VLA CONFIGURATION STUDY - STATUS REPORT

February 27, 1968

<u>Summary of Work for the Period January 1967 - February 1968</u>

The work done during the period under review can be divided into four categories:

(i) Further work has been done using the empirical approach on the lines of the work described in the VLA Proposal. Various distributions of elements along the arms of the Wye were studied. It was felt that although better distributions could be found for specific source declinations, the supplemented Wye was still the overall best configuration. The effect of antenna failure was investigated and found to be small for failure of one antenna out of 36. These results are summarized in VLA Scientific Memorandum No. 7 [1]

(ii) A detailed study was made of the performance of linear correlator arrays. In the supersynthesis mode such arrays have very good performance in a limited declination range. This study has given a good understanding of the working of correlator arrays and has pointed out the limitations of linear arrays. The results are summarized in various VLA Memoranda [2,3].

(iii) The use of complementary arrays has been considered. The required number of antennas can be substantially reduced if the source is observed twice with two different configurations. By superimposing the transfer functions of the two array configurations, a well-filled transfer function can be obtained. However, the total time required per source is greatly increased both because two observations are needed and because of the time expended in changing the configuration, calibration, etc. It was found empirically that 24 antennas used in complementary array configurations could produce a beam pattern comparable to that produced by 36 antennas in a single observation. The results are given elsewhere[4].

(iv) A new technique for finding the optimum configuration using the computer has been developed. This technique, termed Pseudo-Dynamic Programming, selects the best position of an antenna from a given set of possible positions. Thus by using the best configuration found by the empirical approach as the input, and using a set of pre-specified positions as the possible antenna locations, the computer keeps improving the configuration until an optimum configuration is found. A computer program using this technique is now operational. The details of the technique and some results are given in the Appendix.

## Future Work

It is planned to fully exploit the Pseudo-Dynamic Programming technique to get the best possible configuration. As before, both the distribution of elements along the arms, as well as the total number of elements required, are being considered.

Some other factors affecting the array performance, such as, for example, the effect of double sideband operation with a 50 MHz bandwidth, need further study. Some theoretical work has already been done on this subject during the period under review[5]. It is proposed to carry out detailed studies of these effects.

## REFERENCES

[1] Mathur, N. C., "Some Results on the VLA Performance with Less Than 36 Elements", VLA Scientific Memorandum No. 7, National Radio Astronomy Observatory, Charlottesville, Va. (September 6, 1967)

[2] Mathur, N. C., "A Study of an Eight-Element Correlator Array", VLA Scientific Memorandum No. 4, National Radio Astronomy Observatory, Charlottesville, Virginia. (August 10, 1967)

[3] Mathur, N. C., "Aperture Synthesis With the Green Bank Array", VLA Scientific Memorandum No. 10, National Radio Astronomy Observatory, Charlottesville, Va. (December 15, 1967)

[4] Mathur, N. C., "Complementary Array Configurations for the VLA", VLA Scientific Memorandum No. 5, National Radio Astronomy Observatory, Charlottesville, Va. (August 10, 1967)

[5] MacPhie, R. H., "On the Use of Space-Frequency Equivalence in Long Baseline Interferometers", VLA Scientific Memorandum No. 3, National Radio Astronomy Observatory, Charlottesville, Va. (May 1967)

## APPENDIX

## THE USE OF PSEUDO DYNAMIC PROGRAMMING FOR VLA CONFIGURATION DESIGN

### Introduction

The VLA is an antenna array having a beam narrow enough to resolve sources separated by one second-of-arc and yet pure enough to keep the sidelobe response to a minimum. Specifically, it has four resolutions, 1", 3" 9", and 27", and has mean sidelobe level far from the beam but within the field of view less than -30 db, and with the peak sidelobe level less than -15 db within the field of view. The configuration study is aimed at finding the antenna configuration which will produce such a beam at the minimum cost.

The design of antenna arrays to achieve specific beam characteristics has been the subject of intensive research in the past decade.[1-7] In a linear additive array, the characteristics of the beam depend upon the locations and excitations of the elements of the array. No unique technique has yet been developed for finding the optimum array configuration. The various approaches used in array synthesis work can be broadly classified into three categories: (i) analytical approach, (ii) statistical approach, and (iii) computer optimization.

The analytical approach has been tried by several workers.[1,5,8,9,10] Although partial success has been achieved, the arrays designed by the analytical approach do not perform better than arrays designed by other approaches.[7] The chief difficulty is that analytical methods cannot handle a large number of parameters and that the designs produced are often impractical due to very close antenna spacings which result in mutual coupling.

The statistical or probabilistic approach in general leads to arrays with performance comparable to those designed analytically.[7,12,13] They bring out the statistics of the performance of different types of arrays without actually specifying an array which could be termed optimum in some sense.

Computer techniques used in optimization have used empirical approaches and perturbation methods.[4,6] Since computers can handle large numbers of antennas and also take care of many parameters simultaneously, they are well-suited for large arrays. Within a certain restricted sense they lead to optimum array configurations. However, for any rigorous optimization, the computing time soon becomes very long. For similar sized arrays, computer techniques have yielded array configurations which perform as well as arrays designed by any other technique.

It is clear that in the present state of the art the design of an optimum antenna array (even a linear additive array) remains a challenging unsolved problem. The problem of designing the VLA configuration is more involved than that of the linear additive array since the VLA is to be used as a correlator array in the supersynthesis mode. The two main complicating factors are:

(i) The beam pattern, as a function of the element locations, is much more involved for the correlator array than for the linear additive array.

(ii) When used in the supersynthesis mode, the synthesized beam depends not only upon the configuration but also upon the declination of the source and the hour angle range over which it is observed.

It is not possible to write a simple mathematical expression giving the beam pattern as a function of antenna locations, declination of source and tracking time. This makes any analytical approach towards optimization hopelessly difficult. The best approach is the use of a large sized high speed computer which can be programmed to compute the beam pattern of a given configuration for specified declination and tracking time. Such an approach has been used for the VLA configuration study using the IBM/360 computer. A pseudo-dynamic programming technique is being used which produces an "optimum" configuration within the framework of restrictions discussed below:

<u>Pseudo-Dynamic Programming</u>

The pseudo-dynamic programming technique efficiently utilizes the capabilities of the computer to find an optimum configuration. The optimum configuration defines the locations of the antennas which produce the best beam characteristics. In general, if N antennas can occupy any of P possible positions, the number of different possible arrangements is

$$^{P}C_{N} = \frac{P\ (P-1)\ (P-2)\cdots(P-N+1)}{N!}$$

The value of P is governed by the resolution and field of view specifications. Even for relatively small values of P, the above number becomes astronomically large and it is obviously impossible to study all these configurations. One can, therefore, choose the optimum configuration from these ${}^{P}C_{N}$ possibilities by one of three ways:

(i) Random selection: In this approach N numbers are selected at random out of the given P numbers and the transfer function corresponding to these N positions is computed. By considering a large number of random choices, the one yielding the best sampling in the spatial frequency plane (tranfer function) can be chosen as the final configuration.

(ii) Empirical approach: In this approach the N elements are placed in different geometrical forms (for example, along the circumference of a circle or along radial arms, etc.) and the geometrical form giving the best transfer function is selected. Further improvement of the transfer function is then effected by trying various distributions of the elements on the basic geometrical form selected and choosing the best distribution for the configuration.

(iii) Perturbation method: In this approach one starts with an arbitrary configuration. The position of each element is then perturbed by small amounts and the element is then left in a position which improves the transfer function. The final configuration incorporates the result of these perturbation studies.

The first approach obviously lacks any logic and even if a very large number of random configurations is tried, the final configuration cannot be termed optimum in any sense. The second approach points out the classes of geometrical forms which are better than other classes. The third approach improves a given configuration by introducing small perturbations. However, in effecting a small perturbation about the mean position of an antenna, only a local optimum position can be obtained.

The pseudo-dynamic programming combines the second approach with an improved version of the third approach. Basically, the computer selects the optimum location of an antenna from (P-N+1) possible locations for a given arrangement of (N-1) antennas, a given declination and a given tracking time. The starting positions of the N antennas are selected using approach (ii). By optimizing the positions of all antennas successively, a configuration is obtained which is optimum over the P positions for the declination and tracking time considered.

The shortcomings of this technique are (i) that the final optimum configuration obtained depends upon the starting configuration and (ii) that the optimization of one antenna depends upon the location of all others. Thus after optimizing the first antenna, when the second is moved to the optimum position, the position of the first may no longer be optimum.

In actual practice it is found that these two shortcomings have very insignificant effect on the result. It is found that the performance of beams of several optimized configurations for which different starting configurations were chosen are similar to within a fraction of a db. Also, once all N antenna positions have been optimized, we can start from the first antenna again and thus keep optimizing by performing several runs of the N antenna loop until no further improvement is possible. It is found that, depending upon how good the starting configuration is, one reaches a configuration, after about two runs, which cannot be substantially improved by further runs. It is also of interest to note the result of this pseudo-dynamic programming when applied to a linear, non-tracking array. The optimum positions for such an array are given from the theory discussed by Leech[14]. The program came out with these optimum positions after two runs only.

Procedure and Results

An exhaustive empirical study of various types of configurations has been done using digital computers. The results of this study are summarized in the VLA Proposal[15]. That study forms the starting point for the use of pseudo-dynamic programming. It is apparent that in view of the requirement of four configurations giving resolutions of 1", 3", 9", and 27", expandability of the array is very important. It has been shown that a three arm array in the form of a symmetrical Wye offers the best and cheapest layout for the track on which the antennas can be moved. The empirical study concluded that 36 antennas will be needed to achieve the desired beam characteristics in one day's observation.

With this background, the basic layout of the track has been taken as a symmetrical wye with one arm rotated $5^{\circ}$ east of the north-south direction. On each arm 42 possible positions (equispaced) have been taken (making P = 126). Optimization has been carried out mainly at a declination of $0^{\circ}$ although other declinations have also been used. This is because a well-filled transfer function is almost difficult to achieve at $0^{\circ}$. Several different starting configurations have been tried.

The computation of the beam pattern from a given transfer function is a time consuming process on the computer. Therefore, we cannot use the sidelobe levels as a criterion for optimization. The sidelobe levels depend both upon the number and distribution of the unsampled spatial frequencies (holes) in the transfer function. In general, holes near the center of the transfer function cause greater sidelobes than holes far from the center. A count of the number of holes in which greater weighting is given to the holes near the center, serves as a good criterion for optimization. Such a criterion using a gaussian weighting (termed weighted holes) has been used as a criterion and Figure 1 shows the decrease of the weighted holes as more and more antennas are optimized in a 30 element array. It is clear that no further improvement results after the 30 antennas have been run twice through the optimizing loop.

Two approaches have been used to arrive at a final configuration. In one, one starts with a large number of antennas (N = 30, was chosen) and keeps optimizing until no further improvement is possible. Thus each antenna is optimized with respect to the 29 others. In the other approach, one starts with a small number of antennas and optimizes their configuration. Then additional antennas are added, one at a time, each placed at the optimum position as given by the computer. Thus one gets the beam characteristics as a function of the number of elements. This relationship is shown in Figure 2 for $0^{o}$ declination. It should be noted that Figure 2 is strictly true for $0^{o}$ only and gives little information about performance at other declinations.

The performance of a 30-element array, optimized according to the first approach, is shown as a function of declination in Figure 3. For comparison, the performance of the 36-element array (termed supplemented Wye) as proposed in the VLA Proposal is also shown in Figure 3.

The pseudo-dynamic programming technique is a simple procedure for getting the best of a set of possible configurations. Its potentialities are vast and are limited mainly by the size of the computer and the amount of computer time available. This procedure, while bearing some similarity to the techniques of dynamic programming as proposed by Bellman[16], is not really the same since the Principle of Optimality is not satisfied. Nevertheless, with reference to the problem of VLA configuration, it appears to be the best approach.

## REFERENCES


1. Unz, H., "Linear Arrays with Arbitrarily Distributed Elements", IRE Trans. Ant. and Prop., AP-8, 222(1960)

2. King, D. D., R. F. Packard, and R. K. Thomas, "Unequally Spaced Broadband Antenna Arrays, IRE Trans. Ant. and Prop., AP-8, 380(1960)

3. Lo. Y. T. and Swenson, G. W., "The University of Illinois Radio Telescope", IRE, Trans. Ant. and Prop., AP-9(1961)

4. Andreason, M. G., "Linear Arrays with Variable Interelement Spacings", IRE Trans. Ant. and Prop., AP-10, 137(1962)

5. Ishimaru, A., "Theory of Unequally Spaced Arrays", IRE Trans. Ant and Prop., AP-10, 691(1962)

6. Skolnik, M. I., G. Nemhauser, and J. W. Shermann III, "Dynamic Programming Applied to Unequally Spaced Arrays", IEEE Trans. Ant. and Prop., AP-12, 35(1964)

7. Lo, Y. T. and S. W. Lee, "A Study of Space-Tapered Arrays", IEEE Trans. Ant. and Prop. AP-14, 22(1966)

8. Harrington, R. F., "Sidelobe Reduction by Nonuniform Element Spacing", IRE Trans. Ant. and Prop. AP-9, 187(1961)

9. Ishimaru, A. and Y. S. Chen, "Thinning and Broadbanding Antenna Arrays by Unequal Spacing", IEEE Trans. Ant. and Prop., AP-13, 34(1965)

10. Baklanov, Y. V., V. L. Pokrovishi, and G. I. Surdotovich, "A Theory of Linear Antennas with Unequal Spacing", Radio Engg. and Electronic Physics, 6, 905(1962)

11. Lo, Y. T., "A Probabilistic Approach to the Problem of Large Antenna Arrays" Radio Science, 68D, 1011(1064)

12. Skolnik, M. I., J. W. Shermann III, F. C. Ogg, "Statistically Designed Density-Tapered Arrays", IEEE Trans. Ant. and Prop. AP-12, 408(1964)

13. Rabinowitz, S. J. and R. F.Kolar, "Statistical Design of Space Tapered Arrays", presented at the 1962 12th Annual Symposium on USAF Antenna Research and Development Program, Univ. of Illinois, Urbana, Ill.

14. Leech, J., "On the Representation of 1, 2,...n By Differences", J. of London Mathematical Society, 31, 160(1956)

15. The VLA--A Proposal for a Very Large Array Radio Telescope, National Radio Astronomy Observatory, Charlottesville, Virginia. (1967)

16. Bellman, R., "Dynamic Programming", Princeton University Press, Princeton, (1957)

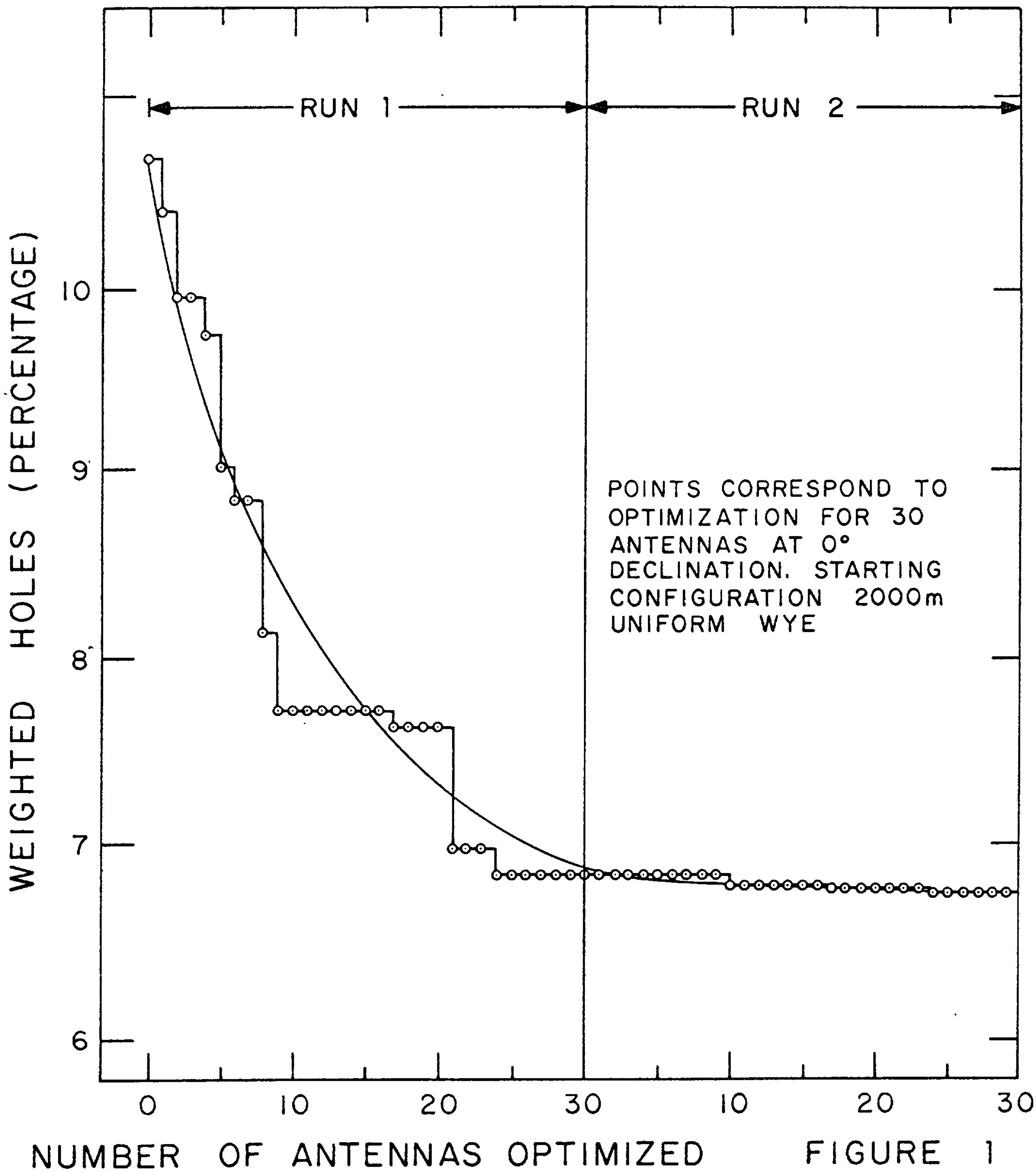
RUN 1
RUN 2
POINTS CORRESPOND TO OPTIMIZATION FOR 30 ANTENNAS AT 0° DECLINATION. STARTING CONFIGURATION 2000m UNIFORM WYE
WEIGHTED HOLES (PERCENTAGE)
6
7
8
9
10
0
10
20
30
10
20
30
NUMBER OF ANTENNAS OPTIMIZED

FIGURE 1

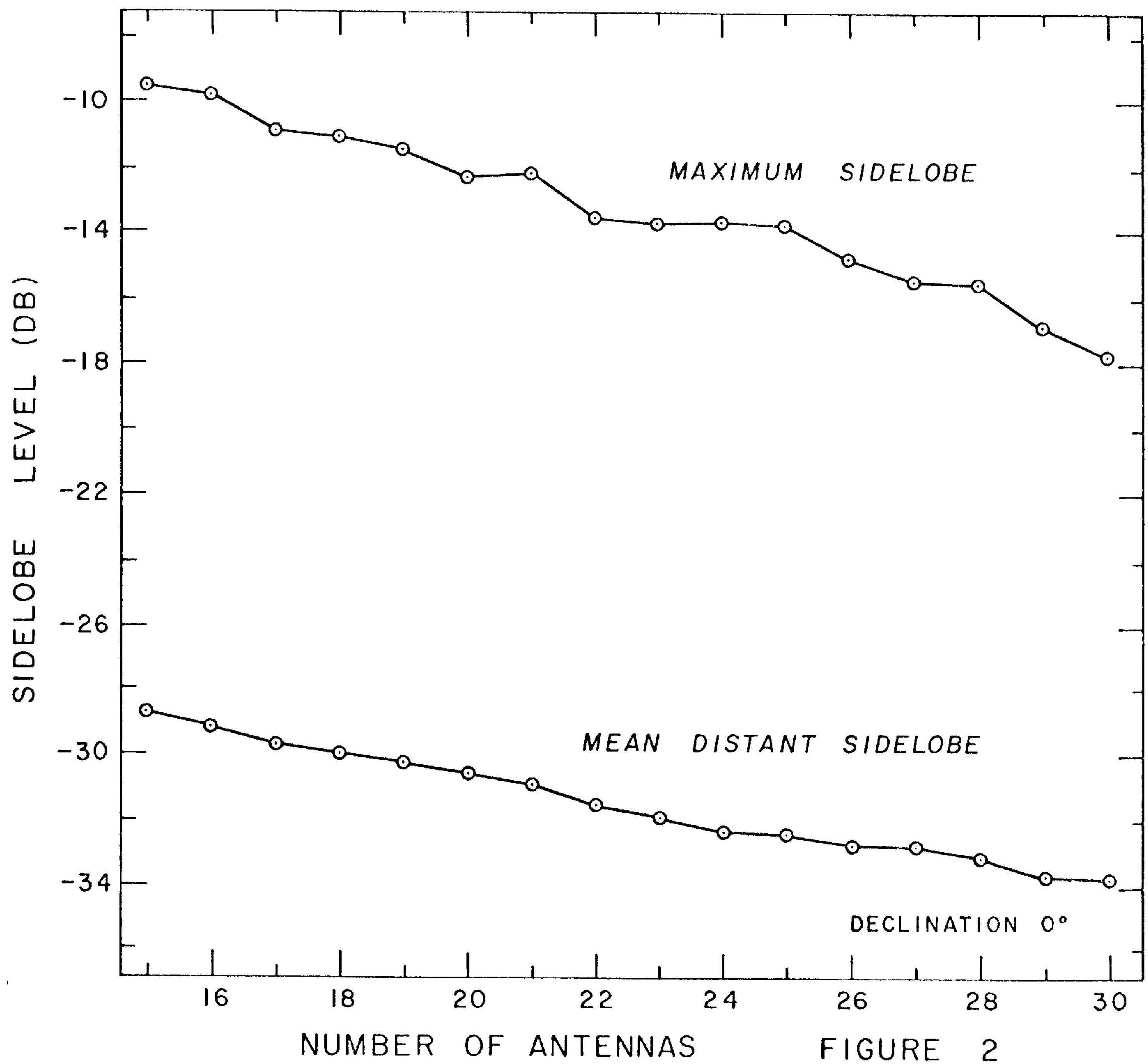
SIDELOBE LEVEL (DB)
-10
-14
-18
-22
-26
-30
-34
MAXIMUM SIDELOBE
MEAN DISTANT SIDELOBE
DECLINATION 0°
16
18
20
22
24
26
28
30
NUMBER OF ANTENNAS

FIGURE 2

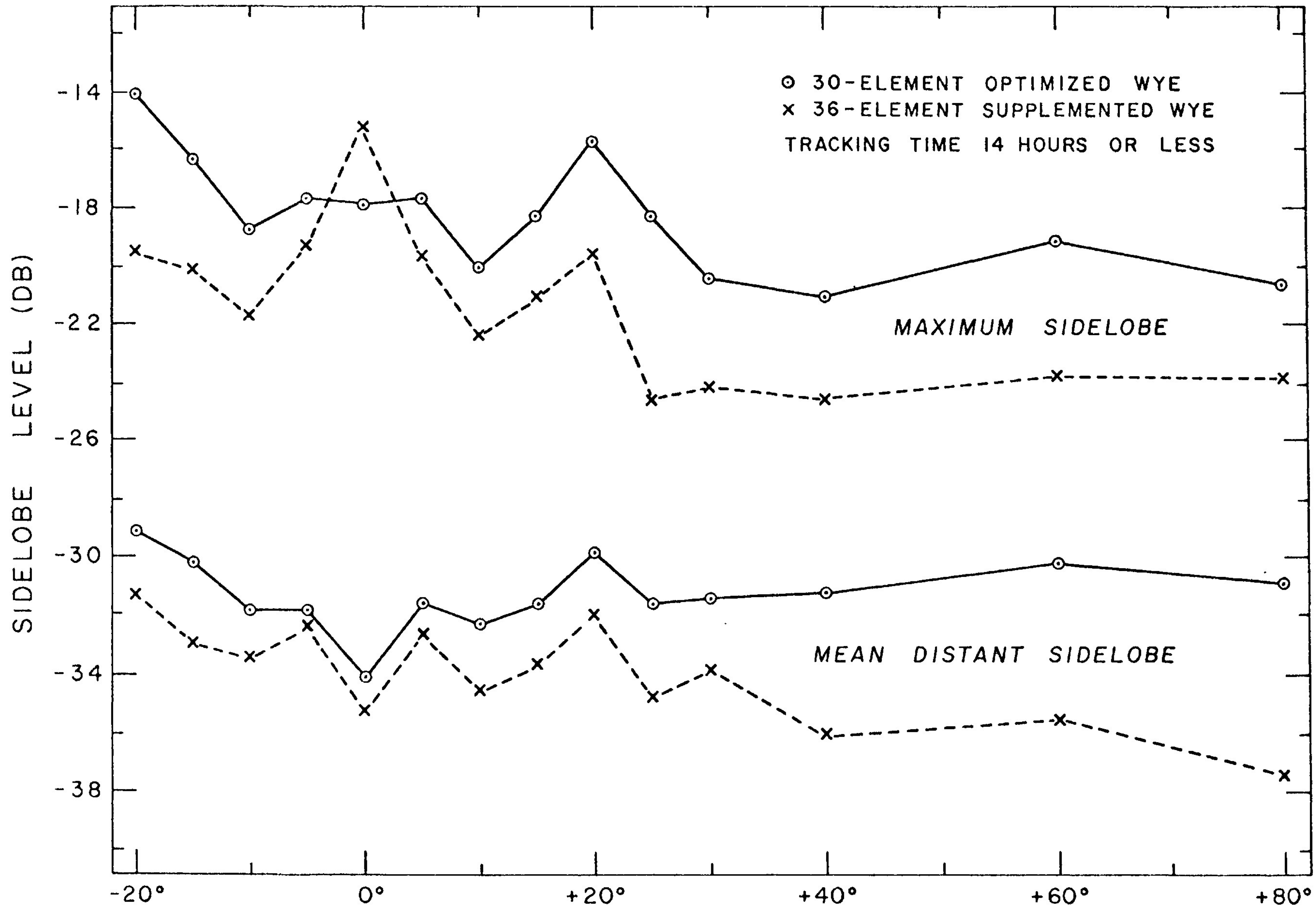
30-ELEMENT OPTIMIZED WYE
36-ELEMENT SUPPLEMENTED WYE
TRACKING TIME 14 HOURS OR LESS
MAXIMUM SIDELOBE
MEAN DISTANT SIDELOBE
SIDELOBE LEVEL (DB)
-14
-18
-22
-26
-30
-34
-38
-20°
0°
+20°
+40°
+60°
+80°
DECLINATION

FIGURE 3