon_\beta \qquad (41)$$

From Equation (27) and Table 1 we see that a reduction in the on-axis beam by a factor of about 10^3 would be required for Cas-A, the worst case. From Equation (41) we see that this implies the ability to control optical density with an accuracy of 0.002. The 100 equal stars case would require the accuracy of the optical density to be about 0.02 to 0.05.

In the case of using a pure-phase material (see J. R. Fienup, "Phase Modulation Encoding Method," ERIM memo, 2 September, 1976) the same type of accuracy in setting

$\beta = B_o$ is required as in the amplitude-transmittance case considered above. Further complications arise, however, if a real-time material with a bandpass spatial frequency response is used. Although a lack of a zero-frequency response forces the mean non-track phase to be the same as the mean phase in the track areas, the on-axis dirty beam term is not automatically eliminated. In fact, even when the bias exposures in the track areas and non-track areas are matched perfectly, there may be a residual dirty beam term. This results from the fact that wherever on a track the film has increased thickness (a hump), then on both edges of the track will be depressions, and vice versa, since the phase mechanism is bulk transport.

Another problem with the two-exposure method of eliminating the on-axis dirty beam is that of mask misregistration. Suppose that the mask has the binary transmittance $1 - A'(u,v) = \{0$ in track areas; 1 in non-track areas$\}$; further suppose for the sake of simplicity, that the film amplitude transmittance is linear with exposure and that its MTF is unity for all spatial frequencies. Then if the mask is offset by an amount ΔU_m in the u direction, the resulting transmittance would be, replacing Equation (3)

$$H(u,v) = A_o(u,v)\{A'(u,v)[B_o + B_1|V(u,v)|\cos(\omega_o u + \phi(u,v))] + \beta[1 - A'(u - \Delta U_m, v)]\} \quad (42)$$

This is the same as Equation (3) except that the usual on-axis dirty beam term $(B_o - \beta)A'(u,v)$ is replaced by the residual dirty beam term $B_oA'(u,v) - \beta A'(u - \Delta U_m, v)$, neglecting the $A_o(u,v)$ term. In the image domain the corresponding residual dirty beam is given by

$$N' = B_o a'(x,y) - \beta a'(x,y) \exp(-j2\pi\Delta U_m x/\lambda f)$$

$$= B_o a'(x,y)[1 - \exp(-j2\pi\Delta U_m x/\lambda f)] - B_o\varepsilon_\beta a'(x,y)\exp(-j2\pi\Delta U_m x/\lambda f)$$

$$= -2jB_o a'(x,y)\sin(\pi\Delta U_m x/\lambda f)\exp(-j\pi\Delta U_m x/\lambda f)$$

$$- B_o\varepsilon_\beta a'(x,y)\exp(-j2\pi\Delta U_m x/\lambda f) \qquad (43)$$

Expanding the exponential, we have

$$N' \simeq B_o a'(x,y)\{1 - (1 - \varepsilon_\beta)[1 - j2\pi\Delta U_m x/\lambda f - \tfrac{1}{2}(2\pi\Delta U_m x/\lambda f)^2]\}$$

$$= B_o a'(x,y)[\varepsilon_\beta + (1 - \varepsilon_\beta)\tfrac{1}{2}(2\pi\Delta U_m x/\lambda f)^2 + j(1 - \varepsilon_\beta)2\pi\Delta U_m x/\lambda f]$$

$$\simeq B_o a'(x,y)[\varepsilon_\beta + \tfrac{1}{2}(2\pi\Delta U_m x/\lambda f)^2 + j2\pi\Delta U_m x/\lambda f] \qquad (44)$$

Let w_T be the width of a track. Then the width of the image is about $W_B = \lambda f/w_T$, or $\lambda f = W_B w_T$. The approximation above assumes that $2\pi \Delta U_m x/\lambda f = 2\pi(\Delta U_m/w_T)(x/W_B) << 1$. If we consider the tracks to be formed by a convolution of elliptical paths with a recording beam of width w_T, then the on-axis dirty beam will be multiplied by a taper of half-width $W_B = \lambda f/w_T$. Considering the factors in Equation (44) that increase with x, combined with the decrease in a'(x,y) with increasing x due to the taper function, it is reasonable to assume that Equation (44) is maximized at about $x = W_B/2$, where the taper is starting to attenuate a'(x,y) strongly. There we have

$$N' \simeq B_o a'(x,y)\left[\varepsilon_\beta + \frac{\pi^2}{2}\left(\frac{\Delta U_m}{w_T}\right)^2 + j\pi\left(\frac{\Delta U_m}{w_T}\right)\right] \qquad (45)$$

The first term in the expression above is the same as that occurring when no misregistration is present, and leads to the sensitometry requirement discussed previously. If the brightness output is made real-valued, then the third term in Equation (45) will not be detected. In order to get the 10^3 improvement required for Cas-A, we would need $(\pi^2/2)(\Delta U_m/w_T)^2 \simeq 10^{-3}$, or

$$\Delta U_m \simeq (2 \times 10^{-3}/\pi^2)^{\frac{1}{2}} w_T = 0.014\ w_T \qquad (46)$$

For $w_T = 25$ µm, the registration would have to be accurate to within 0.35 µm. If combined with the method of making the brightness distribution imaginary, then of all the terms in Equation (45), only the last term would be detected. Then, to get the 10^3 improvement for Cas-A, the registration requirement would be

$$\Delta U_m \simeq (10^{-3}/\pi) w_T \simeq 3 \times 10^{-4} w_T \qquad (47)$$

or $\Delta U_m \simeq 0.8 \times 10^{-2}$ μm for the 25 μm track width, a clearly impossible requirement. Thus, the method of making the brightness output imaginary should not be combined with equalizing the bias levels.

Fortunately, the registration requirements given above are true only if the transmittances of the track and non-track areas have sharp boundaries. Any smoothing of the boundaries will reduce the sensitivity to misregistration. For example, if both track edges in both exposures had a linear (ramp-like) taper over a length ℓ (ℓ would be a fraction of w_T), then it can be shown that the on-axis dirty beam term would be further reduced by a factor of $\Delta U_m/\ell$. Then the requirement for Cas-A would become $(\pi^2/2)(\Delta U_m/w_T)^2 (\Delta U_m/\ell) \simeq 10^{-3}$, or $\Delta U_m = [2 \times 10^{-3}(\ell/w_T)/\pi^2]^{1/3} w_T$. Assuming $\ell/w_T = 1/5$, the requirement becomes $\Delta U_m = 3.4 \times 10^{-2} w_T =$ 0.86 μm. Similarly, if combined with the method of making the brightness distribution imaginary, the requirement would become $\Delta U_m = [10^{-3}(\ell/w_T)/\pi]^{\frac{1}{2}} w_T = 0.2$ μm.

Still another problem with the two-exposure method of eliminating the on-axis dirty beam is that of inexact mask track width. That is, the net average transmittance near the edges of the track may tend to be somewhat higher or lower than B_o or β. To determine the exact effect of this inaccuracy on the image would require further analysis. However, one might expect the resulting noise term to be similar to that of Equation (45) but perhaps without the first-order imaginary term.

2.4 CLIP $|V|_{max}$ TO BOOST η_V

As seen from Equatin (24), the signal-to-noise ratio is proportional to $\sqrt{\eta_V} = \left(\overline{|V|^2}\right)^{\frac{1}{2}}/|V|_{max}$ for a discussion of this, see J. R. Fienup, "Basic Limitations of Encoding Methods," ERIM memo, 21 September 1976, p. 6). Since the brightness distribution is a real, non-negative function, its Fourier transform, the visibility function, is sharply peaked at (u,v) = (0,0), and $\sqrt{\eta_V}$ is typically much less than unity (4×10^{-3} for the worst case, Cas-A). In the case of Cas-A studied at Green Bank, the maximum detected value of $|V|$ was 404.7, compared to the peak at $u = v = 0$, which was calculated to be 1500. Thus we see that $|V(u,v)|$ drops off very quickly with increasing $|u|$ and $|v|$. Thus, if we clip $|V(u,v)|$ near $u = v = 0$, $|V|_{max}$ can be significantly reduced, increasing the signal-to-noise ratio. The clipping can be done in any number of ways: set $|V(u,v)| = |V|_m$ or $= 0$ for all (u,v) for which $|V(u,v)| > |V|_m$ or for which $(u^2 + v^2)^{\frac{1}{2}} \leq u_m$. This clipping would increase $\sqrt{\eta_V}$ by a factor of $|V|_{max}/|V|_m$. The effect of this clipping on the output brightness distribution would be a loss of low spatial frequency information, which may or may not be objectionable, depending on the nature of the sky map and the type of data desired of it. A partial loss of low frequency information already occurs since $V(u,v)$ is not sampled at $u = v = 0$. Clipping $|V(u,v)|$ only extends that loss.

The same signal-to-noise gain as obtained by clipping $|V(u,v)|$ can be obtained without the loss of low spatial frequency information by a method using two transparencies. This is essentially breaking up the signal $V(u,v)$ into two parts and using a different gain for each part. For example, make two transparencies

$$t_1(u,v) = H_1(u,v) \cdot \begin{cases} 0, & (u^2+v^2)^{\frac{1}{2}} < u_m \\ 1, & (u^2+v^2)^{\frac{1}{2}} > u_m \end{cases}$$

and

$$t_2(u,v) = H_2(u,v) \cdot \begin{cases} 1, & (u^2+v^2)^{\frac{1}{2}} < u_m \\ 0, & (u^2+v^2)^{\frac{1}{2}} > u_m \end{cases} \qquad (48)$$

where $H_1(u,v)$ and $H_2(u,v)$ are given by Equation (3) but with $B_1 = \sqrt{\eta_f}/(2|V|_{max})$ and $B_1 = \sqrt{\eta_f}/(2|V|_m)$, respectively, where $|V|_m = \max[|V(u,v)|]$ for $(u^2+v^2)^{\frac{1}{2}} > u_m$. In a manner similar to the case of the real-imaginary encoding method, the wavefronts from t_1 and t_2 can be combined interferometrically, but with different gain factors, to obtain the desired net input. The different gain factors are simple the different amplitudes of the plane or converging spherical wavefronts illuminating the two transparencies in a ratio of $|V|_m/|V|_{max}$. Assuming that u_m is a small fraction of the width of the u-v plane data, then the effective increase in signal-to-noise ratio is $|V|_{max}/|V|_m$; the effective increase in diffraction efficiency is $|V|^2_{max}/|V|^2_m$, allowing a reduction in detector integration time by that factor. Alternatively, with the same increase in signal-to-noise ratio and diffraction efficiency, the two transparencies can be processed one at a time, and their detected outputs summed. Since the transmittance of each transparency is Hermitian, the resulting images will be real-valued. If the low spatial-frequency information is not required, then only $t_2(u,v)$ need be processed. Since $t_2(u,v)$ contains only low spatial frequency information, it might be possible to decrease the readout time since the image might not need to be sampled in as many locations.

2.5 COMPLEMENTARY WEIGHTING

In Equation (3) the aperture function $A_o(u,v)$ can include the effect of a non-uniform illumination beam or a weighting (an apodization) of the input plane. The effect of that weighting is the convolution of the entire output plane by $a_o(x,y)$, the Fourier transform of $A_o(u,v)$. In order to reduce the sidelobes of the on-axis terms, we would want to heavily weight the input plane; however, that would ordinarily result in an undesired loss of resolution in the image. Fortunately, it is possible to heavily weight the input without affecting the image terms by "complementary weighting", by replacing the transmittance of Equation (3) by

$$H(u,v) = A_1(u,v)A_o(u,v)\{\beta + A'(u,v)[(B_o - \beta) + B_1 \frac{|V(u,v)|}{A_1(u,v)} \cos(\omega_o u + \phi(u,v))]\} \quad (49)$$

where $A_1(u,v)$ is the complementary weighting function. In order to not exceed the available dynamic range of the film, we require that

$$\frac{|V(u,v)|}{|V|_{max}} \leq A_1(u,v) \leq 1 \quad \text{for all } (u,v) \tag{50}$$

From Equation (49), we see that the visibility term is totally unaffected, although both on-axis terms are weighted by $A_1(u,v)$. In the output domain the result is the convolution of the on-axis terms with $a_1(x,y)$, the Fourier transform of $A_1(u,v)$.

For example, suppose that

$$A_1(u,v) = \beta_1 + (1 - \beta_1) \exp \left[-\pi(u^2 + v^2)/(aD)^2\right] \tag{51}$$

where $\beta_1 << 1$ so that $(1 - \beta_1) \simeq 1$, and $D = \lambda f/\Delta x$ is approximately the width of the u-v plane coverage, and Δx is the peak-to-null half-width of the dirty beam $a'(x,y)$. Then, from Equation (1),

$$a_1(x,y) \simeq \lambda f \beta_1 \, \delta(x,y) + \lambda f(a/\Delta x)^2 \exp \left[-\pi(x^2 + y^2)(a/\Delta x)^2\right] \tag{52}$$

and the resulting on-axis terms are

$$\beta a_1(x,y) * a_o(x,y) + (B_o - \beta)\, a_1(x,y) * a'(x,y)$$

$$= \beta\beta_1 a_o(x,y) + \beta_1(B_o - \beta) a'(x,y)$$

$$+ \frac{1}{\lambda f} \iint [\beta a_o(\zeta + x,\ \eta + y) + (B_o - \beta) a'(\zeta + x, \eta + y)]\ .$$

$$\cdot\ \lambda g(a/\Delta x)^2 \exp\, [-\pi(\zeta^2 + \eta^2)(a/\Delta x)^2]\ d\zeta d\eta \qquad (53)$$

For an extended object with a sharply-peaked visibility function, we could have $\beta_1 << 1$ and $a << 1$. Then the Gaussian term in the equation above would be much wider than $a_o(x,y)$ and $a'(x,y)$, so that the convolution would essentially take the form of the Gaussian. This can be seen by approximating $a_o(x,y) \simeq a_{omax}(\Delta x)^2 \delta(x,y)$ and $a'(x,y) \simeq a'_{max}(\Delta x)^2 \delta(x,y)$ in Equation (53). Then the on-axis terms become

$$\beta_1\beta\ a_o(x,y) + \beta_1(B_o - \beta) a'(x,y)$$

$$+ a^2 [\beta a_{omax} + (B_o - \beta) a'_{max}] \exp\, [-\pi(x^2 + y^2)(a/\Delta x)^2] \qquad (54)$$

If β_1 were, say, 0.02 then the first two on-axis terms in the equation above would be reduced by 17 dB from their usual value. If a were, say, 1/25, then the Gaussian term would be down 28 dB from the usual peak value at x = y = 0. Away from the optical axis the Gaussian would decrease slowly at first, until it reached about x = 25x, at which point it would be down 14 dB from its peak (down 42 dB altogether), after which point it decreases rapidly to 54 dB down from its peak at x = 50 Δx and 123 dB down from its peak at x = 75 Δx. However, at these low levels the approximation used for Equation (54) may not be valid. Therefore, to see if this analysis is valid, it will be necessary to compute the dirty beam sidelobes, given a strong Gaussian weighting in the u-v plane, using the discrete Fourier transform.

If the Gaussian term does behave as described above, then it is the constant term β_1 in Equations (51) and (54) that determines the degree of reduction of the on-axis dirty beam. The degree of reduction of the on-axis terms by complementary weighting depends heavily upon the particular weighting $A_1(u,v)$ used. The example of Equation (51) above, a Gaussian on a pedestal, is probably not the optimum choice of the dozens of well-known weighting functions. Further study would be required to determine the optimum complementary weighting function for various classes of visibility data both in terms of sidelobe reduction and sensitivity to errors. Further study is also required to determine the expected increase in signal-to-noise ratio obtainable by complimentary weighting, which depends on the detailed description of $|V(u,v)|$ over the entire input plane.

In order to allow the weighting function to go very near zero near the edge of the (u,v) plane, it might be worthwhile to consider an apodization of the visibility data near the edge of the (u,v) plane.

Complementary weighting can be achieved by a weighting of the bias transmittance of the transparency, as suggested at the December 8, 1976 meeting between NRAO and ERIM. However, due to problems of noise and repeatability at those low transmittance levels, the weighting (multiplication) by $A_1(u,v)$ in Equation (49) would be better achieved by making $A_1(u,v)$ the illumination pattern impinging on the transparency of constant bias.

2.6 SUMMARY; COMBINATION OF METHODS

Since a 1% accuracy map of Cas-A requires a 30 dB reduction of the dirty beam, and since it may not be possible to achieve that great a reduction from any one of the five methods described above, it is necessary to consider which methods can be combined successfully. Most methods can be combined with the following exceptions. It is not beneficial to use both clipping of $|V|$ and complementary weighting simultaneously. Subtraction of the dirty beam cannot be accomplished while equalizing the bias levels or while making the brightness and reference beams imaginary. When equalizing the bias levels, the sensitivity to mask misregistration is much greater when the brightness is imaginary than when it is real.

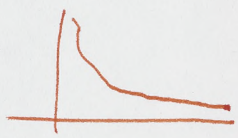
If the bias levels are equalized in a one-step raster scan recording, then it is expected that the on-axis dirty beam term can be made to vanish, and no other methods would be required.

Assuming a two-step recording process, the reduction of the on-axis dirty beam by equalizing bias levels is limited to the greater of the following factors: $\Delta D/1.7$, where ΔD is the mean difference in optical density between the track and non-track areas; for a real brightness $(\pi^2/2) \cdot (\Delta U_m/w_T)^2 (\Delta U_m/\ell)$; and for imaginary brightness, $\pi(\Delta U_m/w_T)(\Delta U_m/\ell)$, where ΔU_m is the misregistration of the mask, w_T the track width, and ℓ the length of the taper of the track edge, assuming $\Delta U_m/\ell \geq 1$. Thus, if we assume $\Delta D = 0.01$, $w_T = 25$ μm, $\ell = 5$ μm, and $\Delta U_m = 1$ μm, then the on-axis dirty beam would be reduced by 22 dB for the real brightness case and by 16 dB for the imaginary brightness case.

The reduction of the error due to the on-axis dirty beam by making the brightness imaginary is limited to a factor of $2\pi\varepsilon_r$, where ε_r is the phase error of the reference beam relative to the signal beam in wavelengths. Assuming control to $\varepsilon_r = 10^{-3}$ (about 1/3 degrees), the noise due to the dirty beam sidelobes would be reduced by 22 dB.

Clipping of $|V|$ would increase the signal-to-noise ratio by 4 to 14 dB, depending on the clippling level. If the two-transparency method were used, then the clipping would be done at the 14 dB level.

The reduction of the sidelobes of the on-axis dirty beam by complementary weighting seems to be limited by the ratio of $|V|_{max}$ to the maximum of $|V(u,v)|$ for large $(u^2 + v^2)^{\frac{1}{2}}$. For Cas-A that ratio is about 0.02, allowing a reduction by 17 dB.



The analysis needed to predict the performance of the method of subtracting the known sidelobes of the dirty beam has not been performed. It could reduce the error by as much as 10 or 20 dB.

The 31 dB improvement in signal-to-(on-axis dirty beam sidelobe)-noise ratio required for the case of Cas-A appears to be easily achieved by combining complementary weighting

(17 dB) with bias equalization (using a two-exposure process and a real brightness: 22 dB), for a total improvement of 39 dB. A gridded raster-scan recording would perform even better. Also possible would be complementary weighting combined with making the brightness imaginary (22 dB), for a total improvement by 37 dB.

The 15 to 20 dB improvement required for the 100 equal stars case appears to be achievable using bias equalization alone or making the brightness imaginary alone (but not together).

3. THE LARGE-APERTURE IMPULSE RESONSE

In this section we consider the sidelobes due to the on-axis large-aperture impulse response term $\beta a_o(x,y)$ of Equation (4). For bias equalization, β will generally be ½. Without bias equalization β equals 1 for a negative film and for phase-only film and is nearly zero for a positive film. Two typical aperture functions $A_o(u,v)$ and their impulse responses $a_o(x,y)$ are as follows

Table 2
Large Square and Circular Apertures

TYPE	SQUARE (area W_o^2)	CIRCULAR (area W_o^2)
$A(u,v)$	$\mathrm{rect}(u/W_o)\,\mathrm{rect}(v/W_o)$	$\mathrm{rect}\left[(u^2+v^2)^{\frac{1}{2}}/(2W_o/\sqrt{\pi})\right]$
$a_o(x,y)$	$\left(\frac{W_o^2}{\lambda f}\right)\mathrm{sinc}\left(\frac{W_o x}{\lambda f}\right)\mathrm{sinc}\left(\frac{W_o y}{\lambda f}\right)$	$(W_o/\sqrt{\pi}r)J_1(2\pi r W_o/\sqrt{\pi}\lambda f)$
a_{omax}	$W_o^2/\lambda f$	$W_o^2/\lambda f$
envelope, large x,y	$\left(\frac{W_o^2}{\lambda f}\right)\cdot\frac{1}{\pi^2}\left(\frac{\lambda^2 f^2}{W_o^2 xy}\right)$	$\left(\frac{W_o^2}{\lambda f}\right)\cdot\frac{1}{\pi^{\frac{1}{4}}}\left(\frac{\lambda f}{W_o r}\right)^{3/2}$

where $x^2 + y^2 = r^2$. Figure 1 shows curves of constant $a_o(x,y)$ envelope for the square aperture (solid line) and circular aperture (dotted line) cases. Along the x (and y) axis, the sinc sidelobes drop off as $1/r$, whereas along the line $x = y$, they drop off as $1/r^2$; and the $J_1(r)/r$ sidelobes drop off as $1/r^{3/2}$ in all directions. Thus, to minimize the effects of these sidelobes, a square aperture $A_o(u,v)$ should be used. The brightness map should be placed in the area of minimum sidelobes by using the carrier $\omega_o(u+v)$ or by orienting $A_o(u,v)$ at a 45^o angle to the portion of the output being detected.

The peak of the signal amplitude is given by Equation (21). Similar to the derivation resulting in Equation (18), we see that the peak amplitude of the on-axis large-aperture term is given by

$$a^2_{omax} = NW_B^{-2}W_o^2 \quad (55)$$

Thus, the ratio of the peak of the signal to the on-axis peak of this noise term is

$$\frac{S_p}{N_p} = \frac{S_p}{\beta a_{omax}} = \left(\frac{\eta_c\eta_V\eta_f\eta_{uv}}{16N\beta^2}\right)^{\frac{1}{2}} \quad (56)$$

where

$$\eta_{uv} \equiv W_T^2/W_o^2. \quad (57)$$

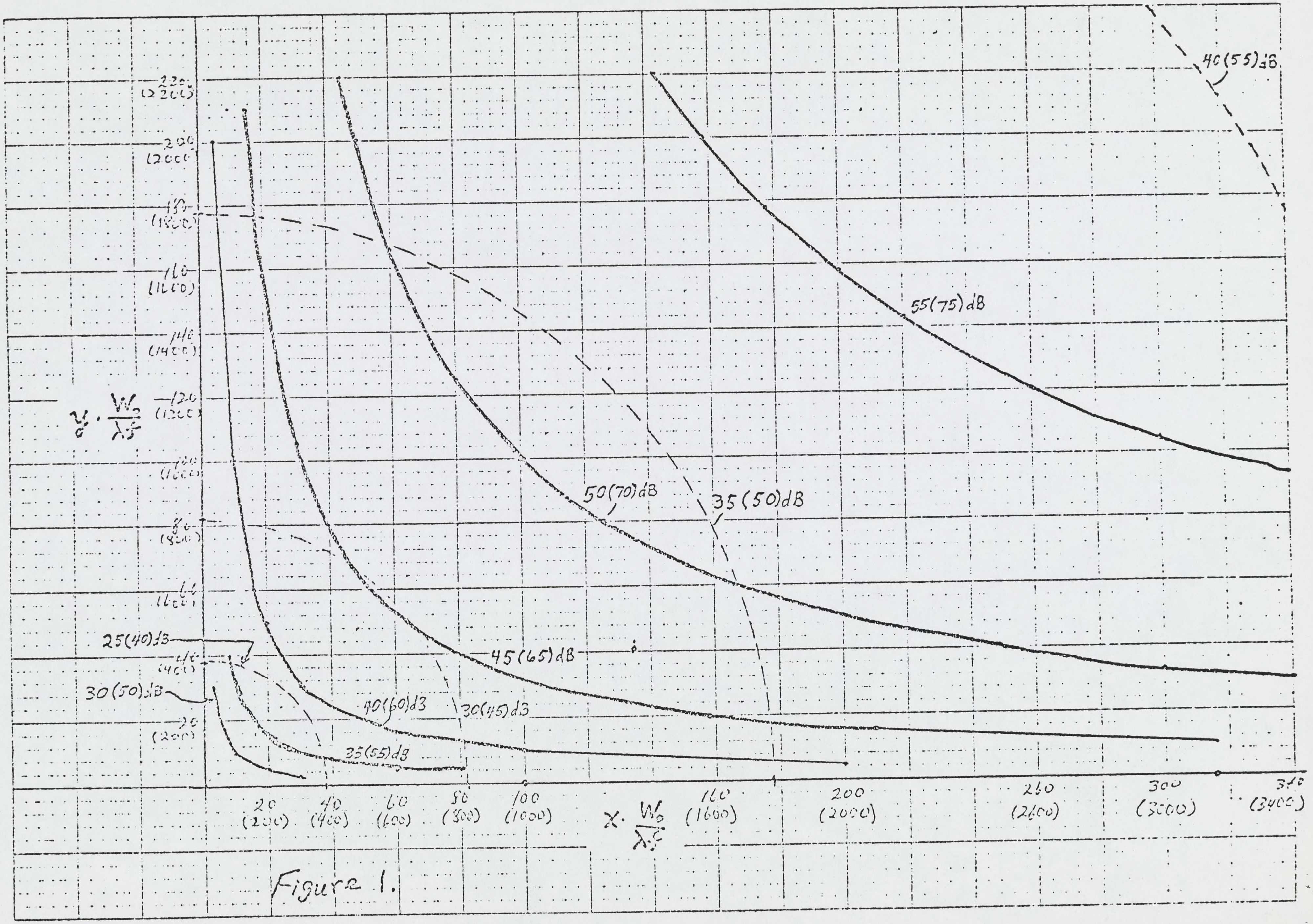

40(55)dB
55(75)dB
50(70)dB
35(50)dB
45(65)dB
25(40)dB
30(50)dB
40(60)dB
30(45)dB
35(55)dB
20 (200)
40 (400)
60 (600)
80 (800)
100 (1000)
160 (1600)
200 (2000)
260 (2600)
300 (3000)
340 (3400)
200 (2000)
160 (1600)
140 (1400)
120 (1200)
80 (800)
40 (400)
20 (200)

Figure 1.

Comparing Equations (56) and (24), we see that the peak signal to peak noise ratio for the large-aperture impulse response is $\sqrt{\eta_{uv}}(B_o - \beta)/\beta$ times that for the on-axis dirty beam. Thus, if we assume $\beta = \frac{1}{2}$ and $\sqrt{\eta_{uv}} = \frac{1}{2}$, then S_p/N_p can be determined from Table 1 by dividing $(\eta_f \eta_V \eta_c/N)^{\frac{1}{2}}/4B_o$ by 2 and adding 3 dB to the "signal level" and "noise allowed" rows. Then we see that for the case of Cas-A, the on-axis large-aperture term sidelobes must be down 59 dB from the peak in order to satisfy the 1% criterion.

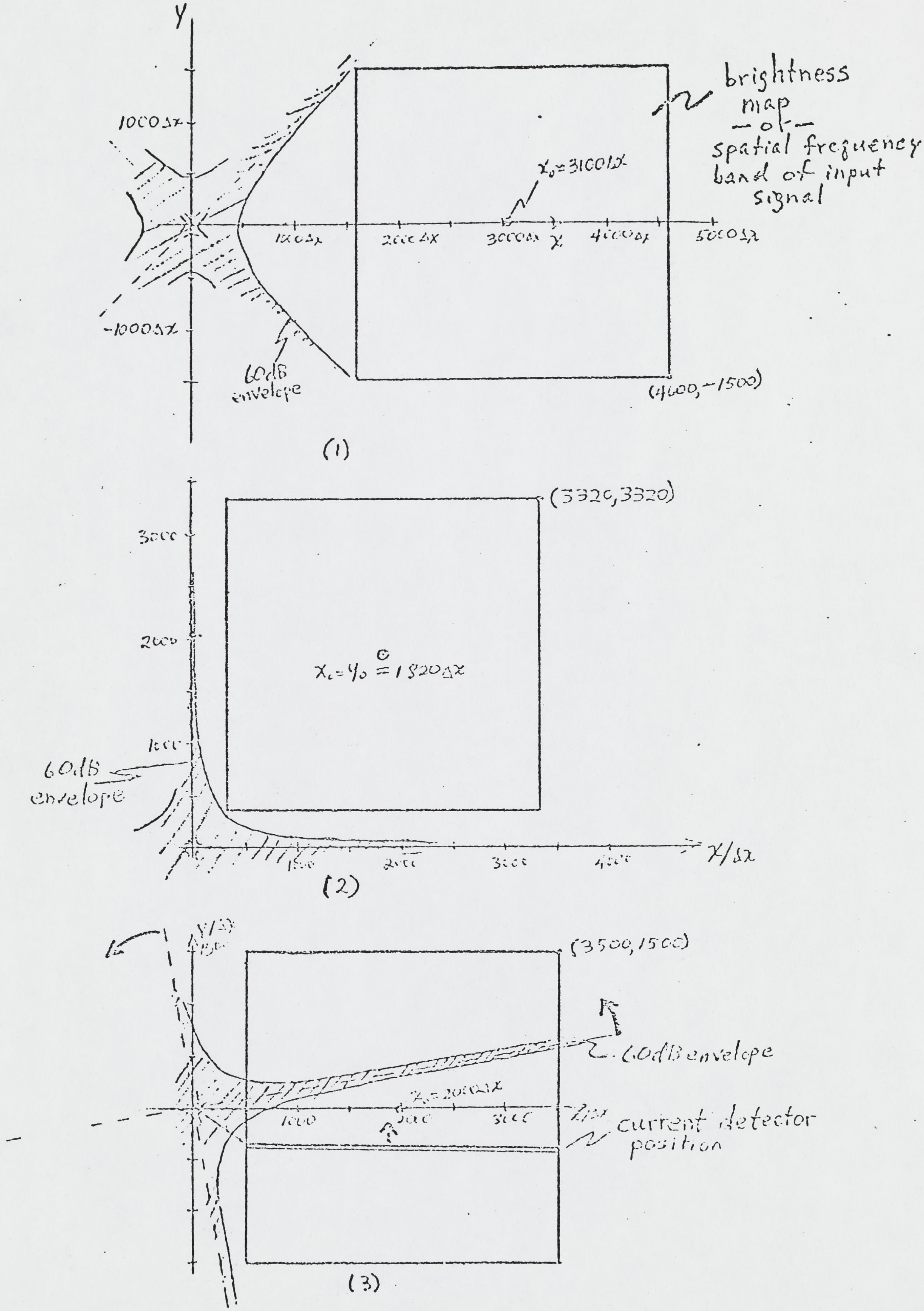
Three possible ways of orienting the sinc x sinc y sidelobes and the brightness map with respect to one another are as follows, as indicated in Figure 2.

(1) Use a carrier $\omega_o u$ and orient the square aperture $A_o(u,v)$ at a 45^o angle with respect to the u-v axes;

(2) Use a carrier $\omega_o(u+v)$ and orient the square aperture along the u-v axes;

(3) Use a carrier $\omega_o u$, and have the orientation of the square aperture track the sweep of the detector array.

If we assume the worst case, namely, that the entire $(3000\ \Delta x)^2$ brightness map must avoid sidelobe levels above the 60 dB level, then the following carrier frequencies and maximum spatial frequencies (in units of W_o^{-1}) are required (see Figure 2):



y
1000Δx
-1000Δx
1000Δx
2000Δx
3000Δx
x
4000Δx
5000Δx
x₀=3100Δx
brightness map -of- spatial frequency band of input signal
60dB envelope
(4600,-1500)
(1)
(3320,3320)
3000
2000
1000
x₀=y₀=1820Δx
60dB envelope
1000
2000
3000
4000
x/Δx
(2)
y/Δx
1500
(3500,1500)
60dB envelope
x₀=2000Δx
1000
2000
3000
current detector position
(3)

Figure 2

Table 3
Spatial frequencies of input for various output geometries (see Fig. 2). Case (0) is to just separate the twin images along x.

Case	x_o	y_o	$\sqrt{x_o^2 + y_o^2}$	x_{max}	y_{max}	$\sqrt{x_{max}^2 + y_{max}^2}$
(0)	1500	0	1500	3000	1500	3354
(1)	3100	0	3100	4600	1500	4840
(2)	1820	1820	2570	3320	3320	4700
(3)	2000	0	2000	3500	1500	3800

For the 100 equally bright stars case, for which the sidelobes must be down 43 to 48 dB, a 10% reduction of the carrier frequency requirement is obtained for cases (2) and (3) in Figure 2. The effect is negligible for case (1). If we consider the brightness map of interest to be a circle of diameter 3000 Δx, (i.e., we can chop off the corners), then we can realize a reduction of the carrier frequency requirement by 10% in case (2) and by about 30% in case (1).

Fortunately, except for bias equalization, all the methods that reduce the on-axis dirty beam term also reduce the large-aperture impulse response by the same amount. However, for the square large aperture, we see from Figure 2 that it is more a problem of geometry than one of the required sidelobe level that puts requirements on the spatial frequency content of the input transparency.

The sidelobes from the on-axis large-aperture impulse response, $\beta a_o(x,y)$, do not appear to limit the performance of the system, even if no attempt is made to reduce them.

4. SCATTERED LIGHT

In this section we consider the noise caused by light scattering from the film and from dust and imperfections in the optical system. The scattered light noise tends to be maximum near the optical axis and drops off away from the optical axis. It can be avoided by using a sufficiently high carrier frequency to move the image away from the optical axis.

4.1 FILM NOISE

As discussed in a previous memo (J. R. Fienup, "Basic Limitations of Encoding Methods," ERIM memo, 21 September 1976), for a number of holographic materials, the mean amplitude of the light scattered light is given by

$$[I_n(\nu)]^{\frac{1}{2}} = I_o^{\frac{1}{2}} \frac{1\text{mm}}{W_o} \sqrt{a}\ \nu^{-b/2} \qquad (58)$$

where I_o is the on-axis intensity, W_o is the width of the u-v plane aperture, ν is the spatial frequency in ℓp/mm, and a and b are constants for a given film. For Kodak 649-F plates, $a = 2.6 \times 10^{-4}$ and $b = 2.26$. The spatial frequency ν corresponds to a position in the output plane a distance $r = \nu\lambda f$ from the optical axis, which corresponds to $r/\Delta x = \nu\lambda f(\lambda f/W_o)^{-1} = \nu \cdot W_o$ beam half-widths from the optical axis. For the proposed processor, $W_o = 50$ mm, $\lambda \simeq 0.5 \times 10^{-3}$ mm and $f \simeq 3.3$ m. Then for 649-F plates, Equation (58) becomes

$$[I_n(r/\Delta x)]^{\frac{1}{2}} = 2.7 \times 10^{-2}\ I_o^{\frac{1}{2}}\ (r/\Delta x)^{-1.13} \qquad (59)$$

If bias equalization is used, then the relation between the peak of the signal and $I_o^{1/2}$ is given by Equation (56). Thus, for the 100 equal stars case, $(I_n/I_o)^{1/2}$ must be down to 5.5×10^{-5} to 1.8×10^{-5} (43 to 48 dB); and for Cas-A $(I_n/I_o)^{1/2}$ must be down to 1.35×10^{-6} (59 dB). Inverting Equation (59),

$$(r/\Delta x) = 4.1 \times 10^{-2} \left[(I_n/I_o)^{-1/2}\right]^{1/1.13} \qquad (60)$$

we see that for the 100 equal stars case, the edge of the image should be $r/\Delta x = 240$ to 650 beam half-widths from the optical axis in order to satisfy the 1% criterion, which is comparable to the effect of the on-axis large-aperture impulse response. For Cas-A it would be $r/\Delta x = 6400$, which is much greater than that required by the on-axis large aperture impulse response. Complementary weighting would reduce the film noise by a factor of

$$\left(\frac{\iint |A_o(u,v)A_1(u,v)|^2 \, dudv}{\iint |A_o(u,v)|^2 \, dudv}\right)^{1/2} \qquad (61)$$

assuming bias equalization. For the example of Equation (51),

$$\iint |A_oA_1|^2 \, dudv = \beta_1^2 D^2 + \left(\int_{-\infty}^{\infty} \exp[-2\pi u^2/(aD)^2] du\right)^2$$

$$= \beta_1^2 D^2 + (aD/\sqrt{2})^2 = (\beta_1^2 + a^2/2)D^2 \qquad (62)$$

and $\iint |A_o|^2 \, dudv = D^2$, so the film noise would be reduced by a factor of $(\beta_1^2 + a^2/2)^{\frac{1}{2}}$ in amplitude. For $\beta_1 = 0.02$ and a = 0.04, that factor would be 0.035. With this additional factor, for Cas-A the edge of the image should be $r/\Delta x$ = 330 beam half-widths from the optical axis in order to satisfy the 1% criterion, imposing no greater restrictions that that required due to the on-axis large-aperture impulse response.

Film noise from 649-F plates does not appear to limit the performance of the system, assuming that complementary weighting is employed when necessary.

4.2 OPTICAL SYSTEM NOISE

Scattered light measurements were performed on ERIM's Spotlight processor for a 1200 mm focal length multi-element system with a 32 mm aperture. The measurements were made with a probe of area of about $4\Delta x^2$, where Δx is the peak-to-null spot width. In order to normalize those results to make them relative to the peak intensity and to include the effect of the larger 50 mm aperture for the VLA optical processor, the relative noise level reported should be decreased by a factor of $4 \times (50/32)^2 \simeq 10$ in intensity. Allowing for this factor of 10, the scattered light was as follows

Table 4
Measured scattered light for Spotlight processor

ν(cyc/mm)	r/Δx	scattered amplitude
2.6	130	40 dB
5.3	260	42 dB
7.3	360	45 dB
10.6	530	47 dB
14.5	730	49 dB

Since the Fourier transforming optics was designed for an astronomical telescope rather than for a coherent optical system, and due to the presence of additional optics not required for the VLA optical processor, and considering the fact that no special precautions were made for keeping the system clean, it is reasonable to expect that the scattered light could be reduced by at least another 10 dB in intensity (or 5 dB in amplitude). Then, for the 100 equal stars case, the edge of the brightness map would have to be up to about $\nu \simeq 6$ cyc/mm or $r/\Delta x \simeq 300$ beam half-widths from the optical axis in order to just satisfy the 1% criterion (the 48 dB noise level). For Cas-A the 1% criterion could not be met unless either clipping of $|V|_{max}$ or complementary weighting is performed. Assuming a reduction of the noise amplitude by a factor of 0.035 for Cas-A by complementary weighting (as in Section 4.1), then the edge of the brightness map would have to be $\nu = 7$ cyc/mm or $r/\Delta x = 350$ beam half-widths from the optical axis in order to just satisfy the 1% criterion.

Scattered light does not appear to limit the performance of the optical system, assuming that complementary weighting is employed when necessary.

5. SUMMARY AND CONCLUSIONS

The scattered light and sidelobes associated with the undiffracted beam can be reduced sufficiently to meet the 1% criterion even for the worst case of Cas-A <u>if</u> certain techniques are employed. For Cas-A type maps, complementary weighting (or clipping of $|V|_{max}$) is an absolute necessity. Further study of candidate weighting functions is warranted. It is expected that the choice of weighting function parameters for a given visibility function will be a relatively simple calculation not requiring the intervention of the operator. The first weighting term $A_1(u,v)$ in Equation (49) would be most easily implemented by inserting a precision weighting mask before the input transparency. A set of precision masks would be fabricated once, and the one with the parameters appropriate to a given visibility function would be selected by a computer. Complementary weighting does not appear to be necessary for maps like the 100 equal stars case.

The on-axis large-aperture impulse response and scattered light noise are avoided by using a carrier frequency sufficiently high to move the edge of the brightness map far enough away from the optical axis. Unless the large aperture is weighted or rotated, the presence of the large aperture impulse response requires a carrier frequency 70% to 107% greater than that necessary to simply separate the twin images (however, the maximum spatial frequencies are increased only by 40% to 44%). Scattered light does not appear to be as important as the on-axis large-aperture impulse response, assuming complementary weighting is used when necessary.

The on-axis dirty beam is the most bothersome term considered here. It does not require the use of a larger carrier frequency, but it does require one or more of the techniques in Section 2 to be used. For the 100 equal stars case, either bias equalization or making the brightness imaginary is sufficient to reduce the noise to below the 1% level. For Cas-A, complementary weighting must be combined with one other method, either bias equalization or making the brightness imaginary.

In summary, complementary weighting is a necessity for Cas-A type maps. In addition either bias equalization or making the brightness imaginary is required for all maps.

JF/pw

cc: C. Aleksoff
I. Cindrich
W. Colburn
A. Klooster