FINAL REPORT ON THE ANALYSIS OF THE ELECTRICAL DISTRIBUTION SYSTEM AND EMERGENCY GENERATOR CAPABILITY OF THE VERY LARGE ARRAY SITE UNDER SUBCONTRACT VLA-373 OF NSF AST 79-08925

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TABLE OF CONTENTS

I. INTRODUCTION

The purpose of this report is to present the final results of the analysis of the electrical distribution system and standby generator capabilities of the Very Large Array (VLA) site. This analysis was initiated to study a problem of parallel generator instability at the site during emergency power service to the VLA antennas when operated in certain configurations. The distribution network consists primarily of three 7200/12470 volt three phase underground cable runs of length between 11 and 12 miles each. During loss of commercial service, two 500 KW diesel engine driven generators are utilized for standby power. Past experience has indicated that these two generators can supply the emergency load satisfactorily when one or more of the three cable runs are disconnected or shortened by the opening of sectionalizing switches. When all three complete cable runs are connected, site personnel have reported that one generator consistently trips off the line generally through the initiation of a reverse power relay. The remaining generator has been able to supply the critical site load although it was reportedly in an overload condition. Attempts to document a typical sequence of events during loss of commercial service have led to somewhat contradictory information although some test data has been verified through simulation in this analysis.

The primary objective of this work is to determine the most economical system modification which will permit parallel generator operation under site critical load with all cable runs in service and all antennas on line in any of the four configurations. VLA site consultants have recommended the installation of two 100 KVA shunt reactors at two locations on each of the three cable runs. Complete analysis of the generator control functions must be made to determine if an alternative method could permit stable parallel operation.

This work was scheduled in two phases. The first phase consisted of a complete steady state analysis of the electrical distribution system to determine the potential size and location for shunt reactive compensation of the cable system. In addition, the generator and generator control systems were analyzed from a preliminary simplified model to obtain a feeling for the need of the shunt reactors and any potential problems for their installation. The results were documented in Reference [17]. The second phase consisted of a full transient stability analysis of the two generating systems with and without shunt reactors. The purpose of the second phase was to confirm the preliminary conclusion that shunt reactors are required, and to detect any possible problems which may be created by the shunt compensation. This report is a comprehensive summary of the first and second phases and contains a full description of the analysis and conclusions.

II. STEADY STATE SYSTEM ANALYSIS

This chapter is concerned with the steady state analysis of the VLA electrical distribution network and emergency generator systems. The analysis assumes sinusoidal balanced steady state with constant speed, constant voltage generator supply. The simulation and analysis are divided by components in the following sections.

2.1 Cable Model

The three cable runs serving the VLA antenna loads consist of 3-1/c #2 AWG Alum, 175 mils high molecular weight Polyethylene 15 KV primary concentric UD cable [1,2]. The electrical characteristics of this cable were computed as follows using the methods of references [3, 5, 6]. Line charging shunt capacitance per phase equivalent,

$$C = \frac{0.00736\ K}{\log_{10} \frac{D}{d}}$$

where C = capacitance in μf/1000 ft

K = dielectric constant of insulation

D = diameter over insulation

d = diameter over conductor.

Using D = 0.71", d = 0.36", K = 2.3, the single phase equivalent line charging capacitance is computed as:

$$\underline{C = 0.0574\mu f/1000\ ft}$$

This number agrees with typical values in reference [5], and the manufacturers' data provided in reference [4].

The single phase series impedance equivalent is computed as the positive sequence impedance from the single phase self impedance minus the mutual impedance of the conductors.

$$Z_{series} = Z_{self} - Z_m$$

From references [3, 5, 6],

$$Z_{self} = [R_a + 4.788 \times 10^{-5} \times 2\pi f]$$

$$+ j\ 2\pi f[6.618 \times 10^{-4} + 6.096 \times 10^{-5} \ln \frac{1}{24\ GMR_a} \sqrt{\rho/f}\ ]$$

where R_a = conductor resistance

GMR_a = geometric mean radius in feet

ρ = earth resistivity

f = frequency

using R_a = 0.317 ohm/1000 ft, GMR_a = 0.0083', ρ = 100 meter ohms, f = 60 Hz

$$Z_{self} = 0.335 + j\ 0.292 \text{ ohm/1000 ft.}$$

Similarly,

$$Z_m = [4.788 \times 10^{-5} \times 2\pi f] + j\ 2\pi f[4.681 \times 10^{-4} + 6.096 \times 10^{-5} \ln\left(\frac{1}{S_m} \sqrt{\rho/f}\right)]$$

where S_m depends on conductor installation. Considering the direct burial to be flat trench spacing of 7.5" between conductors,

$$S_m = \sqrt[3]{7.5 \times 7.5 \times 15} = 9.45'' = 0.7875'$$

so that

$$Z_m = 0.018 + j\ 0.1878 \text{ ohm/1000 ft.}$$

Thus the series single phase equivalent impedance at 60 Hz is,

$$Z_{series} = 0.317 + j\ 0.1042 \text{ ohm/1000 ft}$$

or

$$\underline{R_{series} = 0.317 \text{ ohm/1000 ft}}$$

and

$$\underline{L_{series} = 0.276 \times 10^{-3} \text{ h/1000 ft.}}$$

Examination of the frequency variation of the self and mutual impedances clearly shows that the resistance and inductance values above are independent of frequency. The following single phase equivalent PI section was used to simulate the cable runs.

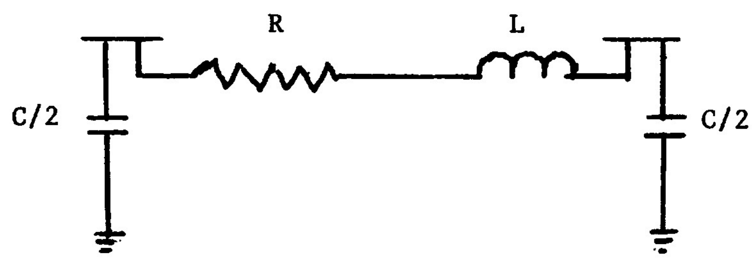


Figure 1

where $R = 0.317$ ohm/1000 ft

$L = 0.2764 \times 10^{-3}$ h/1000 ft

$C = 0.0574$ μf/1000 ft.

All of these values agree with typical values of references [5, 6] and the manufacturer's data of reference [4].

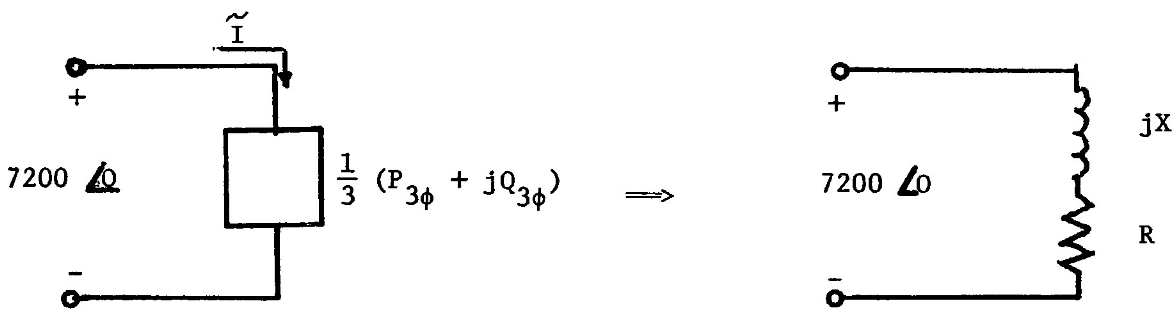
Several cable sections are of the order of two to three miles in length. The error in using a lumped parameter model for these large sections was computed. In reference to the distributed parameter model of a ten mile line, the error of the lumped parameter model was found to be less than 1% [6]. The cable network was thus simulated by the single phase equivalent circuit of Figure 1 with each R, L and C being equal to the distributed parameter value times the appropriate line length.

2.2 Load Model

The system was analyzed in response to existing full load and proposed site critical load. The existing full service load was estimated from the transformer installations shown in reference [1]. Proposed site critical loads were taken from data provided in reference [7]. The loads used were:

	Load Name	Existing Full Load (.85 pf lag)	Proposed Site Critical Load (.85 pf lag)
a)	MD1 - 750 KVA TFMR Pump house, etc.	300 KVA	12.8 KVA
b)	MD5 - 500 KVA TFMR Cafeteria, etc.	200 KVA	0.0 KVA
c)	MD6 - 750 KVA TFMR Control Bldg., etc.	200 KVA	8.6 KVA
d)	W1D - 275 KVA Antenna Assembly	100 KVA	0.0 KVA
e)	Each of 27 Antenna Locations (plus 10 KVA during stow, 5 antennas at a time)	25 KVA	11.0 KVA

A constant impedance load model was used in the analysis as follows:



$$\bar{S}_{\substack{IN\\1\phi}} = \tilde{V}_{1n}\,\tilde{I}^{*} = \frac{1}{3}\,(P)_{3\phi} + jQ_{3\phi})$$

$$\bar{S}_{\substack{IN\\1\phi}} = \tilde{V}_{1n}\,\tilde{I}^{*} = \tilde{V}_{1n}(\bar{Y}\,\tilde{V}_{1n})^{*} = |\tilde{V}_{1n}|^{2}\,\bar{Y}^{*} = \frac{|\tilde{V}_{1n}|^{2}}{R - jX}$$

Figure 2

The equivalent single phase constant impedance load model used for given $P_{3\phi} + jQ_{3\phi}$ was

$$R + jX = \frac{3|\tilde{V}_{1n}|^{2}}{P_{3\phi} - jQ_{3\phi}}$$

For example, the MD1 proposed site critical load was simulated as a constant impedance tie to ground of

$$\overline{Z} = \frac{277^2 \times 3}{(12.8 \times 0.85 - j\ 12.8 \times 0.53) \times 10^3}$$

$$= 15.29 + j\ 9.47 \text{ ohms/phase.}$$

This type of load modeling is acceptable provided the line to neutral voltage magnitude does not change substantially between no load and full load.

2.3 Generator Model

For purposes of the steady state analysis, the two 500 KW generators and associated control systems were simulated as an ideal voltage source as follows:

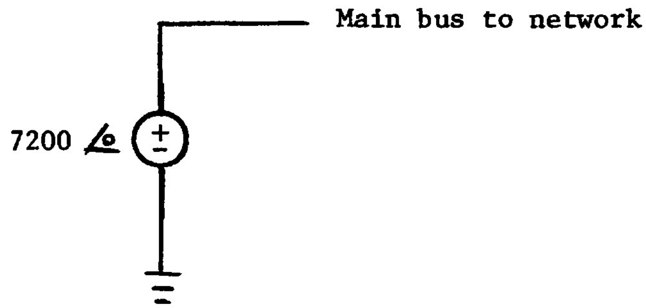


Figure 3

2.4 Network Model

The entire VLA site was modelled using a 74 Bus 73 Line single phase equivalent with bus numbers assigned to bus locations as follows:

Table 1

Bus #	Bus Description
1	MD1 - 277 volt generator terminals
2	MD2 -7200 volt side at 750 KVA Gen. TFMR
3	MD3 - 7200 volt side at 750 KVA Control Bldg.
4	MD4 - 7200 volt side at 500 KVA Cafeteria
5	MD5 - 277 volt side at 500 KVA Cafeteria
6	MD6 - 277 volt side at 750 KVA Control Building

Table 1 (continued)

Bus #	Bus Description
7	TW1 - 7200 volt side
8	TW2 - "
9	TW3 - "
10	TW4 - "
11	TW5 - "
12	TW6 - "
13	TW7 - "
14	TW8 - "
15	TW9 - "
16	TW10 - "
17	TW11 - "
18	TW12 - "
19	TW13 - "
20	TE1 - "
21	TE2 - "
22	TE3 - "
23	TE4 - "
24	TE5 - "
25	TE6 - "
26	TE7 - "
27	TE8 - "
28	TE9 - "
29	TE10 - "
30	TE11 - "
31	TE12 - "
32	TN1 - "
33	TN2 - "
34	TN3 - "
35	TN4 - "
36	TN5 - "
37	TN6 - "
38	TN7 - "
39	TN8 - "
40	TN9 - "
41	TN10 - "
42	TN11 - "
43	TN12 - "
44	TW14 - "
45	TW15 - "
46	TW16 - "
47	TW17 - "
48	TW18 - "
49	TW19 - "
50	TW20 - "
51	TW21 - "
52	TW22 - "
53	TW23 - "
54	TE13 - "
55	TE14 - "
56	TE15 - "
57	TE16 - "

Table 1 (continued)

Bus #	Bus Description
58	TE17 - 7200 volt side
59	TE18 - "
60	TE19 - "
61	TE20 - "
62	TE21 - "
63	TE22 - "
64	TN13 - "
65	TN14 - "
66	TN15 - "
67	TN16 - "
68	TN17 - "
69	TN18 - "
70	TN19 - "
71	TN20 - "
72	TN21 - "
73	TN22 - "
74	TW10 - 277 volt side (Assembly Bldg.)

This definition of the system buses corresponds to the one line diagram shown in Figure 4. The numbers between the buses are the line lengths in feet. The line and load impedances were computed and per unitized to a common system power base of 1.0×10^6 watts three phase. For the 7200/12470 volt impedances,

$$V_{Base_{HV}} = 7200 \text{ volts}$$

$$I_{Base_{HV}} = \frac{10^6/3}{7200} = 46.3 \text{ amps}$$

$$Z_{Base_{HV}} = \frac{7200}{46.3} = 155.5 \text{ ohms.}$$

For example, the line parameters for the line between MD2 and MD3 are computed from its 1450 foot length as follows:

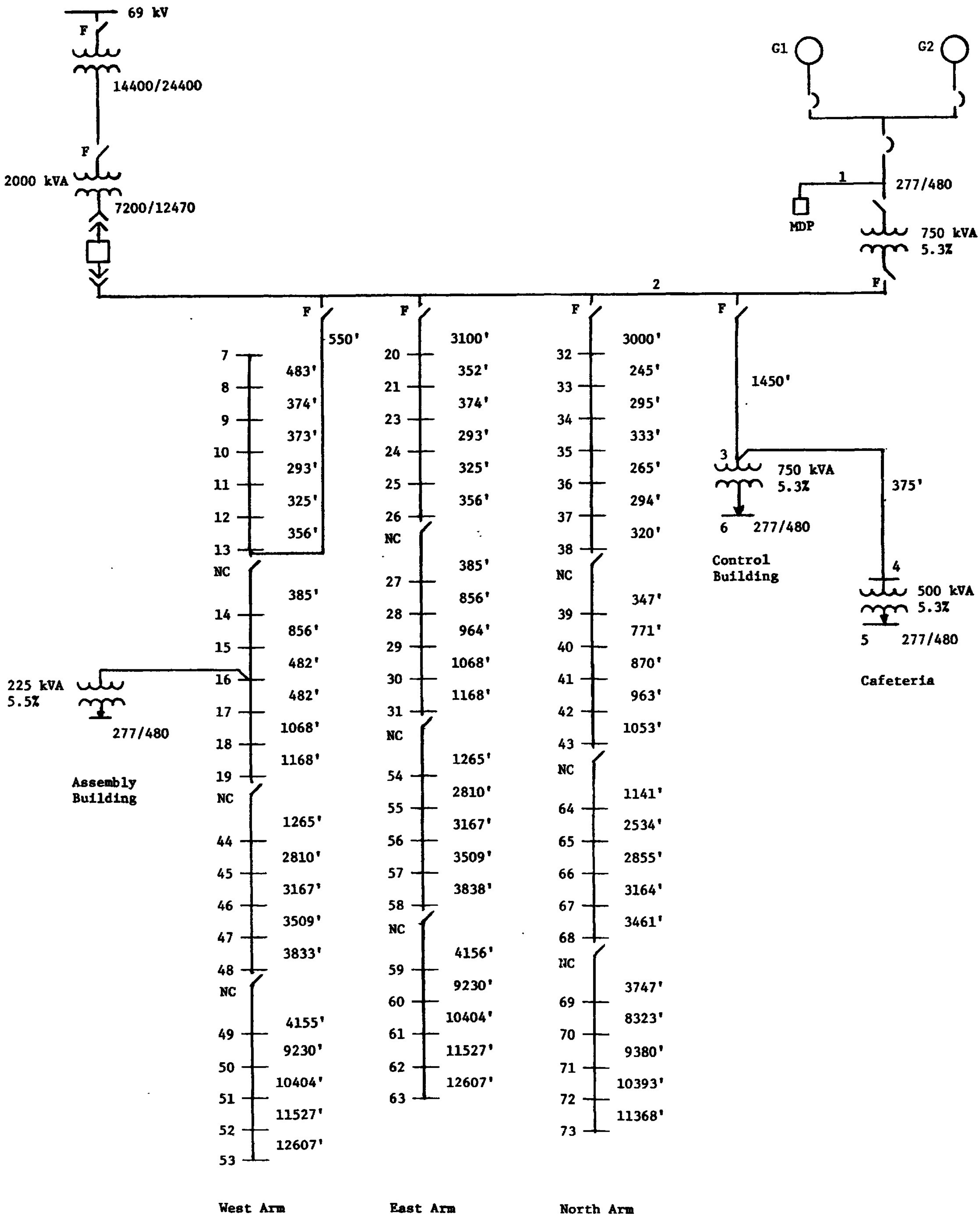

69 kV
F
14400/24400
F
2000 kVA
7200/12470
G1
G2
1
277/480
MDP
750 kVA
5.3%
F
2
F
F
F
F
550'
3100'
3000'
7
483'
8
374'
9
373'
10
293'
11
325'
12
356'
13
NC
385'
14
856'
15
482'
16
482'
17
1068'
18
1168'
19
NC
1265'
44
2810'
45
3167'
46
3509'
47
3833'
48
NC
4155'
49
9230'
50
10404'
51
11527'
52
12607'
53
20
352'
21
374'
23
293'
24
325'
25
356'
26
NC
385'
27
856'
28
964'
29
1068'
30
1168'
31
NC
1265'
54
2810'
55
3167'
56
3509'
57
3838'
58
NC
4156'
59
9230'
60
10404'
61
11527'
62
12607'
63
32
245'
33
295'
34
333'
35
265'
36
294'
37
320'
38
NC
347'
39
771'
40
870'
41
963'
42
1053'
43
NC
1141'
64
2534'
65
2855'
66
3164'
67
3461'
68
NC
3747'
69
8323'
70
9380'
71
10393'
72
11368'
73
1450'
3
750 kVA
5.3%
6
277/480
375'
Control
Building
4
500 kVA
5.3%
5
277/480
Cafeteria
225 kVA
5.5%
277/480
Assembly
Building
West Arm
East Arm
North Arm

Figure 4

$$R_{Line} = (0.317 \times 10^{-3} \frac{ohm}{Ft})\ (1450\ Ft)/155.5 = 2.96 \times 10^{-3}\ \text{P.U. Ohms}$$

$$\omega L_{Line} = 2\pi \times 60 \times 0.2764 \times 10^{-6} \frac{h}{Ft} \times 1450\ Ft/155.5$$

$$= 9.72 \times 10^{-4}\ \text{P.U. ohms}$$

$$\omega C_{Shunt} = 2\pi \times 60 \times 0.0574 \times 10^{-9} \frac{f}{Ft} \times 1450\ Ft \times 155.5$$

$$= 4.879 \times 10^{-3}\ \text{P.U. ohms}$$

For the 277/480 volt impedances,

$$V_{Base_{LV}} = 277\ \text{volts}$$

$$I_{Base_{LV}} = \frac{10^6/3}{277} = 1203.4\ \text{amps}$$

$$Z_{Base_{LV}} = \frac{277}{1203.4} = 0.23\ \text{ohms.}$$

For example, the single phase constant impedance load equivalent for the proposed site critical power at bus MD1 is

$$\bar{Z}_{gnd} = (15.29 + j\ 9.47)/0.23$$

$$= 66.48 + j\ 41.17\ \text{P.U. ohms}$$

The main transformer impedances were modelled by their per cent impedance as follows:

$$\bar{Z}_{series} = j\left(\frac{\%\ Z}{100}\right) \frac{10^6}{VA_{Rated}}\ \text{P.U. ohms}$$

For example, the line between buses MD1 and MD2 is a 750 KVA 5.3% impedance transformer. It was modelled as a series line impedance of:

$$\overline{Z}_{series} = 0 + j\left(\frac{5.3}{100} \times \frac{10^6}{750\times10^3}\right)$$

$$= 0 + j\ 0.0707 \text{ P.U. ohms}$$

The small transformers serving the antenna load were not included in the analysis. The antenna loads were considered to be connected directly to the 7200 volt line. This was done to reduce the number of system buses, and does not introduce significant error in the steady state analysis. The voltage on the secondary side of each antenna transformer will be with 99.5% of the referred primary voltage. The location of each antenna load for the four configurations is given in Table 2.

Table 2

A	B
Loads at Buses	Loads at Buses
12, 18, 44, 47, 49, 50, 51, 52, 53, 25 30, 55, 57, 59, 60, 61, 62, 63, 37, 42, 65, 67, 69, 70, 71, 72, 73	9, 12, 15, 18, 44, 45, 46, 47, 48, 22, 25, 28, 30, 54, 55, 56, 57, 58, 34, 37, 40, 42, 64, 65, 66, 67, 68

C	D
Loads at Buses	Loads at Buses
8, 9, 10, 12, 14, 15, 17, 18, 19, 21, 22, 23, 25, 27, 28 29, 30, 31, 33, 34, 35, 37, 39, 40, 41, 42, 43	7, 8, 8, 9, 9, 10, 11, 12, 13, 20, 21, 21, 22, 22, 23, 24, 25, 26, 32, 33, 33, 34, 34, 35, 36, 37, 38

2.5 Voltage and Power Flow Analysis

The distribution system was analyzed under three loading conditions. Using the estimated normal full load values given in section 2.2, the analysis yielded the voltage profile and source requirements shown in Table 3a. Using the proposed site critical loads given in section 2.2, the analysis yielded the voltage profile and source requirements shown in Table 3b. The no-load analysis results are shown in Table 3c. All studies were performed with the antennas located in the "A" configuration, assuming the source is the standby generator pair. Under full load, the site power is nearly unity power factor, but neither the generator pair nor the step up 750 KVA transformer could supply this load. Full load system voltages are ideal. Under the proposed critical site load, the main step up transformer is marginal, although this is not a serious problem. It is important to note that the KVA requirement is 655, which exceeds the KVA rating of one generator. System voltages are acceptable, and could be brought to ideal by adjustment of the automatic voltage regulator set point. Under no load conditions, the reactive power requirements alone exceed one generator's KVA rating by more than 20%. System voltages again could be brought down if the regulator set point could be reduced.

Table 3

	Minimum voltage percent(1)	Maximum voltage percent(1)	Maximum line load percent(2)	Source real power required(3)	Source reactive power required(4)
a. (full load)	98.7	100.1	144.4	1,339.0	+ 184.6
b. (proposed critical load)	100.0	104.2	91.3	300.5	- 583.4
c. (no load)	100.0	106.2	109.6	9.6	- 778.9

(1) 100% = 7200 volts or 277 volts
(2) Based on transformer rating or 116 amp cable load (70% of 165 amp Cable rating)
(3) KW 3ϕ
(4) KVAR 3ϕ
(+) Lag - Inductive
(-) Lead - Capacitive

The previous analysis was performed without the installation of any shunt reactive compensation. If shunt compensation is made at the low voltage side of the 750 KVA transformer, the voltage profiles and line power flow results would remain unchanged. The source reactive power requirements would of course be reduced. If shunt compensation is made anywhere on the high voltage cable system, the main 750 KVA step up transformer would be considerably unloaded. This is an argument favoring high voltage compensation and will be discussed later.

2.6 Resonant Frequency Analysis

Reactive compensation of electrical networks can often lead to undesirable resonance conditions. This is particularly true when series compensation is considered. Series compensation of the leading power factor cable was ruled out as an alternative solution early in the analysis. This is primarily due to the fact that the leading power factor is a shunt load, virtually independent of system load. As stated earlier, any shunt compensation on the low voltage side of the main step up transformer (i.e. generator terminals) will not affect the steady state performance of the distribution network. Nevertheless, the resonant condition must be considered in the transient analysis.

The driving point impedance as seen by the generators was computed for various loading conditions and for frequencies between 0.5 and 250 Hz. The results of this analysis are shown in the following figures. Figures 5 and 6 show the impedance with critical load and no shunt inductance. This is the existing system response. Figure 7 and 8 show the existing system no load response. Figures 9 and 10 show the system response at critical load with an inductance corresponding to 200 KVAR per phase reactance at 60 Hz at the generator terminals. Figures 11 and 12 show the no load response with the same inductance in shunt with the generators. This frequency response shows a sharp peak at nearly 53 Hz. While this will have virtually no effect on the steady state system performance, it should be analyzed further in the transient state.

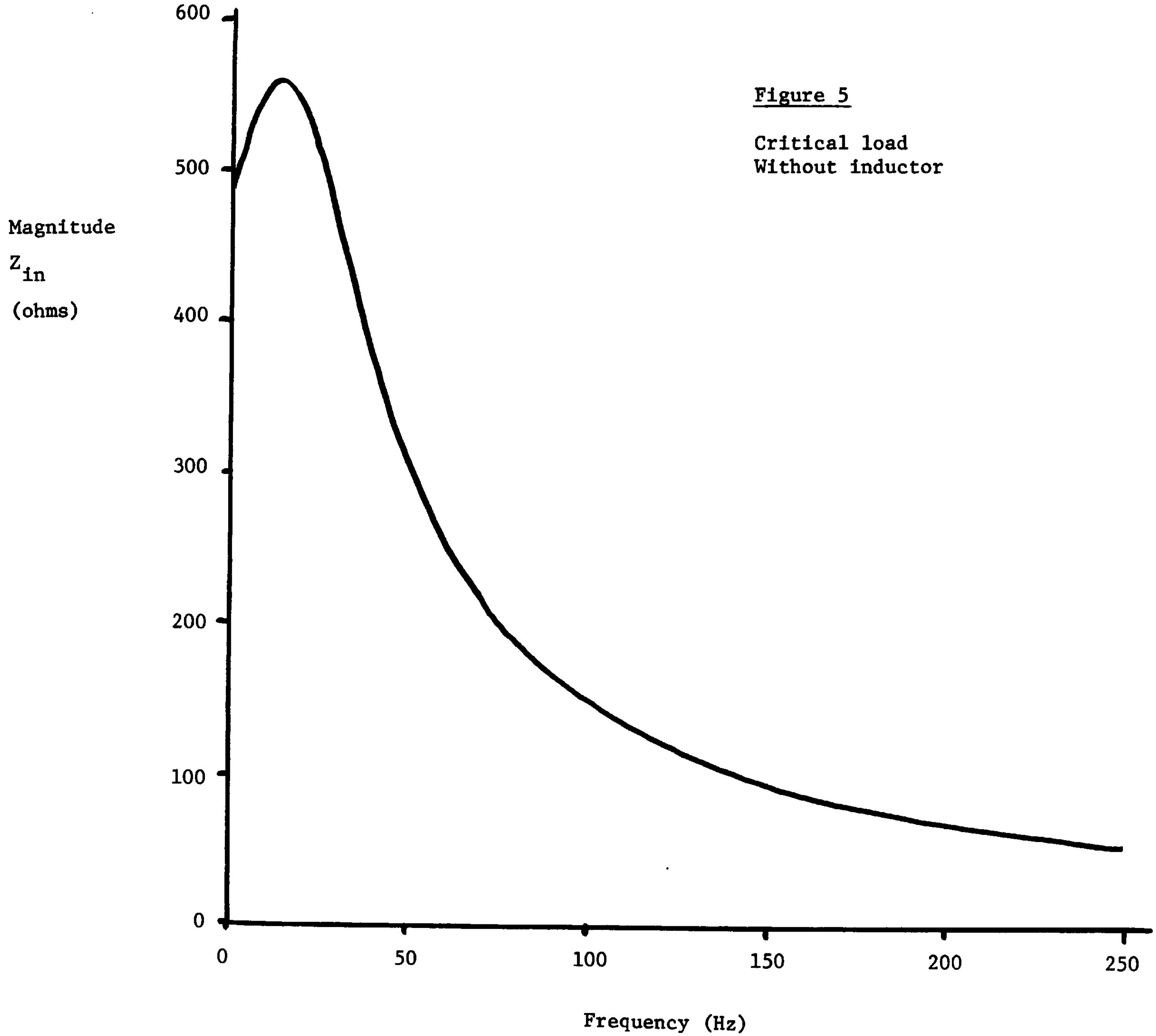


Figure 5

Critical load
Without inductor

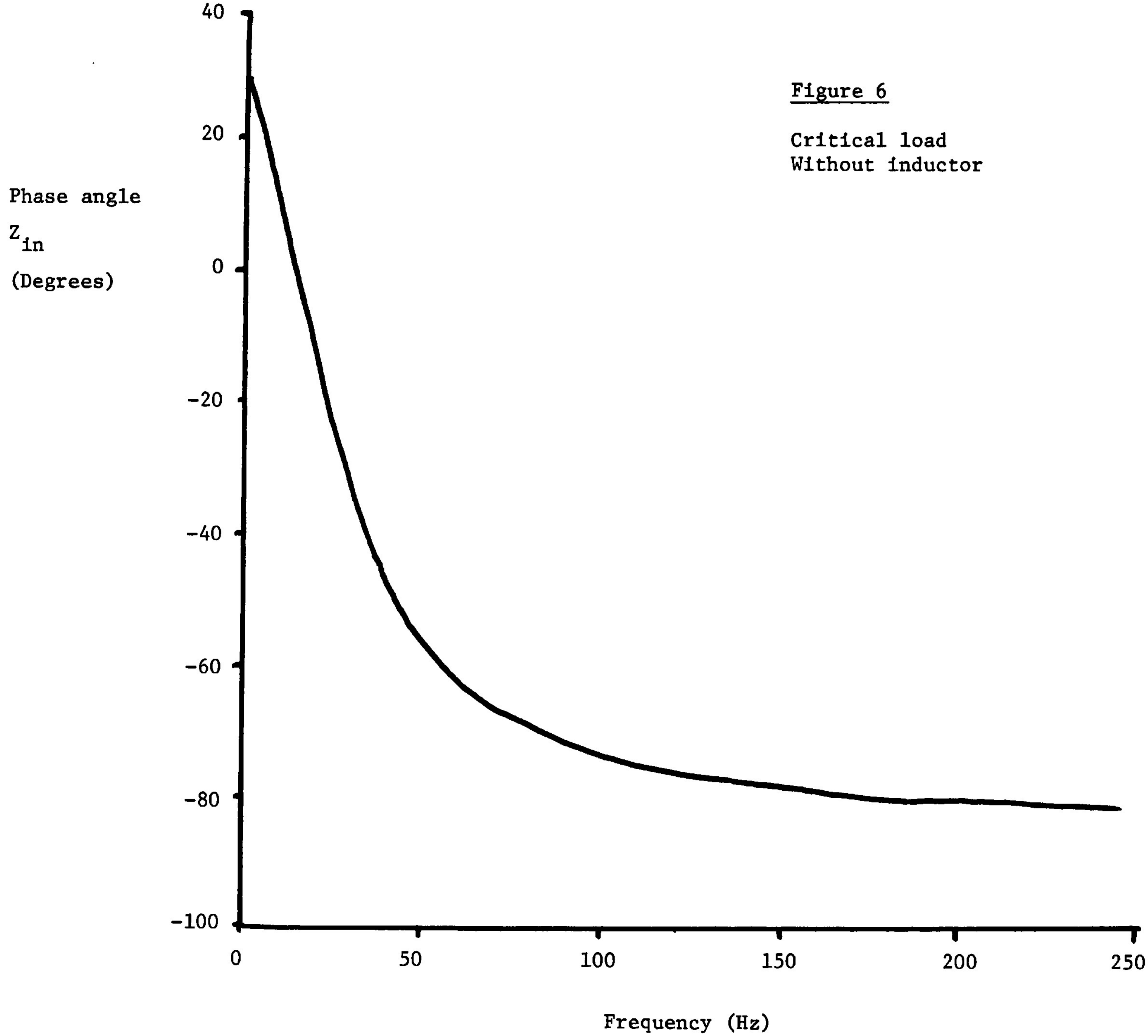


Figure 6

Critical load
Without inductor

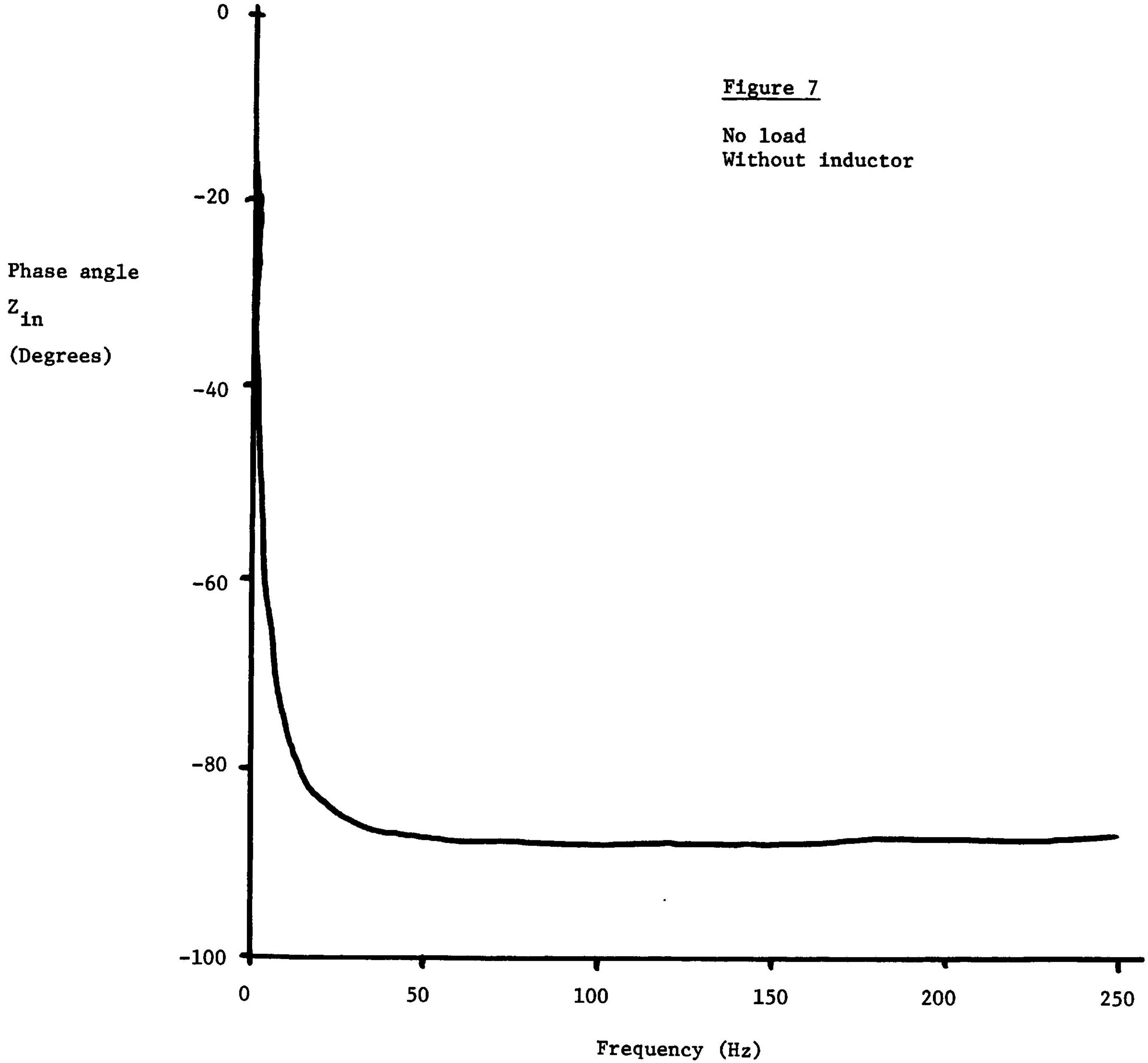


Figure 7

No load
Without inductor

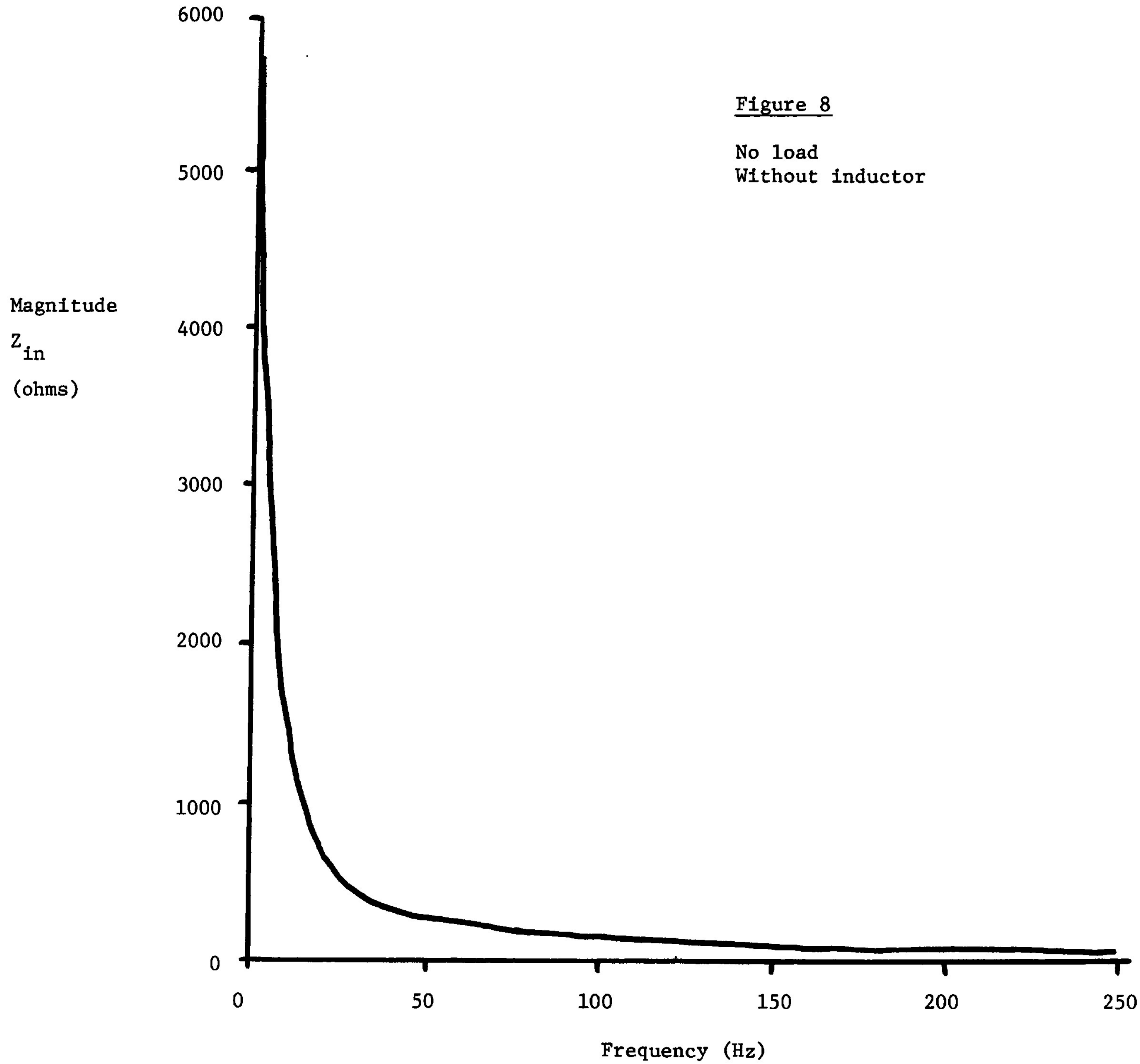


Figure 8

No load
Without inductor

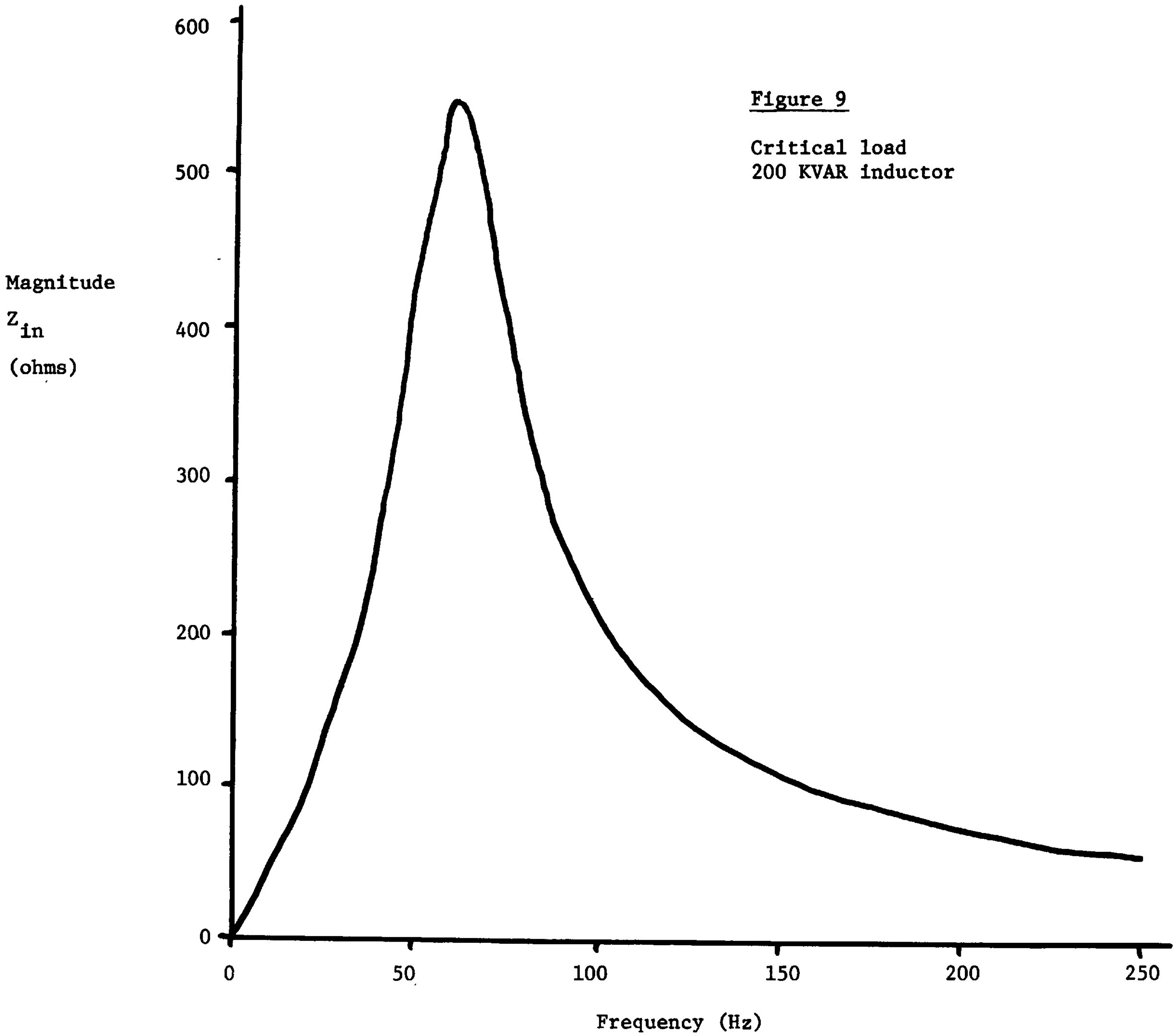


Figure 9

Critical load
200 KVAR inductor

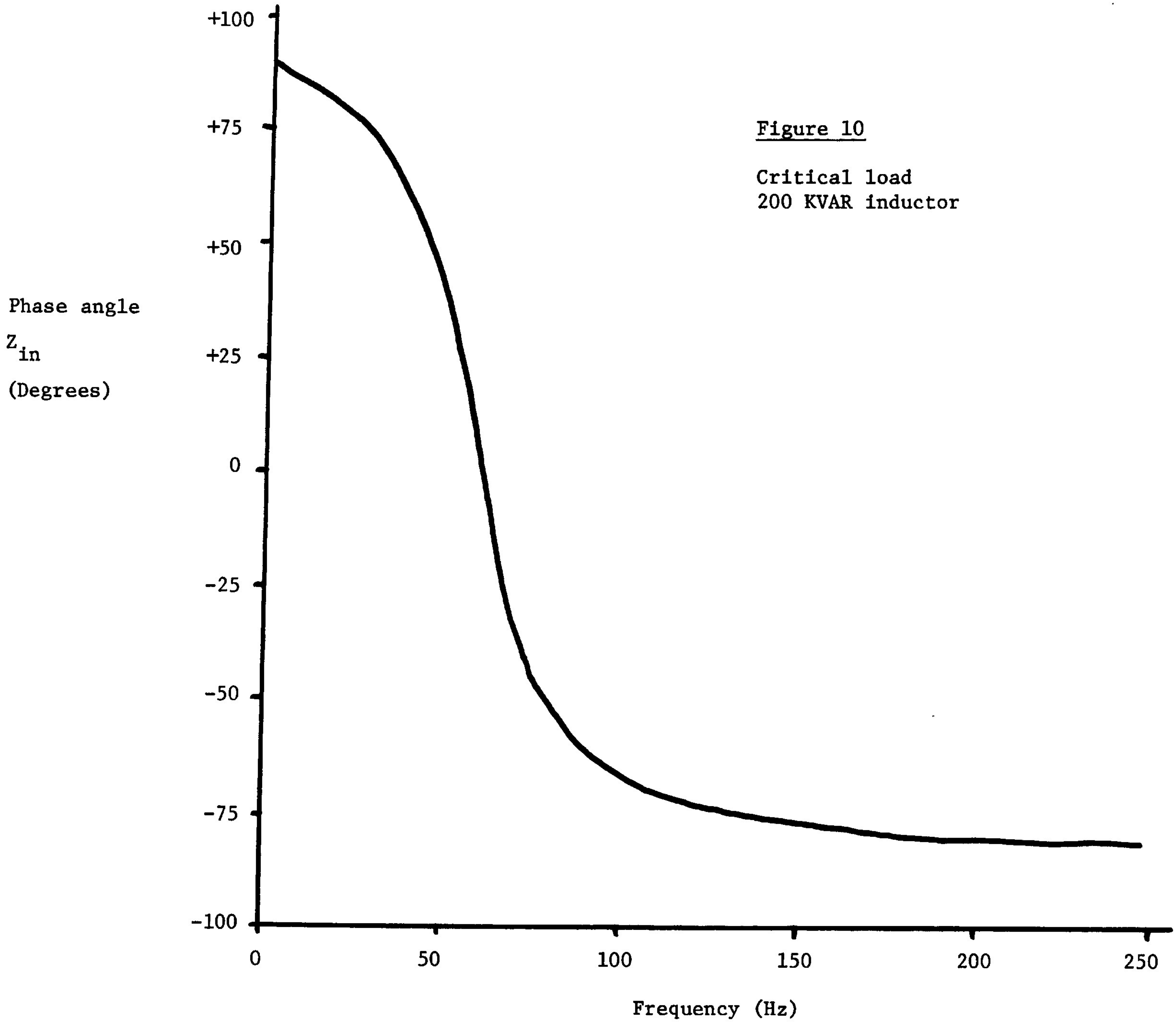
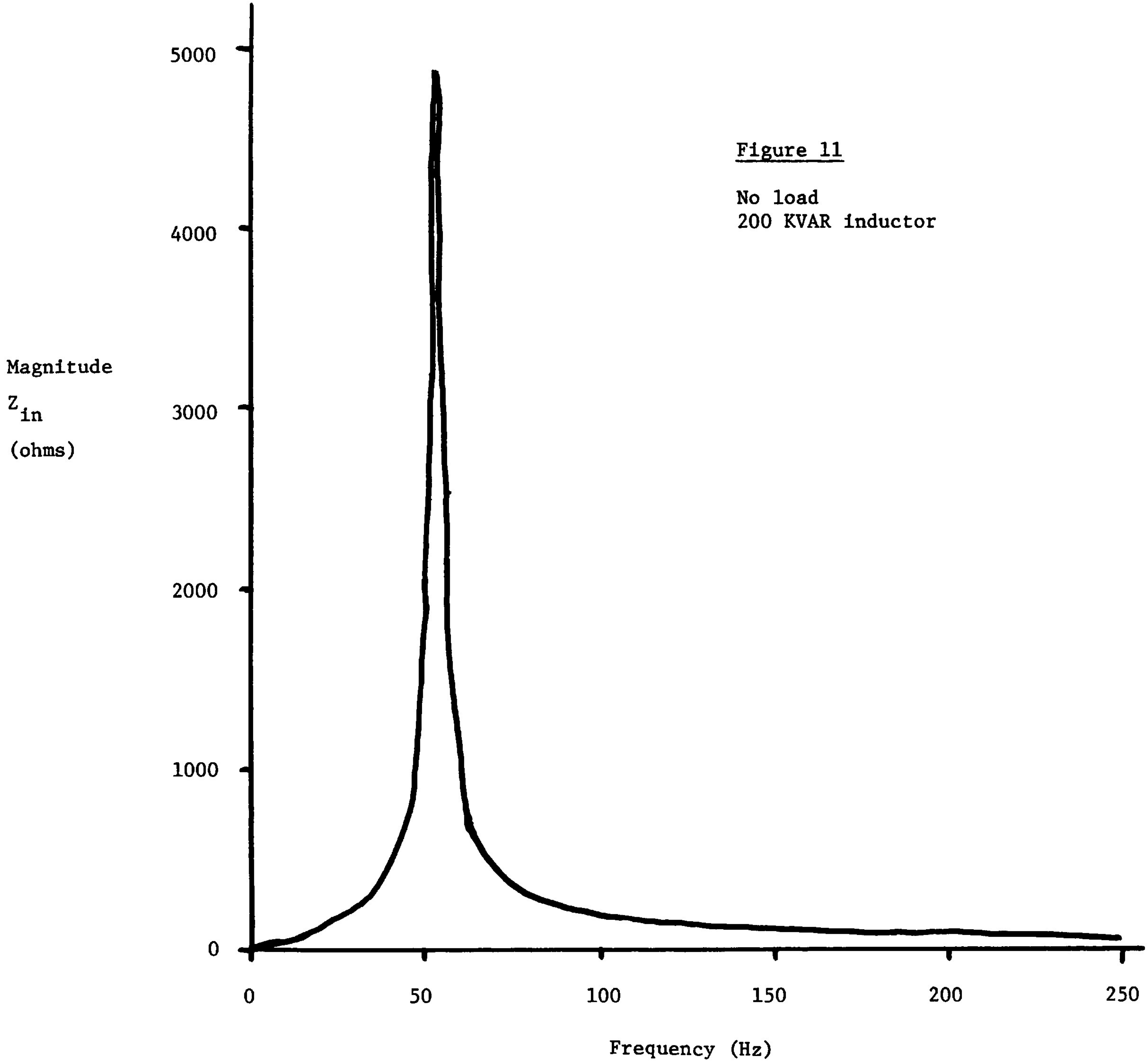


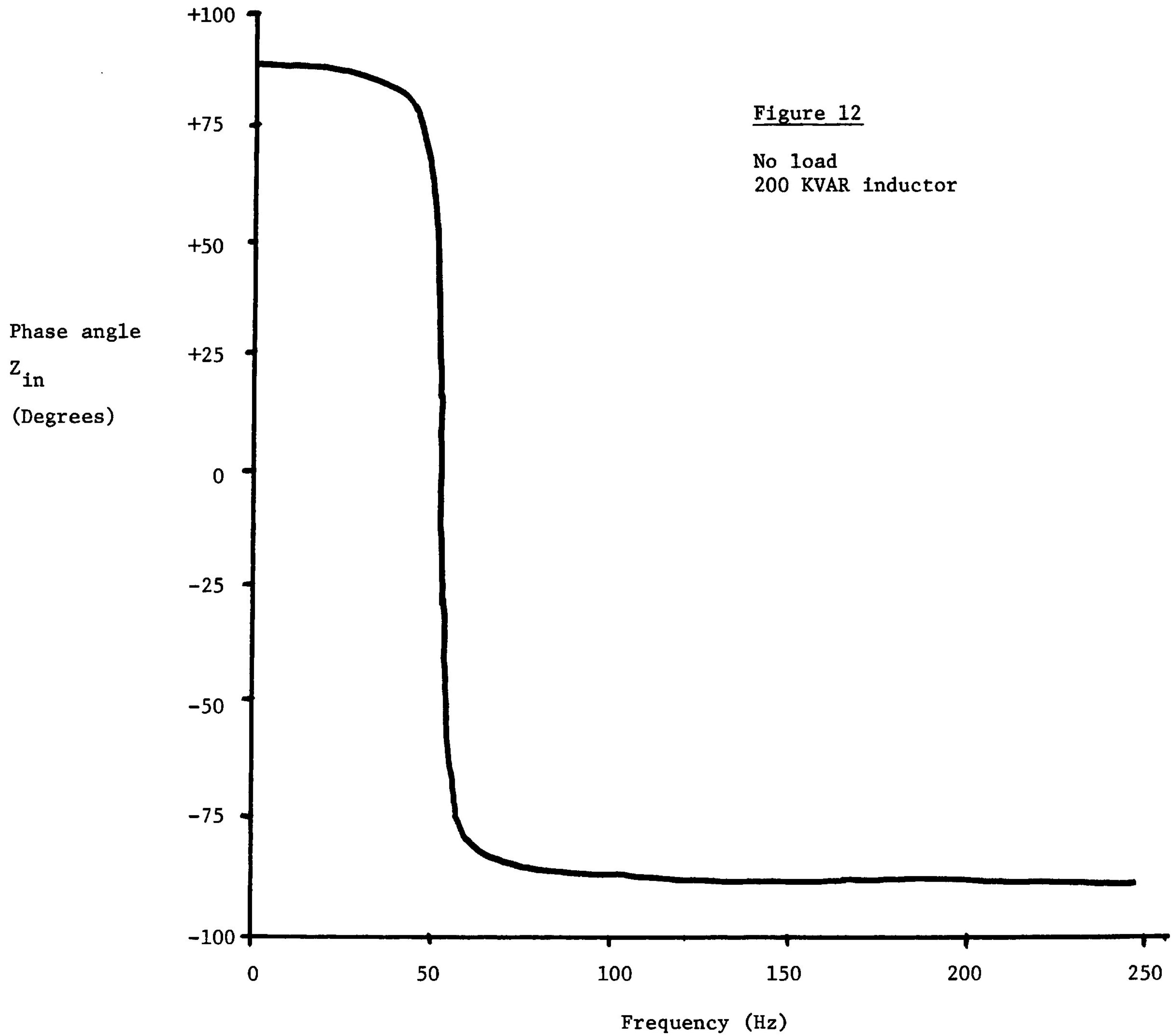
Figure 10

Critical load
200 KVAR inductor



<u>Figure 11</u>

No load
200 KVAR inductor



<u>Figure 12</u>

No load
200 KVAR inductor

2.7 Generator Reactive Power Capabilities

The following circuit represents the single phase equivalent of a balanced symmetrical three phase linear synchronous generator with negligible stator resistance and round rotor [8].

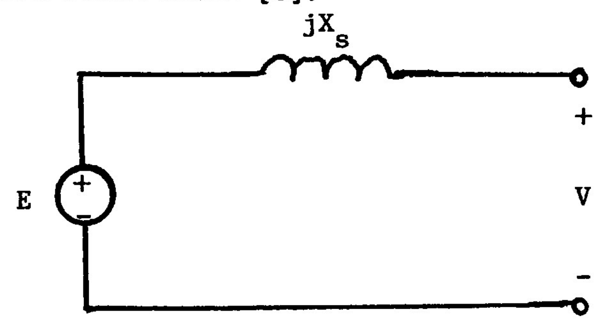


Figure 13.

where X_S = synchronous reactance

V = terminal line to neutral voltage

E = internal voltage whose magnitude is proportional to the field current.

Assume that the automatic voltage regulator is controlling the terminal voltage to 1.0 per unit on a 277 volt base. While the generators are salient pole with direct axis synchronous reactance of 1.41 per unit and quadrature axis synchronous reactance of 0.83 per unit (on 277/480 volt, 500 KW 3ϕ base), a first order analysis of the machine capabilities can be achieved by assuming the machine to be round rotor with X_S = 1.41 per unit.

The total three phase power delivered by one machine is then,

$$P_{out} = \frac{|E|\ |V|}{X_S} \sin\delta \qquad \text{P.U. watts } 3\phi$$

$$Q_{out} = \frac{|E|\ |V|}{X_S} \cos\delta - \frac{|V|^2}{X_S} \qquad \text{P.U. vars } 3\phi$$

where $\delta = \measuredangle E - \measuredangle V$. When the machine is supplying a capacitive load, the automatic voltage regulator will reduce the excitation current in order to maintain constant $|V|$. The machine will absorb reactive power in this case, and Q_{out} will be negative. The absolute maximum value of reactive power which one machine can absorb is $|V|^2/X_S$ which is 0.71 P.U. on its own base of 500 KW 3ϕ. Thus each machine can absorb an absolute maximum of 355 KVAR three phase provided one of the two regulators is maintaining constant $|V|$ = 277 volts line to neutral. This could only occur at a value of $\delta \simeq 90°$. If one generator excitation was driven to its lower limit before the other, one internal voltage $|E|$ would be fixed, and the governor could drive the fixed voltage machine to the unstable mode of fixed excitation, fixed terminal voltage and $\delta = 90°$. At this point, the fixed excitation generator would lose synchronism with the fixed voltage terminal bus. The generator would likely trip out because of loss of field excitation (if such a relay exists) or possibly due to reverse power as the rotor swings. When the salient rotor is considered, the absolute maximum value of reactive power which one machine can absorb would be $|V|^2/X_q$ or 1/0.83 = 1.2 per unit on a base of 500 KW. This consideration "increases" the minimum VAR capability to -600 KVAR. It is also important to note that the KVA rating of one machine is 625 KVA. Thus if one machine was delivering zero real power, the minimum VAR capability as a single machine would be -625 KVAR. However, since the machine rating is based on 0.8 power factor, this load would probably be considered excessive.

III. TRANSIENT SYSTEM ANALYSIS

The steady state analysis of the previous chapter assumed sinusoidal constant voltage and frequency supply from the two generators. In order to verify the postulated theory for the instability of one generator, a full transient analysis must be made. Based on the previous analysis, one generator cannot supply the entire real plus reactive critical site load. While it may be possible to modify the voltage and speed control systems to permit parallel operation without shunt reactive compensation, this would appear pointless since the loss of either generator would require serious load curtailment when operated in the "A" configuration. Thus a shunt reactor appears necessary and should improve the stability of the system. The near complete compensation of the cable capacitance will not create any adverse situations in steady state. However, this tuned circuit may introduce problems in the transient states, and is therefore analyzed in this chapter. A simplified block diagram of the entire system is provided in Figure 14. The mathematical model for each component is presented and interfaced in the following sections. Several introductory remarks are necessary to clarify the models. The network and generators are 4-wire grounded systems. Since they will be assumed balanced and symmetrical serving a balanced 3-phase load, the neutral wire should not carry current in steady state or during balanced symmetrical transients. Each component and interface will utilize a balanced symmetrical 3-phase 3-wire model. A linear magnetic circuit will be utilized in the machine models. Justification for this is given in a later section. Similar assumptions and approximations for each component and the interfacing are also given in the appropriate sections.

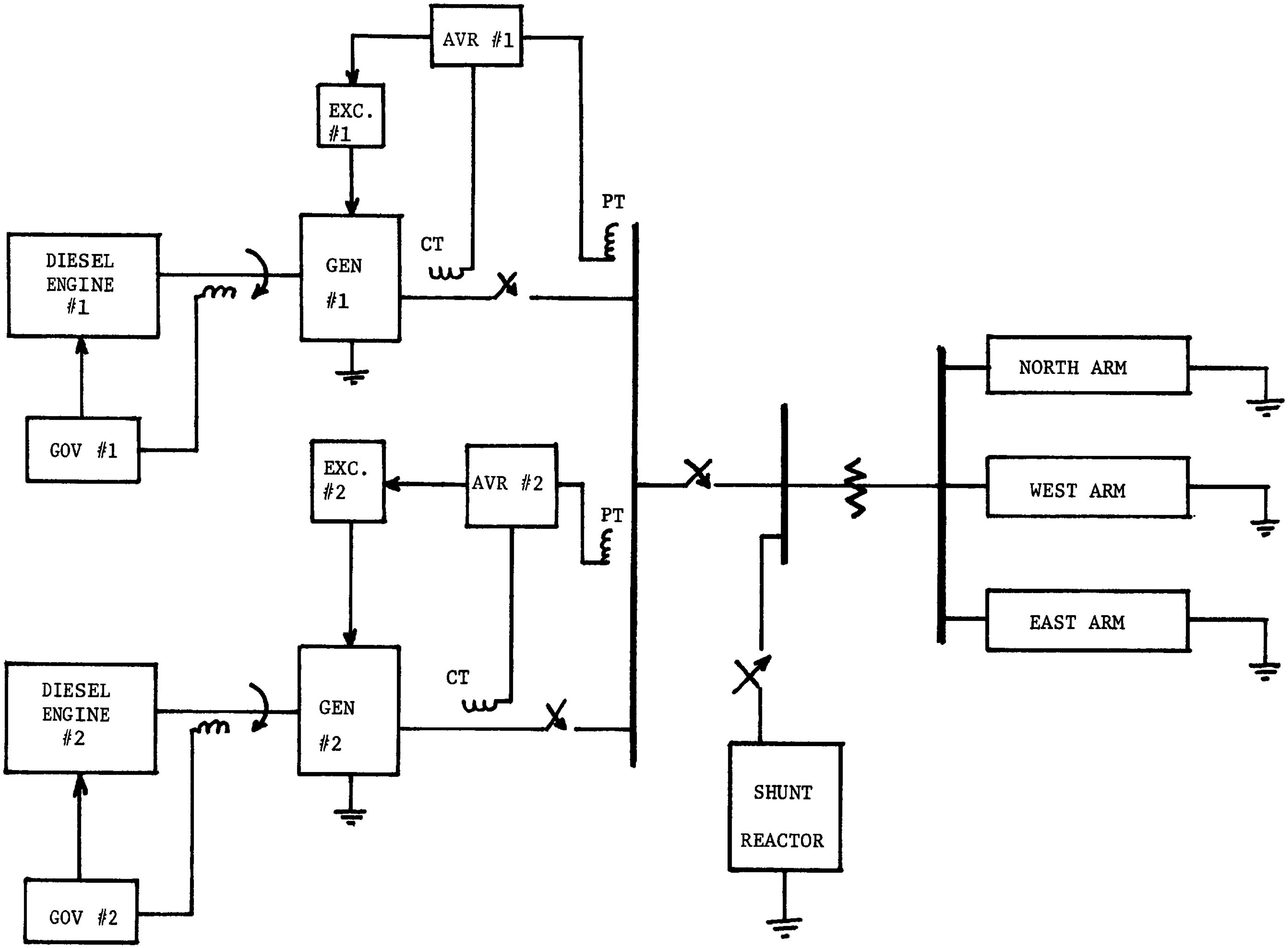

AVR #1
EXC. #1
PT
CT
DIESEL ENGINE #1
GEN #1
GOV #1
EXC. #2
AVR #2
PT
NORTH ARM
WEST ARM
EAST ARM
CT
DIESEL ENGINE #2
GEN #2
GOV #2
SHUNT REACTOR

Figure 14

3.1 Network and Load Model

The network and load model used is essentially the same as that given in Sections 2.1, 2.2 and 2.4. While all three phases are modelled, the phase A diagram is shown in Figure 15. For each arm taken as 13 miles:

$$
\begin{aligned}
R_1 &= 0.317 \text{ ohms/1000' at 7200 V L-N} \\
&= 21.8 \text{ ohms at 7200 V L-N} \\
&= \underline{0.032} \text{ ohms at 277 V L-N}
\end{aligned}
$$

$$
\begin{aligned}
L_3 &= 0.2764 \times 10^{-3} \text{ h/1000' at 7200 V L-N} \\
&= 1.9 \times 10^{-2} \text{ h at 7200 V L-N} \\
&= \underline{2.81 \times 10^{-5} \text{ h}} \text{ at 277 V L-N}
\end{aligned}
$$

$$
\begin{aligned}
C_2 &= \tfrac{1}{2} \times 0.057 \times 10^{-6} \text{ f/1000' at 7200 V L-N} \\
&= 1.96 \times 10^{-6} \text{ f at 7200 V L-N} \\
&= \underline{1.32 \times 10^{-3} \text{ f}} \text{ at 277 V L-N}
\end{aligned}
$$

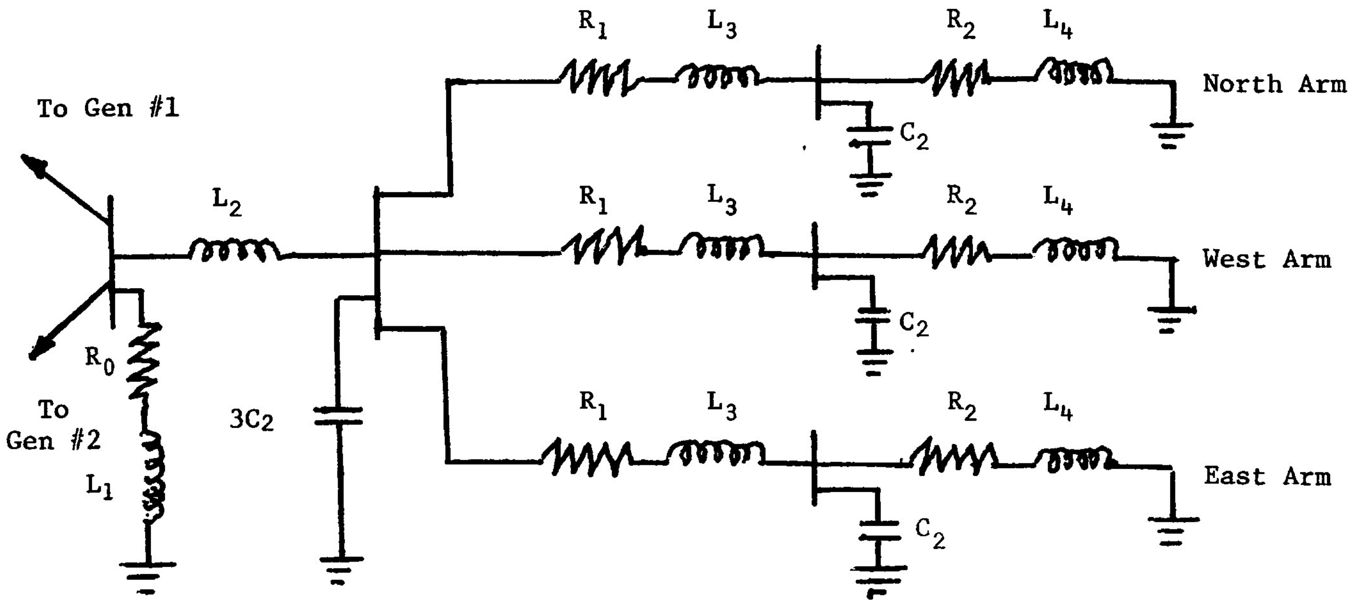
$$
\begin{aligned}
L_1 &= \text{200 KVAR inductor per phase at 277 V L-N} \\
&= 0.38/377 \text{ h at 277 V L-N} \\
&= \underline{0.001 \text{ h}} \text{ at 277 V L-N}
\end{aligned}
$$

$$
\begin{aligned}
R_o &= \text{Resistance of 200 KVAR inductor per phase at 277 V L-N} \\
&= \underline{0.0038 \text{ ohms}} \text{ at 277 V L-N}
\end{aligned}
$$

$$
\begin{aligned}
L_2 &= 5.3\% \text{ Z 750 KVA transformer on 750 KVA base} \\
&= \frac{5.3}{100} \times \frac{625 \times 10^3}{750 \times 10^3} \text{ ohms P.U. on 625 KVA, } \sqrt{2}\ 277 \text{ V base} \\
&= .04417 \text{ ohms P.U.} \quad \left(Z_{base} = \frac{\sqrt{2}^2\ 277^2 \times 3/2}{625 \times 10^3} = 0.37 \text{ ohms}\right) \\
&= \underline{4.3 \times 10^{-5} \text{ h}} \text{ at 277 V L-N}
\end{aligned}
$$

$$
\begin{aligned}
R_2 &= \text{Load equivalent suppose 100 KVA .8 P.F. lag at 277 V L-N} \\
&= 277^2 \text{ Cos } (37°)/33 \times 10^3 \\
&= \underline{1.86 \text{ ohms}} \text{ at 277 V L-N}
\end{aligned}
$$

$$
\begin{aligned}
L_4 &= \text{load equivalent suppose 100 KVA .8 P.F. lag at 277 V L-N} \\
&= 277^2 \text{ Sin } (37°)/(33 \times 10^3 \times 377) \\
&= \underline{0.0037} \text{ h at 277 V L-N}
\end{aligned}
$$



R_o = Shunt reactor resistance per phase

L_1 = Shunt reactor inductance per phase

L_2 = Main transformer leakage inductance per phase

R_1 = Cable series resistance per phase

L_3 = Cable series inductance per phase

R_2 = Load equivalent resistance per phase

L_4 = Load equivalent inductance per phase

C_2 = One half of total cable arm shunt capacitance per phase.

Figure 15

The three network and load arms are in parallel and may be reduced to minimize computation. The parallel equivalent for the network and load phase A model is given in Figure 16. The corresponding phase A network and load equations are:

$$\frac{di_1}{dt} = \frac{1}{L_1} v_1 - \frac{R_o}{L_1} i_1$$

$$\frac{di_2}{dt} = \frac{1}{L_2} v_1 - \frac{1}{L_2} v_2$$

$$\frac{dv_2}{dt} = \frac{1}{3C_2} i_2 - \frac{1}{3C_2} i_3$$

$$\frac{di_3}{dt} = \frac{3}{L_3} v_2 - \frac{R_1}{L_3} i_3 - \frac{3}{L_3} v_4$$

$$\frac{dv_4}{dt} = \frac{1}{3C_2} i_3 - \frac{1}{3C_2} i_4$$

$$\frac{di_4}{dt} = \frac{3}{L_4} v_4 - \frac{R_2}{L_4} i_4$$

where R_1, L_3, C_2, L_1, R_o, L_2, R_2, L_4 are as previously defined.

The network and load model can be perunitized by employing the following base quantities:

$$V_{BS} = \sqrt{2}\ 277 \text{ volts peak L-N}$$

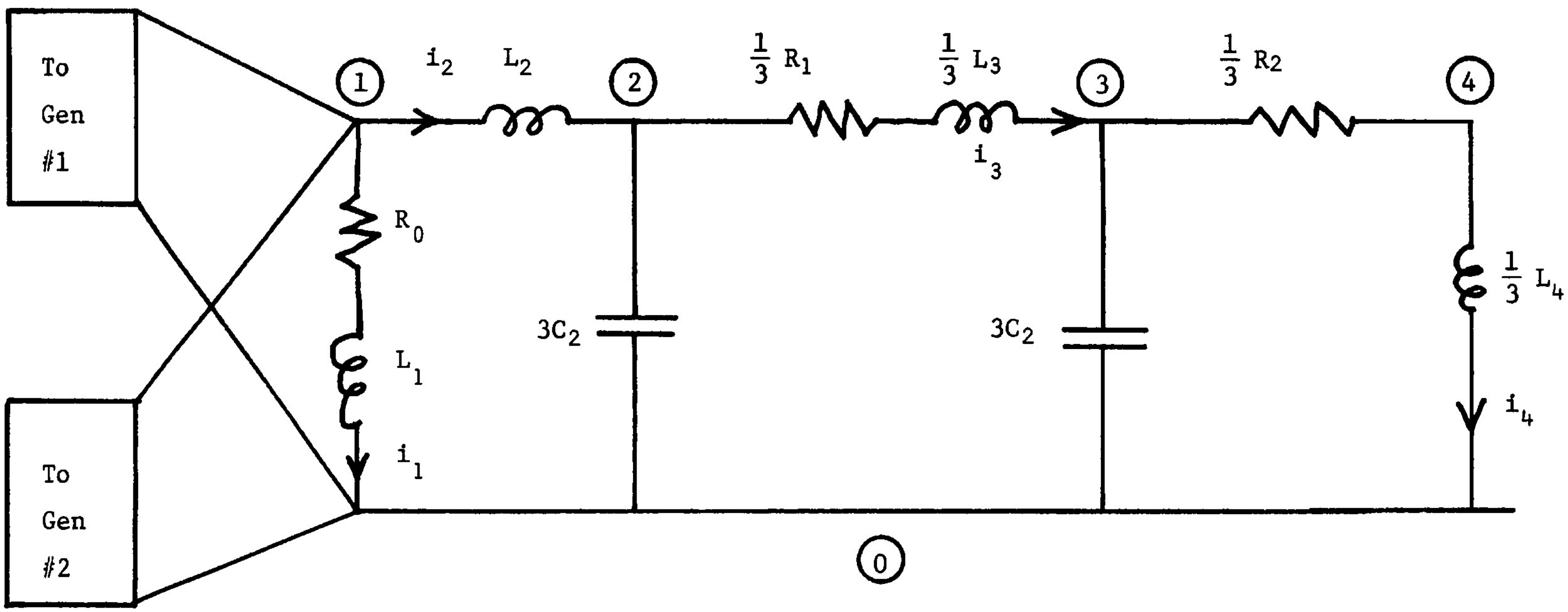
$$P_B = 625 \times 10^3 \text{ voltamps 3 phase}$$

$$I_B = \sqrt{2}\ 752 \text{ amps peak}$$

$$Z_{BS} = 0.37 \text{ ohms}$$

$$\omega_B = 377 \text{ Electrical radians/seconds}$$

$$t_B = (1/377) \text{ seconds}$$


To Gen #1
To Gen #2
1
2
3
4
0
i_2
L_2
$\frac{1}{3} R_1$
$\frac{1}{3} L_3$
i_3
$\frac{1}{3} R_2$
R_0
L_1
i_1
$3C_2$
$3C_2$
$\frac{1}{3} L_4$
i_4

<u>Figure 16</u>

$$L_{BS} = 9.8 \times 10^{-4}\ h$$

$$C_{BS} = (139.49)^{-1} f$$

NOTE: $P_B = \frac{3}{2} V_{BS} I_{BS}$ $\qquad Z_{BS} = V_{BS}/I_{BS}$

$L_{BS} = Z_{BS}/\omega_B$ $\qquad C_{BS} = 1/(Z_{BS}\omega_B)$

$t_B = 1/\omega_B$

The perunitized network and load equations are:

$$\frac{d\bar{i}_1}{d\bar{t}} = \frac{1}{\bar{L}_1}\bar{v}_1 - \frac{\bar{R}_o}{\bar{L}_1}\bar{i}_1 \qquad \text{(N-1)}$$

$$\frac{d\bar{i}_2}{d\bar{t}} = \frac{1}{\bar{L}_2}\bar{v}_1 - \frac{1}{\bar{L}_2}\bar{v}_2 \qquad \text{(N-2)}$$

$$\frac{d\bar{v}_2}{d\bar{t}} = \frac{1}{3\bar{C}_2}\bar{i}_2 - \frac{1}{3\bar{C}_2}\bar{i}_3 \qquad \text{(N-3)}$$

$$\frac{d\bar{i}_3}{d\bar{t}} = \frac{3}{\bar{L}_3}\bar{v}_2 - \frac{\bar{R}_1}{\bar{L}_3}\bar{i}_3 - \frac{3}{\bar{L}_3}\bar{v}_4 \qquad \text{(N-4)}$$

$$\frac{d\bar{v}_4}{d\bar{t}} = \frac{1}{3\bar{C}_2}\bar{i}_3 - \frac{1}{3\bar{C}_2}\bar{i}_4 \qquad \text{(N-5)}$$

$$\frac{d\bar{i}_4}{d\bar{t}} = \frac{3}{\bar{L}_4}\bar{v}_4 - \frac{\bar{R}_2}{\bar{L}_4}\bar{i}_4 \qquad \text{(N-6)}$$

where, $\bar{i}_k = i_k/I_{BS}$ $\qquad \bar{v}_k = v_k/V_{BS}$

$\bar{R}_k = R_k/Z_{BS}$ $\qquad \bar{L}_k = L_k/L_{BS}$

$\bar{C}_k = C_k/C_{BS}$ $\qquad \bar{t} = t/t_B$

<u>NOTE:</u> These equations are the perunitized network and load equations for phase A only. There are identical equations which must be added to include phases B and C. These equations will be integrated with the other system dynamic models in a later section. Note that $\overline{v}_1$ is also the terminal voltage of the two machines and that $\overline{i}_1$ plus $\overline{i}_2$ must equal the sum of the two machine currents. This means that not all four currents can be independent state variables. This will be satisfied in the interface section.

3.2 Generator Model

The generators are salient pole synchronous machines with the following data (reference [11]):

> 277/480 volt, 3ϕ 4W Y, 60 Hz, 6 Pole, 625 KVA, 500 KW,
> 1200 RPM, 752 Amps, Model 500MSS6, Type 10461, SN50234,
> Temp. rise 50°C, 0.8 PF lag, Field current 34 amps,
> Kato AC Gen A-38.

The units were manufactured by Kato Engineering Company in Mankato, Minnesota. Mr. Forrest Nelson and Kevin Becker furnished the following data as well as the machine open circuit characteristics (contacted at 507-625-4011).

$\overline{X}_d = 1.411$ P.U. ohms	$T_d'' = 0.02$ sec
$\overline{X}_d' = 0.168$ P.U. ohms	$T_{d_o}'' = 0.03$ sec
$\overline{X}_d'' = 0.068$ P.U. ohms	$T_{d_o}' = 4.4$ sec
$\overline{X}_2 = 0.066$ P.U. ohms	$T_d' = 0.52$ sec
$\overline{X}_o = 0.025$ P.U. ohms	$T_A = 0.02$ sec
$\overline{X}_q = 0.830$ P.U. ohms	$H = 0.2118$ KWsec/KVA
$\overline{X}_q'' = 0.063$ P.U. Ohms	

Based on this given data, the remaining machine parameters were computed from reference [18] as:

$\overline{R}_S = 8.675 \times 10^{-3}$ P.U. ohms

$\overline{X}_d = 1.41$ P.U. Ohms

$\overline{X}_{dm} = 1.366$ P.U. ohms

$\overline{X}_q = 0.83$ P.U. ohms

$\overline{X}_{qm} = 0.785$ P.U. ohms

$\overline{R}_D = 0.01338$ P.U. ohms

$\overline{X}_D = 1.39329$ P.U. ohms

$\overline{R}_Q = 0.053278$ P.U. ohms

$\overline{X}_Q$ = 0.80342 P.U. ohms

$\overline{R}_{fd}$ = see later analysis

$\overline{X}_{fd}$ = 1.50 P.U. ohms

The synchronous machines are currently very underexcited due to the large cable capacitance. These machines would behave linearly under such conditions, and saturation would not be a significant factor. With the shunt reactor, the load may require higher machine excitation. Saturation will still be neglected. This is necessary in part due to the lack of sufficient machine data.

The nameplate data indicated a maximum field excitation of 34 amps. The open circuit characteristic indicated a no-load open circuit field excitation of about 15 amps for rated voltage. This is consistent with Kato's claim that the excitation at full load should be about twice that at no load.

Consider the six single machine equations in motor ABC notation:

$$v_{as} = R_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = R_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{cs} = R_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$v_{fd} = R_f i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$0 = R_D i_D + \frac{d\lambda_D}{dt}$$

$$0 = R_Q i_Q + \frac{d\lambda_Q}{dt}$$

where λ's are flux linkages, D & Q represent the direct and quadrature axis damper windings. The flux linkages are functions of the six machine currents plus inductances which vary as a function of rotor position θ. For the two machines

$$\theta_1 = \int_o^t \omega_1 \, dt + \theta_1(0) \tag{G-1}$$

$$\theta_2 = \int_o^t \omega_2 \, dt + \theta_2(0) \tag{G-2}$$

The angles of Equations (G-1) and (G-2) are measured from the stator A phase axis to the rotor direct axis. Since the inductances which make up the flux linkage to current relationship vary with θ_1 (or θ_2), the coefficients of the state differentials will be time varying in the abc coordinate system. Consider the transformation to the odq coordinate system through:

$$\chi_{odqs_1} = T_1 \, \chi_{abcs_1}$$

$$\chi_{odqs_2} = T_2 \, \chi_{abcs_2}$$

where χ may be current, voltage, or flux linkage, and

$$T_i = \sqrt{2/3} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \text{Cos } \theta_i & \text{Cos}(\theta_i - \frac{2\pi}{3}) & \text{Cos}(\theta_i + \frac{2\pi}{3}) \\ -\text{Sin } \theta_i & -\text{Sin}(\theta_i - \frac{2\pi}{3}) & -\text{Sin}(\theta_i + \frac{2\pi}{3}) \end{bmatrix} \tag{G-3}$$

$$T_i^{-1} = \sqrt{2/3} \begin{bmatrix} 1/\sqrt{2} & \text{Cos } \theta_i & -\text{Sin } \theta_i \\ 1/\sqrt{2} & \text{Cos}(\theta_i - \frac{2\pi}{3}) & -\text{Sin}(\theta_i - \frac{2\pi}{3}) \\ 1/\sqrt{2} & \text{Cos}(\theta_i + \frac{2\pi}{3}) & -\text{Sin}(\theta_i + \frac{2\pi}{3}) \end{bmatrix} \tag{G-4}$$

For a 3 wire or 4 wire balanced symmetrical system, it can be shown that $\chi_{os_i} = 0$. When the above transformations are applied to the respective machine equations, the following five equations result:

$$v_{ds_1} = R_{s_1} i_{ds_1} + L_{ds_1} \frac{di_{ds_1}}{dt} + M_{ds_1,fd_1} \frac{di_{fd_1}}{dt} + M_{ds_1,D_1} \frac{di_{D_1}}{dt} - (L_{qs_1} i_{qs_1} + M_{qs_1 Q_1} i_{Q_1})\omega_1$$

$$v_{qs_1} = R_{s_1} i_{qs_1} + L_{qs_1} \frac{di_{qs_1}}{dt} + M_{qs_1,Q_1} \frac{di_{Q_1}}{dt} + (L_{ds_1} i_{ds_1} + M_{ds_1,fd_1} i_{fd_1} + M_{ds_1,D_1} i_{D_1})\omega_1$$

$$v_{fd_1} = R_{fd_1} i_{fd_1} + L_{fd_1} \frac{di_{fd_1}}{dt} + M_{fd_1,ds_1} \frac{di_{ds_1}}{dt} + M_{fd_1,D_1} \frac{di_{D_1}}{dt}$$

$$0 = R_{D_1} i_{D_1} + L_{D_1} \frac{di_{D_1}}{dt} + M_{D_1,ds_1} \frac{di_{ds_1}}{dt} + M_{D_1,fd_1} \frac{di_{fd_1}}{dt}$$

$$0 = R_{Q_1} i_{Q_1} + L_{Q_1} \frac{di_{Q_1}}{dt} + M_{Q_1,qs_1} \frac{di_{qs_1}}{dt}$$

where ω_1 is in electrical radians per second. The torque of electrical origin in the positive θ_1 direction is found from the coenergy as:

$$T_{e_+} = \frac{P}{2}\left[-(L_{qs_1} i_{qs_1} + M_{qs_1,Q_1} i_{Q_1}) i_{ds_1} + (L_{ds_1} i_{ds_1} + M_{ds_1,D_1} i_{D_1} + M_{ds_1 fd_1} i_{fd_1}) i_{qs_1}\right]$$

where P = # of poles.

Since the data provided is given in per unit, it is necessary to formulate the machine equations in per unit. The per unit equations are [18]:

$$\bar{v}_{ds_1} = \bar{R}_{s_1} \bar{i}_{ds_1} + \bar{X}_{d_1} \frac{d\bar{i}_{ds_1}}{d\bar{t}} + \bar{X}_{dm} \frac{d\bar{i}_{fd_1}}{d\bar{t}} + \bar{X}_{dm} \frac{d\bar{i}_{D_1}}{d\bar{t}} - \frac{\omega_1}{\omega_B} (\bar{X}_{q_1} \bar{i}_{qs_1} + \bar{X}_{qm_1} \bar{i}_{Q_1}) \tag{G-5}$$

$$\bar{v}_{qs_1} = \bar{R}_{s_1} \bar{i}_{qs_1} + \bar{X}_{q_1} \frac{d\bar{i}_{qs_1}}{d\bar{t}} + \bar{X}_{qm_1} \frac{d\bar{i}_{Q_1}}{d\bar{t}} + \frac{\omega_1}{\omega_B} (\bar{X}_{d_1} \bar{i}_{ds_1} + \bar{X}_{dm_1} \bar{i}_{fd_1} + \bar{X}_{dm_1} \bar{i}_{D_1}) \tag{G-6}$$

$$\bar{v}_{fd_1} = \bar{R}_{fd_1} \bar{i}_{fd_1} + \bar{X}_{dm_1} \frac{d\bar{i}_{ds_1}}{d\bar{t}} + \bar{X}_{fd_1} \frac{d\bar{i}_{fd_1}}{d\bar{t}} + \bar{X}_{dm_1} \frac{d\bar{i}_{D_1}}{d\bar{t}} \tag{G-7}$$

$$0 = \bar{R}_{D_1} \bar{i}_{D_1} + \bar{X}_{dm_1} \frac{d\bar{i}_{ds_1}}{d\bar{t}} + \bar{X}_{dm_1} \frac{d\bar{i}_{fd_1}}{d\bar{t}} + \bar{X}_{D_1} \frac{d\bar{i}_{D_1}}{d\bar{t}} \tag{G-8}$$

$$0 = \bar{R}_{Q_1} \bar{i}_{Q_1} + \bar{X}_{qm_1} \frac{d\bar{i}_{qs_1}}{d\bar{t}} + \bar{X}_{Q_1} \frac{d\bar{i}_{Q_1}}{d\bar{t}} \tag{G-9}$$

The torque of electrical origin in the positive θ_1 direction is:

$$\bar{T}_{e_1} = -\frac{2}{3}\frac{P}{2}\left[\bar{X}_{q_1} \bar{i}_{qs_1} \bar{i}_{ds_1} + \bar{X}_{qm_1} \bar{i}_{Q_1} \bar{i}_{ds_1} - \bar{X}_{d_1} \bar{i}_{ds_1} \bar{i}_{qs_1} - \bar{X}_{dm_1} \bar{i}_{fd_1} \bar{i}_{qs_1} - \bar{X}_{dm_1} \bar{i}_{D_1} \bar{i}_{qs_1}\right] \tag{G-10}$$

Identical equations for machine #2 can be written using subscript 2.

It is possible to compute $\bar{R}_{fd}$ from the previous parameters (see reference [18]). The true value, however, is known in ohms to be $R_{fd} = 3.85$ ohms. The field current required for rated open circuit terminal voltage is also known to be 15 amps. Consider the following analysis: At open circuit with $\omega_1 = \omega_2 = \omega_B$,

$$\bar{v}_{ds_1} = 0$$

$$\bar{v}_{qs_1} = \bar{X}_{dm_1} \bar{i}_{fd_1}$$

and from (IF-24), with $\bar{v}_{ds_1} = 0$,

$$\bar{v}_{T_1}' = \bar{v}_{T_1} = \sqrt{2}\, |\bar{v}_{qs_1}|$$

and since rated $\bar{v}_{T_1} = \sqrt{3}$ P.U. volts ℓ-ℓ

$$|\bar{v}_{qs_1}| = \sqrt{3/2} \text{ P.U. volts}$$

so that

$$|\bar{I}_{fd_1}| = \frac{\sqrt{3/2}}{1.366} = 0.897 \text{ P.U. amps.}$$

Since the actual field current is 15 amps, the current base for the synchronous machine field winding must be

$$I_{BR} = \frac{15}{0.897} = 16.7 \text{ Amps}$$

Therefore, since the stator base current is $752\sqrt{2}$

$$\frac{N_R}{N_S} = \frac{I_{BS}}{I_{BR}} = \frac{1063}{16.7} = 64 \qquad \text{(G-11)}$$

Now since the actual R_{fd} is 3.85 ohms, the actual resistance as seen by the stator is

$$R'_{fd} = 3.85 \left(\frac{N_S}{N_R}\right)^2 = 0.94 \times 10^{-3} \text{ ohms.}$$

Since the impedance base on the stator side is $Z_{BS} = 0.37$

$$\bar{R}_{fd} = \frac{0.94 \times 10^{-3}}{0.37}$$

$$\bar{R}_{fd} = 2.53 \times 10^{-3} \text{ P.U. ohms} \qquad \text{(G-12)}$$

It is important to clarify the significance of θ_1 and θ_2 relative to the machine terminal voltages. Consider steady state operation with positive sequence voltages:

$$v_{as} = \sqrt{2} \times 277 \text{ Cos } (377t + \gamma)$$

$$v_{bs} = \sqrt{2} \times 277 \text{ Cos } (377t + \gamma - 2\pi/3)$$

$$v_{cs} = \sqrt{2} \times 277 \text{ Cos } (377t + \gamma + 2\pi/3)$$

Applying the transformation of Equation (G-3) yields:

$$v_{os_1} = 0.$$

$$v_{ds_1} = \sqrt{2} \times 277 \times \sqrt{3/2} \text{ Cos } (\theta_1 - 377t - \gamma)$$

$$v_{qs_1} = -\sqrt{2} \times 277 \times \sqrt{3/2} \text{ Sin } (\theta_1 - 377t - \gamma)$$

If the machine is in steady state at $t = 0$,

$$\theta_{1_{ss}} = 377t + \theta_{1_{ss}}(0)$$

so that:

$$v_{ds_1} = \sqrt{2} \times 277 \times \sqrt{3/2}\ \mathrm{Cos}\ (\theta_{1_{ss}}(0) - \gamma)$$

$$v_{gs_1} = -\sqrt{2} \times 277 \times \sqrt{3/2}\ \mathrm{Sin}\ (\theta_{1_{ss}}(0) - \gamma)$$

It can be shown that for positive field current, the electrical angle

$$\theta_{1_{ss}}(0) - \gamma + \pi/2$$

is the so-called steady state torque angle [19]. Or for negative field current, the electrical angle

$$\theta_{1_{ss}}(0) - \gamma - \pi/2$$

is the so-called steady state torque angle. If the machines are operating at no load, this angle would be zero so that

$$\underset{\text{no load}}{\theta_{1_{ss}}(0)} = \gamma - \pi/2 \qquad I_f > 0 \tag{G-13}$$

and

$$\underset{\text{no load}}{\theta_{1_{ss}}(0)} = \gamma + \pi/2 \qquad I_f < 0 \tag{G-14}$$

Under these conditions.

$$v_{ds_1} = 0.$$

$$v_{qs_1} = +\sqrt{2} \times 277 \times \sqrt{3/2} \qquad I_f > 0 \tag{G-15}$$

$$v_{qs_1} = -\sqrt{2} \times 277 \times \sqrt{3/2} \qquad I_f < 0 \tag{G-16}$$

Note that the arbitrary selection of $\theta_{1_{ss}}(0)$ will automatically fix the instantaneous values of terminal voltage at no load, time zero. This will be done in the section covering initial conditions.

3.3 Automatic Voltage Regulator and Exciter Models

The automatic voltage regulators are solid state regulators with the following data (reference [10]):

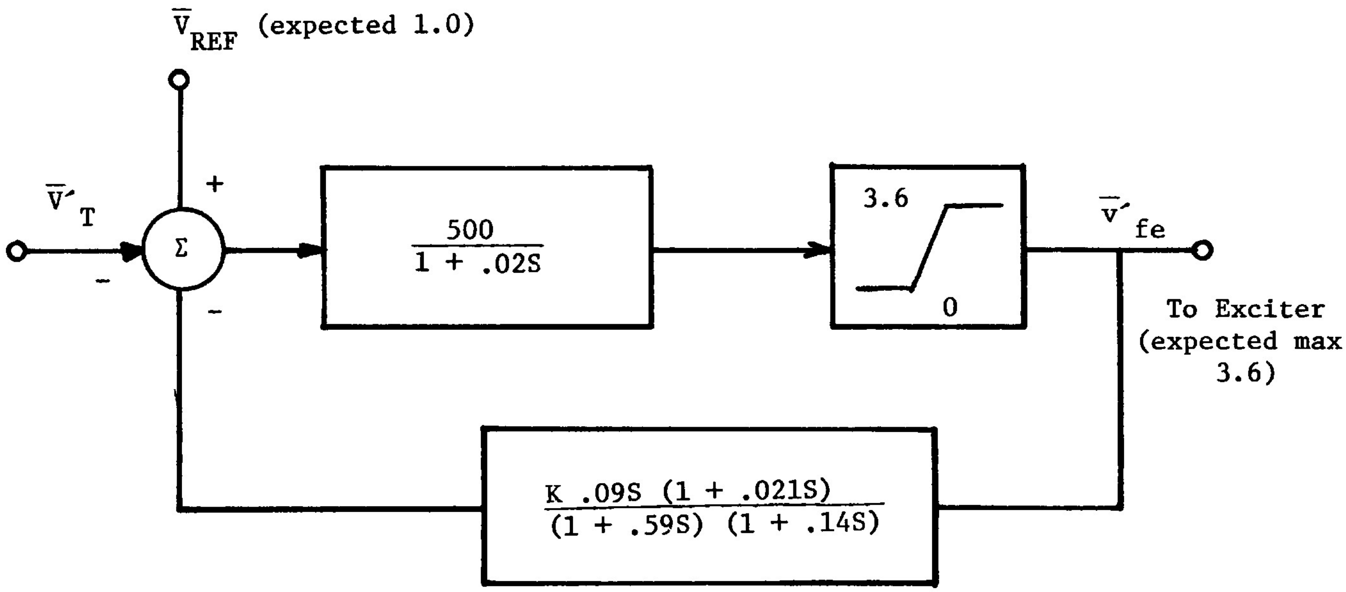
Model SR8A2B03A3A
Response time 16 MS, Droop Adj. to 5%
Parallel compensation circuit included,
minimum field resistance 18 ohms,
maximum continuous output 125 volts,
one minute forcing output 180 volts.

The units were manufactured by Basler Electric in Highland, Illinois. Mr. Charles Hummel furnished the transfer function shown in Figure 17 (618-654-2341). It was assumed that this transfer function was given in per unit quantities real time expecting V_{REF} = 1.0 PU V and providing a maximum field voltage output of 3.6 PU V (3.6 times rated). In addition, the voltage regulator has a parallel compensation circuit which allows reactive power sharing by adjusting the sensed voltage to be

$$V_T' = \text{Peak}\left[v_{AC} - \alpha\, i_{B_{out}}\right] = \text{Peak}\left[v_{AC} + \alpha\, i_{B_{in}}\right]$$

where v_{AC} is the line to line phase A to phase C voltage (positive sequence) and $I_{B_{out}}$ is the generator line current out of the phase B of the machine. Note that if the load is purely capacitive, $i_{B_{out}}$ will be in phase with v_{AC} and thus the sensed voltage is less than the actual line to line terminal voltage. This is consistent with the regulator manual description of "droop" parallel operation. Also note that in steady state, the regulator will attempt to drive the average values of $V_{REF} - V_T'$ to zero. Thus the steady state terminal voltage will have a "droop" (rise for capacitive load) from V_{REF} depending on $i_{B_{out}}$ as:

$$v_{AC} = V_{REF} + \alpha i_{B_{out}}$$



($\overline{V}'_T$ and $\overline{v}'_{fe}$ must be in per unit)

Note: Choosing K = 0.5 (mid range) the feedback block is equivalent to:

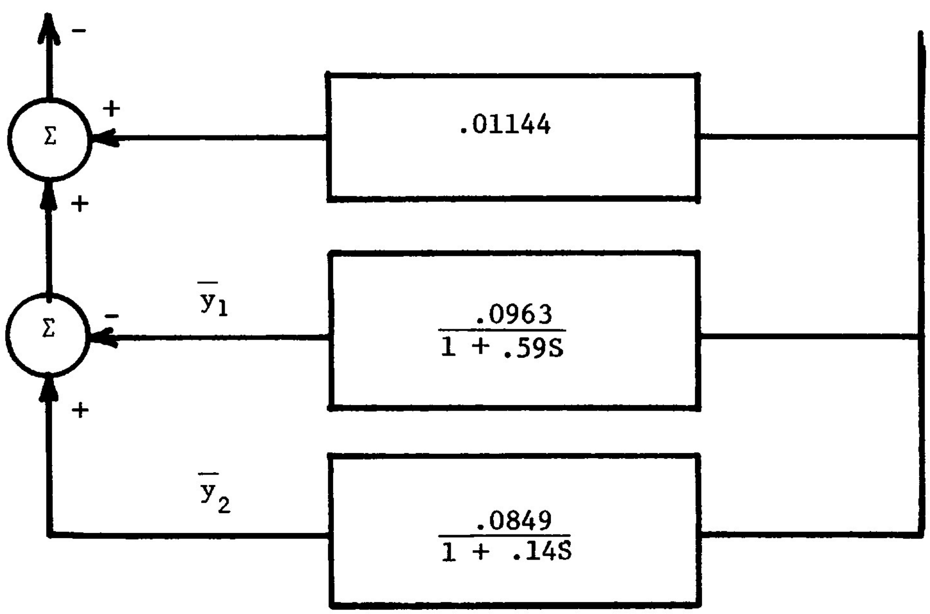


Figure 17

Actual test data yields the following:

V_{REF} = 450 RMS volts $\qquad V_{AC}$ = 459 RMS volts

I_{B_1} = 340 RMS Amps $\qquad I_{B_2}$ = 430 RMS Amps

Q_{Total} = -650 KVAR (meter)

So: $|S_1| = \sqrt{3} \times 459 \times 340 \qquad |S_2| = \sqrt{3} \times 459 \times 430$

$= 270{,}300$ VA $3\phi \qquad = 341{,}850$ VA 3ϕ

Note that these readings are not possible since $|S_1| + |S_2| < Q_{Total}$. Assuming the load is purely capacitive, with

$$Q_{1_{out}} = 270{,}300 \text{ VAR } 3\phi$$

$$Q_{2_{out}} = 341{,}850 \text{ VAR } 3\phi$$

for

$$Q_{Tot_{out}} = 612{,}150 \text{ VAR } 3\phi$$

Since v_{AC} and $i_{B_{out}}$ are in phase,

$$459 = 450 + \alpha_1\, 340 = 450 + \alpha_2\, 430$$

So

$$\alpha_1 = 0.0265 \text{ ohms}$$

$$\alpha_2 = 0.0209 \text{ ohms}$$

In per unit with $Z_{BS} = 0.37$ ohms

$$\underline{\bar{\alpha}_1 = 0.072 \text{ P.U. ohms}}$$

$$\underline{\bar{\alpha}_2 = 0.056 \text{ P.U. ohms}}$$

where $\bar{\alpha}_1 = \alpha_1/Z_{BS}$ and $\bar{\alpha}_2 = \alpha_2/Z_{BS}$.

The transfer function yields the following AVR equations in per unit:

$$\frac{d\bar{v}_{fe}}{d\bar{t}} = -\frac{336}{\omega_B}\bar{v}_{fe}' + \frac{2500}{\omega_B}(\bar{V}_{REF} - \bar{V}_T' + \bar{y}_1 - \bar{y}_2)$$

$$0 \leq \bar{v}_{fe}' \leq 3.6$$

$$\frac{d\bar{y}_1}{d\bar{t}} = -\frac{1}{.59\omega_B}\bar{y}_1 + \frac{.0963}{.59\omega_B}\bar{v}_{fe}'$$

$$\frac{d\bar{y}_2}{d\bar{t}} = -\frac{1}{.14\omega_B}\bar{y}_2 + \frac{.0849}{.14\omega_B}\bar{v}_{fe}'$$

where $\bar{t} = t/t_B$

$\bar{V}_{REF} = V_{REF}/V_{BS}$

$\bar{V}_T' = V_T'/V_{BS}$

$\bar{y}_1$ = Internal AVR per unit quantity

$\bar{y}_2$ = Internal AVR per unit quantity

$\bar{v}_{fe}'$ = Must equal 1.0 for rated v_{fe}

The exciter is a normally self excited shunt DC generator manufactured with the generator by Kato Engineering. The Kato Company furnished the open circuit characteristics plus the following data (reference [11] plus nameplate):

Model 6.5 x UG2, Type 10460
125 volt 52 amps Field amps 1.5
Field resistance 46 ohms at 25°C
Time constant T_E = 0.32 seconds L_E = 15 h

Consider the assumed connection of figure 18. The exciter equations neglecting R_a and L_a are:

$$v_{fe} = R_{fe} i_{fe} + L_{fe}\frac{di_{fe}}{dt}$$

$$v_a = v_{fd} = K_{fe}\, i_{fe} \text{ (Linear machine gain)}$$

These equations also neglect the saturation of the iron. From the open circuit characteristics and Kato data,

R_{fe} = 46 ohms

L_{fe} = 15 henries

K_{fe} = 119 ohms

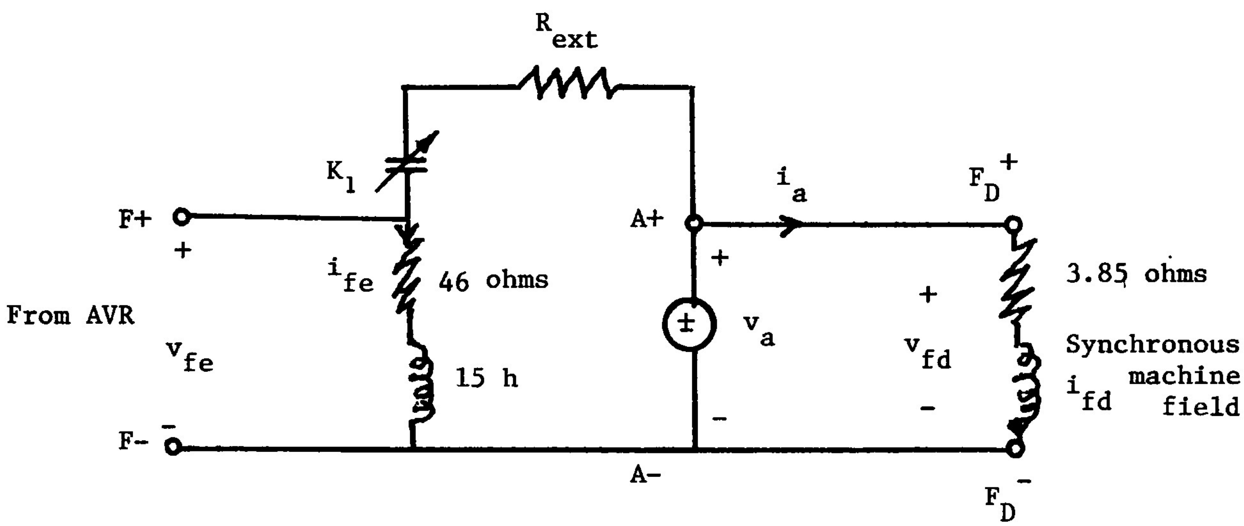


Figure 18

Note for Figure: The open circuit characteristic of the synchronous machine indicates that rated 480 V ℓ-ℓ occurs when i_{fd} = 15 Amps. ($i_{fd} = i_a$ above). For R_{fd} = 3.85 ohms, this gives v_a = 57.75 volts. The open circuit characteristic of the exciter indicates that this occurs when i_{fe} = 0.38 Amps. A field test indicated that the AVR output at open circuit was near v_{fe} = 17 volts. This agrees with the product of i_{fe} = 0.38 amps and R_{fe} = 46 ohms. Note that the generator cannot be self-excited unless an external resistance of R_{ext} = 106 ohms is in the field circuit (v_a = 57.75 volts must equal $i_{fe}\ R_{fe\,total}$ = 0.38 x 152). The AVR allows self excitation until the terminal voltage reaches 75% of rated. The R_{ext} simulates the $L \frac{di}{dt}$ voltage difference.

The exciter equation is then

$$v_{fe} = \frac{R_{fe}}{K_{fe}} v_{fd} + \frac{L_{fe}}{K_{fe}} \frac{dv_{fd}}{dt}$$

This equation can be represented as the following transfer function:

$$v_{fe} \longrightarrow \boxed{\frac{K_E}{1 + T_E\ S}} \longrightarrow v_{fd}$$

where $\underline{T_E = \frac{L_{fe}}{R_{fe}} = 0.32 \text{ seconds}}$

$$\underline{K_E = \frac{K_{fe}}{R_{fe}} = 2.587}$$

The AVR and exciter equations need to be combined and per unitized. Recall that the AVR expects $V_{REF} = 1.0$ P.U. volts and $v_{fe}^{rated} = 1.0$. In order to join these two components, this criteria must be satisfied. Since v_{fe}^{rated} may not be 1.0 per unit, a gain factor will have to be added to the model.

Let $\bar{v}_{ve} = v_{fe}/v_{BF}$ and recall from the machine equations that:

$$\bar{v}_{fd} = v_{fd}/v_{BR}$$

So the exciter equation is:

$$\bar{v}_{fe}\ v_{BF} = \frac{R_{fe}}{K_{fe}} \bar{v}_{fd}\ v_{BR} + \frac{L_{fe}}{K_{fe}} \frac{d\ \bar{v}_{fd}\ v_{BR}}{d\ \bar{t}\ t_B}$$

or

$$\bar{v}_{fe} \frac{v_{BF}}{v_{BR}} = \frac{R_{fe}}{K_{fe}} \bar{v}_{fd} + \frac{\omega_B L_{fe}}{K_{fe}} \frac{d\ \bar{v}_{fd}}{d\ \bar{t}}$$

For $\frac{v_{BF}}{v_{BR}} = \frac{N_F}{N_A}$ (equivalent turns ratio assume 10)

$$\bar{v}_{fe} = \frac{R_{fe}}{10K_{fe}} \bar{v}_{fd} + \frac{\omega_B\ L_{fe}}{10\ K_{fe}} \frac{d\ \bar{v}_{fd}}{d\ \bar{t}}$$

The base voltage for $\bar{v}_{fe}$ must be determined to compute the gain required to yield a 1.0 per unit $\bar{v}_{fe}'$. The base voltages of the machine have been selected such that $\bar{V}_{t_{\ell\ell}} = 1.732$ indicates $V_{t_{\ell\ell}}$ = Rated terminal voltage. Since the D.C. machine and synchronous machine both have speed gains as well as transformer turn ratio gains, the base voltages which are computed by turns ratios alone for the field of the synchronous machine and exciter armature/field do not correspond to rated voltages. For example:

$$V_{BS} = \sqrt{2} \times 277$$

$$V_{BR} = \sqrt{2} \times 277 \frac{N_R}{N_S} = \sqrt{2} \times 277 \times 64$$

Since the armature of the exciter is connected to the field of the synchronous machine, the base voltages of the exciter armature must be V_{BR}, i.e.

$$V_{BA} = \sqrt{2} \times 277 \times 64$$

Thus the base voltage for the exciter field is:

$$V_{BF} = \frac{N_F}{N_A} V_{BA} = 10 \times \sqrt{2} \times 277 \times 64.$$

The exciter field resistance is about one third the total resistance which would be required for rated shunt operation (120 volts). Recall this was determined from the assumed AVR connection in the previous paragraphs. The field alone then must have a rating of 40 volts. For $v_{BF} = 10 \times \sqrt{2} \times 277 \times 64$,

$$\bar{v}_{fe}^{\,rated} = \frac{40}{2.5 \times 10^5} \text{ P.U. volts}$$

$$= 1.6 \times 10^{-4} \text{ P.U. volts.}$$

Since $\bar{v}_{fe}^{\,rated}$ is not equal to 1.0 P.U. volts, a gain of 1.6×10^{-4} must be added to interface $\bar{v}_{fe}'$ and $\bar{v}_{fe}$. Similarly, since $\bar{V}_{REF} = 1.732$ and the AVR expects $\bar{V}_{REF} = 1.0$, a gain reduction must be added to the AVR circuit. If the limiting output is multiplied by $\sqrt{3}$ and the above $\bar{v}_{fe}'$ gain is divided by $\sqrt{3}$, the two components can be joined in per unit to yield the final transfer function of figure 19.

The final AVR/Exciter equations in per unit with per unit time are:

$$\frac{d\bar{v}_{fd}}{d\bar{t}} = -\frac{R_{fe}}{\omega_B L_{fe}} \bar{v}_{fd} + \frac{10\ K_{fe}}{\omega_B L_{fe}} \times \frac{1.6\text{x}10^{-4}}{\sqrt{3}} \bar{v}_{fe}' \quad (0 \leq \bar{v}_{fe}' \leq 3.6\ \sqrt{3}) \tag{AE-1}$$

$$\frac{d\bar{v}_{fe}'}{d\bar{t}} = -\frac{336}{\omega_B} \bar{v}_{fe}' + \frac{25000}{\omega_B} (\bar{V}_{REF} - \bar{V}_T' + \bar{y}_1 - \bar{y}_2) \tag{AE-2}$$

$$\frac{d\bar{y}_1}{d\bar{t}} = -\frac{1}{.59\omega_B} \bar{y}_1 + \frac{.0963}{.59\omega_B} \bar{v}_{fe}' \tag{AE-3}$$

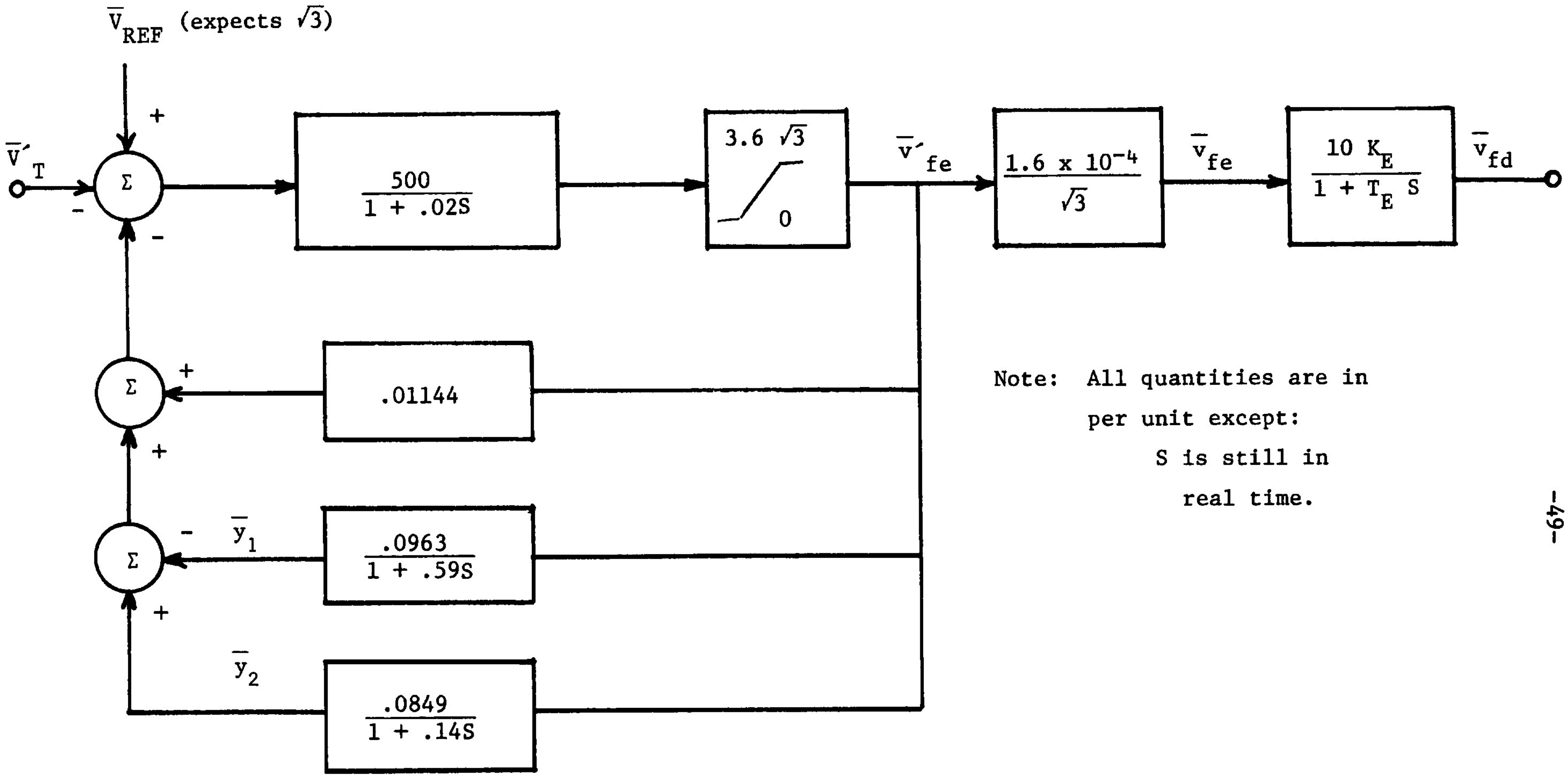
$$\frac{d\bar{y}_2}{d\bar{t}} = -\frac{1}{.14\omega_B} \bar{y}_2 + \frac{.0849}{.14\omega_B} v_{fe}' \tag{AE-4}$$

(Note: R_{fe}, L_{fe}, K_{fe} are not in per unit.)

Note that $\bar{V}_T'$ is expressible as a function of v_A v_C and i_B through the machine equations. This will be done in the interface section. Note also that there will be two sets of three equations (one for each unit). The interface equations are

$$\bar{V}_{T_1}' = \text{Peak} \left[\bar{v}_{A_1} - \bar{v}_{C_1} + \bar{\alpha}_1 \bar{i}_{B_{1\,in}} \right] \tag{AE-5}$$

$$\bar{V}_{T_2}' = \text{Peak} \left[\bar{v}_{A_2} - \bar{v}_{L_2} + \bar{\alpha}_2 \bar{i}_{B_{2\,in}} \right] \tag{AE-6}$$



Note: All quantities are in per unit except: S is still in real time.

<u>Figure 19</u>

3.4 Governor and Diesel Engine Model

The governors are hydraulic governors with the following data (reference [9]):

> Universal governor, 8 foot pounds output, mechanical hydraulic for diesel or steam turbines, max 42° travel of output shaft (recommended 28°). Model UG-8 droop or isochronous.

The units were manufactured by Woodward Governor Company, Fort Collins, Colorado. Messrs. Dan Garvey, Jim Brown and Jim Wimp furnished the transfer function shown in figure 20 (303-482-5811). It was assumed that this transfer function was given in actual units involving RPM and inches. The fuel actuating arm can rotate from zero (no load) to a maximum of 42° (full load). This corresponds to a governor piston motion through $Z_1 = 0$ (no load) to a maximum $Z_1 = 1.5$ inches (full load). The governors have a load limit setting which can adjust this to be less than 42°.

Governor #1 had the load limit set at 8.9 while #2 had a setting at 10.0. (Ten is maximum corresponding to Z max = 1.5 inches.) Thus,

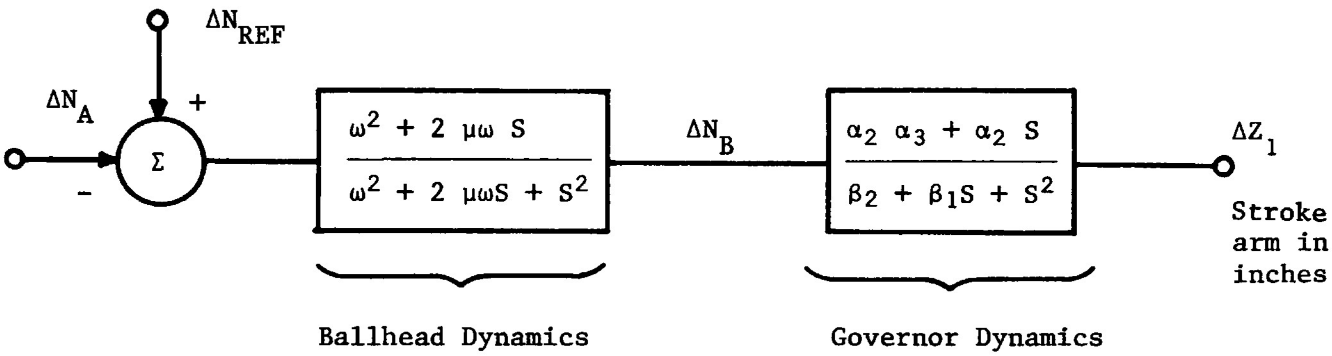
$$\underline{Z_{11}\ \max = 1.335\ \text{inches}}$$

$$\underline{Z_{12}\ \max = 1.500\ \text{inches.}}$$

Note that the D.C. steady state gain for $\Delta F_1/\Delta N_2$ is $1/\alpha_d$. This corresponds to an adjustable droop speed dial setting. A 5% maximum speed droop was assumed. Governor #1 had a droop setting of 77 while #2 had a setting of 80 (100 is maximum). A 5% maximum speed droop assumption means that the engine speed will drop 5% between no load and full load. Unity power factor load tests indicate that on the average $|I_2| \approx 1.2\ |I_1|$. This indicates that the actual droops differ by a ratio nearly the reverse of the droop settings. (A high droop machine will take less load.) Based on this data, assume

$$\underline{\alpha d_1 = .05 \times 1200 \times DS_1/1.5\ \text{RPM/inch} \qquad DS_1 = 0.8}$$

$$\underline{\alpha d_2 = .05 \times 1200 \times DS_2/1.5\ \text{RPM/inch} \qquad DS_2 = 0.8/1.2}$$



For N = 1457 RPM

$\mu = 0.2$ $\qquad \alpha_1 = 77.16$ $\qquad \beta_1 = \alpha_1 + \alpha_2\alpha_d$

$\omega = 10.5$ rad/sec $\qquad \alpha_2 = 0.20$ $\qquad \beta_2 = \alpha_2\alpha_3\alpha_d$

$\alpha_3 = 0.41$

α_d = Proportional to droop setting.

Note: If $\alpha_d = 0$ (Droop = 0) governor is isochronous.

Figure 20

The diesel engines have the following data (reference [12] and nameplate):

Model #VLRDU, Spec #F-279; SN #1 RU12946; SN #2 RU17943 flywheel plus crankshaft inertia 1434 lb ft^2, load response time 0.2 second.

The units were manufactured by Waukesha in Waukesha, Wisconsin. Messrs. Mel Erickson, Steve Moldenauer, Ed Antoniweicz and Warren Herbrand indicated that Waukesha does not currently have mathematical transfer functions to model the engine dynamics. They have hired a Mr. Bob Kern from the Milwaukee School of Engineering to develop a transfer function and model for the diesel engine. These were not available for this simulation. The transfer function of figure 21 was used.

Since the governor gain is $1/\alpha_d$, the steady state gain for the combined transfer function is:

$$\Delta P_m = \frac{K_{DE}}{\alpha_d} \Delta N$$

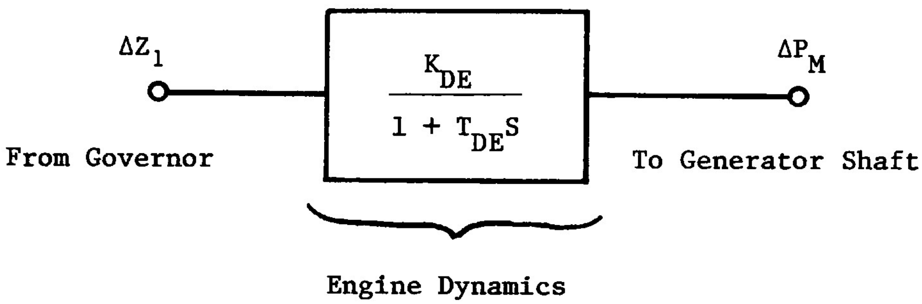
And, when $\Delta P_m = 500 \times 10^3$ watts, the desired change in speed is

$$\Delta N = .04 \times 1200 \times DS \quad \text{RPM}$$

Thus, for $\alpha_d = .04 \times 1200 \times DS/1.5$ RPM/inch, the engine gain is

$$\underline{K_{DE} = 3.33 \times 10^5 \text{ watts/inch}}$$

In order to integrate these components into the system equations, the input must be in electrical radians per second, and the output must be in per unit watts.



T_{DE} = 0.2 second

K_{DE} = (Will be determined in this analysis)

Figure 21

The input can be converted by noting that for a 6 pole machine,

$$\Delta N = \frac{120\ \Delta f}{6} = 20\ \Delta f = \frac{10}{\pi}\ \Delta\omega$$

Neglecting the ballhead dynamics, the equations are

$$\frac{d\Delta P_M}{dt} = -\frac{1}{T_{DE}}\ \Delta P_M + \frac{K_{DE}}{T_{DE}}\ \Delta Z_1$$

$$\frac{d^2\Delta Z_1}{dt^2} + (77.16 + .2\ \alpha_d)\ \frac{d\,\Delta Z_1}{dt} + .082\ \alpha_d\ \Delta Z_1 =$$

$$-\ \frac{10}{\pi}\ \times 0.2\ \frac{d}{dt}\ (\Delta\omega - \Delta\omega_{REF}) - \frac{10}{\pi} \times 0.082\ (\Delta\omega - \Delta\omega_{REF})$$

$$0 \leq \Delta Z_1 \leq \Delta Z_1 \text{ max inches}$$

Let $\qquad \Delta Z_2 = \dfrac{d\Delta Z_1}{dt}$

And also let

$$P_M = P_M^{\circ} + \Delta P_M$$

$$Z_1 = Z_1^{\circ} + \Delta Z_1$$

$$Z_2 = Z_2^{\circ} + \Delta Z_2$$

$$\omega = \omega^{\circ} + \Delta\omega$$

$$\omega_{REF} = \omega_{REF}^{\circ} + \Delta\omega_{REF}$$

with $\dfrac{d\omega_{REF}}{dt} = 0$ (No speed set point change.)

$$P_M{}^\circ = K_{DE}\, Z_1{}^\circ \qquad \text{(constant)}$$

$$Z_2{}^\circ = 0$$

$$\omega_{REF}{}^\circ = \omega^\circ + \frac{10}{\pi}\, \alpha_d\, Z_1{}^\circ \qquad \text{(constant)}$$

The normal unit governor/diesel engine equations are:

$$\frac{dP_M}{dt} = -\frac{1}{T_{DE}} P_M + \frac{K_{DE}}{T_{DE}} Z_1$$

$$\frac{dZ_1}{dt} = Z_2$$

$$\frac{dZ_2}{dt} = -(77.16 + .2\ \alpha_d)\, Z_2 - .082\, \alpha_d\, Z_1$$

$$- \frac{10}{\pi} \times 0.2 \frac{d\omega}{dt} - \frac{10}{\pi} \times 0.082\ (\omega - \omega_{REF})$$

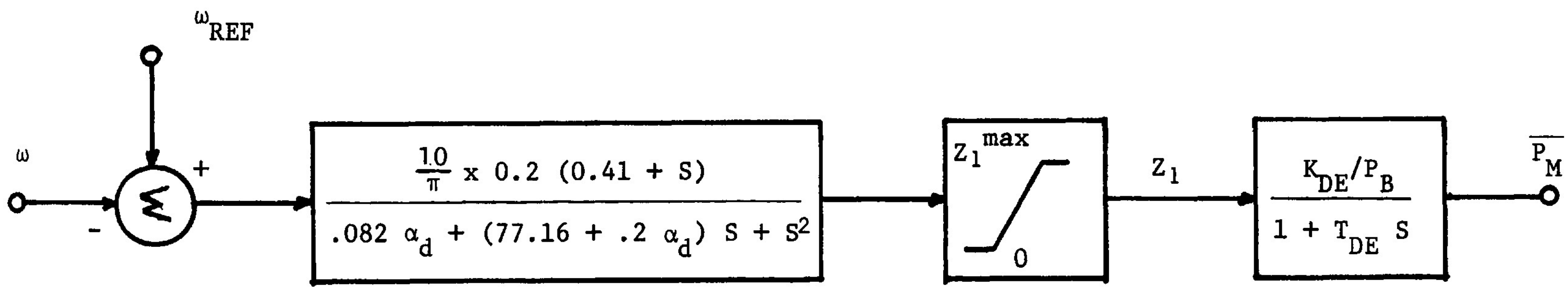
$$0 \le Z \le Z_1{}^{max}$$

The first equation must be per unitized to allow integration with the other system equations. Thus, for

$$P_M = \overline{P_M}\, P_B$$

$$t = \overline{t}/\omega_B$$

The final combined transfer function is shown in figure 22.



Note: S is still in real time units.

Figure 22

The final per unitized governor/diesel engine equations are:

$$\frac{d\,\overline{P_M}}{d\bar{t}} = -\frac{1}{T_{DE}\,\omega_B}\,\overline{P_M} + \frac{K_{DE}}{P_B\,T_{DE}\,\omega_B}\,Z_1 \qquad \text{(GDE-1)}$$

$$\frac{dZ_1}{d\bar{t}} = \frac{1}{\omega_B}\,Z_2 \qquad 0 \le Z_1 \le Z_1^{max} \qquad \text{(GDE-2)}$$

$$\frac{dZ_2}{d\bar{t}} = -\frac{(77.16 + .2\,\alpha_d)}{\omega_B}\,Z_2 - \frac{.082\,\alpha_d}{\omega_B}\,Z_1$$

$$-\frac{10 \times 0.2}{\pi}\,\frac{d\omega}{d\bar{t}} - \frac{10 \times 0.082}{\pi\,\omega_B}\,(\omega - \omega_{REF}) \qquad \text{(GDE-3)}$$

Note that $\overline{P_M}$ is in pu watts, Z_1, Z_2 and Z_2^{max} are in inches, ω is in electrical radians per second, and α_d is in RPM/inch. Note also that $\frac{d\omega}{d\bar{t}}$ is expressible as a function of machine variables. This will be done in the interface section.

Note also that there will be two sets of these equations (one for each unit). The values of α_d and Z_2^{max} will be different as previously defined.

3.5 Interfacing the Component Models

Four component models have been developed. These models must be integrated to represent a consistent set of unified system equations. Consider the following summary of the component states and required inputs. A state is a variable which appears in differential form. An input is a variable which is not a state variable of that component set.

Network and load models

States: $\bar{i}_{1A}\ \bar{i}_{1B}\ \bar{i}_{1C}\ \bar{i}_{2A}\ \bar{i}_{2B}\ \bar{i}_{2C}\ \bar{v}_{2A}\ \bar{v}_{2B}\ \bar{v}_{2C}$

$\bar{i}_{3A}\ \bar{i}_{3B}\ \bar{i}_{3C}\ \bar{v}_{4A}\ \bar{v}_{4B}\ \bar{v}_{4C}\ \bar{i}_{4A}\ \bar{i}_{4B}\ \bar{i}_{4C}$

Equations: 18

Inputs: $\bar{v}_{1A}\ \bar{v}_{1B}\ \bar{v}_{1C}$

Generator #1

States: $\bar{i}_{qs_1}\ \bar{i}_{ds_1}\ \bar{i}_{Q_1}\ \bar{i}_{D_1}\ \bar{i}_{fd_1}$

Equations: 5

Inputs: $\bar{v}_{qs_1}\ \bar{v}_{ds_1}\ \bar{v}_{fd_1}\ \theta_1\ \omega_1$

Generator #2

States: $\bar{i}_{qs_2}\ \bar{i}_{ds_2}\ \bar{i}_{Q_2}\ \bar{i}_{D_2}\ \bar{i}_{fd_2}$

Equations: 5

Inputs: $\bar{v}_{qs_2}\ \bar{v}_{ds_2}\ \bar{v}_{fd_2}\ \theta_2\ \omega_2$

AVR/Exciter #1

States: $\bar{v}_{fd_1}\ \bar{v}'_{fe_1}\ \bar{y}_{11}\ \bar{y}_{12}$

Equations: 4

Inputs: $\bar{V}_{REF}\ \bar{V}'_{T_1}$

where $\bar{V}'_{T_1}$ = Peak $\bar{v}_{A_1} - \bar{v}_{C_1} + \bar{\alpha}_1\ \bar{i}_{B_1\,IN}$

AVR Exciter #2

States: $\bar{v}_{fd_2}\ \bar{v}_{fe_1}\ \bar{y}_{21}\ \bar{y}_{22}$

Equations: 4

Inputs: $\bar{V}_{REF}\ \bar{V}'_{T_2}$

where $\bar{V}'_{T_2}$ = Peak $\bar{v}_{A_2} - \bar{v}_{C_2} + \bar{\alpha}_2\ \bar{i}_{B_2\,IN}$

Governor/Diesel Engine #1

States: $\overline{P}_{m_1}$ Z_{11} Z_{12} ω_1

Equations: 3

Inputs: ω_{REF_1}

Governor/Diesel Engine #2

States: $\overline{P}_{m_2}$ Z_{21} Z_{22} ω_2

Equations: 3

Inputs: ω_{REF_2}

In addition to these state equations for the models, the rotating elements must satisfy Newton's Second Law:

$$J_1 \frac{d\omega_1}{dt} = T_{e_1} + T_{m_1} \quad \text{(IF-1)}$$

$$J_2 \frac{d\omega_2}{dt} = T_{e_2} + T_{m_2} \quad \text{(IF-2)}$$

where

$$\frac{d\theta_1}{dt} = \omega_1 \quad \text{(IF-3)}$$

$$\frac{d\theta_2}{dt} = \omega_2 \quad \text{(IF-4)}$$

Notice also that since the system has been assumed to be a 3-wire network, the 18 network and load equations can be reduced to 12 independent equations through kirchhoff's laws

$$\overline{i}_{1C} = -\overline{i}_{1A} - \overline{i}_{1B} \qquad \overline{i}_{2C} = -\overline{i}_{2A} - \overline{i}_{2B}$$

$$\overline{v}_{2C} = -\overline{v}_{2A} - \overline{v}_{2B} \qquad \overline{i}_{3C} = -\overline{i}_{3A} - \overline{i}_{3B}$$

$$\overline{v}_{4C} = -\overline{v}_{4A} - \overline{v}_{4B} \qquad \overline{i}_{4C} = -\overline{i}_{4A} - \overline{i}_{4B}$$

Similarly, the sum of the two machine currents plus the two network currents at network bus #1 must satisfy Kirchhoff's current law.

Similarly, the Network bus 1 voltages $\bar{v}_{1A}$ $\bar{v}_{1B}$ are the same as the transformed machine voltages $\bar{v}_{qs_1}$ $\bar{v}_{ds_1}$ and $\bar{v}_{qs_2}$ $\bar{v}_{ds_2}$. These two variables must be eliminated from all of the state equations if they are to be solved simultaneously by a numerical integration scheme. This will be done by arbitrarily choosing network equation number two. Again working with phase A only,

$$\bar{v}_1 = \bar{L}_2 \frac{d\bar{i}_2}{d\bar{t}} + \bar{v}_2 \tag{IF-5}$$

This equation will be substituted into every other system equation requiring $\bar{v}_1$.

Kirchhoff's current law at Network bus 1 will be satisfied by substituting the following into every system equation requiring $\bar{i}_1$

$$\bar{i}_1 = -\bar{i}_{\underset{IN}{as_1}} - \bar{i}_{\underset{IN}{as_2}} - \bar{i}_2 \tag{IF-6}$$

The network equations must be appropriately transformed in order to be integrated with the machine equations. Consider the above substitutions and the multiplication of the network equations by transformation matrix T_1 in vector notation, with "$\cdot$" $= \frac{d}{d\bar{t}}$,

$$-T_1 \dot{\bar{i}}_{abcs_1} - T_1 \dot{\bar{i}}_{abcs_2} = (\frac{\bar{L}_2}{\bar{L}_1} + 1)\, T_1 \dot{\bar{i}}_{abc_2} + \frac{1}{\bar{L}_1} T_1 \bar{v}_{abc_2} - \frac{\bar{R}_o}{\bar{L}_1} T_1 (\bar{i}_{abcs_1} + \bar{i}_{abcs_2} + \bar{i}_{abc_2})$$

$$T_1 \dot{\bar{v}}_{abc_2} = \frac{1}{3\bar{C}_2} T_1 \bar{i}_{abc_2} - \frac{1}{3\bar{C}_2} T_1 \bar{i}_{abc_3}$$

$$T_1 \dot{\bar{i}}_{abc_3} = \frac{3}{\bar{L}_3} T_1 \bar{v}_{abc_2} - \frac{\bar{R}_1}{\bar{L}_3} T_1 \bar{i}_{abc_3} - \frac{3}{\bar{L}_3} T_1 \bar{v}_{abc_4}$$

$$T_1 \dot{\bar{v}}_{abc_4} = \frac{1}{3\bar{C}_2} T_1 \bar{i}_{abc_3} - \frac{1}{3\bar{C}_2} T_1 \bar{i}_{abc_4}$$

$$T_1 \dot{\bar{i}}_{abc_4} = \frac{3}{\bar{L}_4} T_1 \bar{v}_{abc_4} - \frac{\bar{R}_2}{\bar{L}_4} T_1 \bar{i}_{abc_4}$$

where T_1 is as previously defined and the interface equation has been transformed as

$$T_1 \bar{v}_{abc_1} = T_1 \bar{v}_{abcs} = \bar{L}_2 T_1 \dot{\bar{i}}_{abc_2} + T_1 \bar{v}_{abc_2}$$

<u>Note:</u> $T_1 X_{abc_1} = X_{odq_1}$

$$\dot{X}_{odq_1} = \dot{T}_1 X_{abc_1} + T_1 \dot{X}_{abc_1}$$

so: $T_1 \dot{X}_{abc_1} = \dot{X}_{odq_1} - \dot{T}_1 X_{abc_1}$

$$\underline{T_1 \dot{X}_{abc_1} = \dot{X}_{odq_1} - \dot{T}_1 T_1^{-1} X_{odq_1}} \qquad \text{(IF-7)}$$

Similarily, if:

$$T_2 X_{abc_2} = X_{odq_2}$$

then $\dot{X}_{odq_2} = \dot{T}_2 X_{abc_2} + T_2 \dot{X}_{abc_2}$

$$= \dot{T}_2 T_2^{-1} X_{odq_2} + T_2 T_1^{-1} T_1 \dot{X}_{abc_2}$$

and

$$T_1 \dot{X}_{abc_2} = (T_2 T_1^{-1})^{-1} X_{odq_2} - (T_2 T_1^{-1})^{-1} \dot{T}_2 T_2^{-1} X_{odq_2}$$

or $\underline{T_1 \dot{X}_{abc_2} = T_1 T_2^{-1} \dot{X}_{odq_2} - T_1 T_2^{-1} \dot{T}_2 T_2^{-1} X_{odq_2}} \qquad \text{(IF-8)}$

The transformed network and load equations are:

$$-\bar{i}_{odqs_1} + \dot{T}_1 T_1^{-1} \bar{i}_{odqs_1} - T_1 T_2^{-1} \dot{\bar{i}}_{odqs_2} + T_1 T_2^{-1} \dot{T}_2 T_2^{-1} \bar{i}_{odqs_2} =$$

$$\left(\frac{\bar{L}_2}{\bar{L}_1} + 1\right) \left[\dot{\bar{i}}_{odq_2} - \dot{T}_1 T_1^{-1} \bar{i}_{odq_2}\right] + \frac{1}{\bar{L}_1} \bar{v}_{odq_2}$$

$$- \frac{\bar{R}_o}{\bar{L}_1} \bar{i}_{odqs} - \frac{\bar{R}_o}{\bar{L}_1} T_1 T_2^{-1} \bar{i}_{odqs_2} - \frac{\bar{R}_o}{\bar{L}_1} \bar{i}_{odq_2} \qquad \text{(IF-9)}$$

$$\dot{\bar{v}}_{odq_2} - \dot{T}_1 T_1^{-1} \bar{v}_{odq_2} = \frac{1}{3\bar{C}_2} \bar{i}_{odq_2} - \frac{1}{3\bar{C}_2} \bar{i}_{odq_3} \qquad \text{(IF-10)}$$

$$\dot{\bar{i}}_{odq} - \dot{T}_1 T_1^{-1} \bar{i}_{odq_3} = \frac{3}{\bar{L}_3} \bar{v}_{odq_3} - \frac{\bar{R}_1}{\bar{L}_3} \bar{i}_{odq_3} - \frac{3}{\bar{L}_3} \bar{v}_{odq_4} \qquad \text{(IF-11)}$$

$$\dot{\bar{v}}_{odq_4} - \dot{T}_1 T_1^{-1} \bar{v}_{odq_4} = \frac{1}{3\bar{C}_2} \bar{i}_{odq_3} - \frac{1}{3\bar{C}_2} \bar{i}_{odq_4} \qquad \text{(IF-12)}$$

$$\dot{\bar{i}}_{odq_4} - \dot{T}_1 T_1^{-1} \bar{i}_{odq_4} = \frac{3}{\bar{L}_4} \bar{v}_{odq_4} - \frac{\bar{R}_2}{\bar{L}_4} \bar{i}_{odq_4} \qquad \text{(IF-13)}$$

and the transformed interface equation is:

$$\bar{v}_{odqs_1} = \bar{L}_2 \left[\dot{\bar{i}}_{odq_2} - \dot{T}_1 T_1^{-1} \bar{i}_{odq_2}\right] + \bar{v}_{odq_2} \qquad \text{(IF-14)}$$

Equations (IF-9) through (IF-13) represent the final network and load state equations. Since the system is a balanced 3-wire system, the "0" component variables are zero. The remaining 10 equations can now be directly appended to the 10 machine equations. The ten new states introduced are:

$\bar{i}_{d2}\ \bar{i}_{q2}\ \bar{v}_{d2}\ \bar{v}_{q2}\ \bar{i}_{d3}\ \bar{i}_{q3}\ \bar{i}_{d4}\ \bar{i}_{q4}\ \bar{v}_{d4}\ \bar{v}_{q4}$

Note that the states $\bar{i}_{ds1}\ \bar{i}_{qs1}\ \bar{i}_{ds2}\ \bar{i}_{qs2}$ are states in the machine equations.

Equation (IF-14) can be directly substituted for the $\bar{v}_{ds1}\ \bar{v}_{qs1}$ inputs to the machine #1 equations.

An expression for $\bar{v}_{ds2}\ \bar{v}_{qs2}$ is required for the input to the machine #2 equations:

$$\bar{v}_{odqs2} = T_2\,\bar{v}_{abcs2} = T_2\,\bar{v}_1 = T_2\,T_1^{-1}\,\bar{v}_{odqs1}$$

Employing Equation (IF-14),

$$\bar{v}_{odqs2} = \bar{L}_2\,T_2\,T_1^{-1}\,\dot{\bar{i}}_{odq_2} - \bar{L}_2\,T_2\,T_1^{-1}\,\dot{T}_1\,T_1^{-1}\,\bar{i}_{odq_2} + T_2\,T_1^{-1}\,\bar{v}_{odq_2} \qquad \text{(IF-15)}$$

Equation (IF-15) can be directly substituted for the $\bar{v}_{ds_2}\ \bar{v}_{qs_2}$ inputs to the machine #2 equations.

In per unit time, the torque equations can be appended to the machine equations with $\bar{J} = P\bar{H}$ (P = # of poles)

$$\frac{d\omega_1}{d\bar{t}} = \frac{1}{P\bar{H}_1}\left[\bar{T}_{e1_+} + \bar{T}_{m1_+}\right] \qquad \text{(IF-16)}$$

$$\frac{d\omega_2}{d\bar{t}} = \frac{1}{P\bar{\bar{H}}_2}\left[\bar{T}_{e2_+} + \bar{T}_{m2_+}\right] \qquad \text{(IF-17)}$$

$$\frac{d\theta_1}{d\bar{t}} = \frac{\omega_1}{\omega_B} \qquad \text{(IF-18)}$$

$$\frac{d\theta_2}{d\bar{t}} = \frac{\omega_2}{\omega_B} \qquad \text{(IF-19)}$$

where ω and θ are electrical quantities. Equations (IF-16) and (IF-17) are required also for the governor equations (GDE-3) (written for both machine #1 and machine #2).

The inertia constants $\overline{H}_1$ and $\overline{H}_2$ are the combined inertias of the synchronous machine plus the diesel engine. These are computed as:

$$\overline{H}_{Total} = \overline{H}_{SM} + \overline{H}_{DE}$$

where $\overline{H}_{SM}$ = 0.20 P.U. sec and $\overline{H}_{DE}$ is found from:

$$J_{DE} = 1434 \text{ lb ft}^2$$

$$= 61 \text{ kg m}^2$$

and

$$\overline{H}_{DE} = \frac{1}{2} J_{DE}\, \omega^2_{Rated}/P_{BS}$$

$$= \frac{1}{2} \times 61 \times (\frac{2\pi}{60} \times 1200)^2/625 \times 10^3$$

$$= 0.76 \text{ P.U. sec}$$

so

$$\underline{\overline{H}_{Total} = 0.96 \text{ P.U. sec.}}$$

Notice that at this point the only inputs to the 10 machine, 10 network and loads plus 4 Newton second law equations are the two mechanical torques $\overline{T}_{m_1}$ and $\overline{T}_{m_2}$ plus the two field voltages $\overline{v}_{fd_1}$ and $\overline{v}_{fd_2}$. These will come from the governor and AVR/Exciter equations respectively. The electrical torques are:

$$\overline{T}_{\underset{+}{e_1}} = -\frac{P}{3}\left[\overline{x}_{q_1}\,\overline{i}_{qs_1}\,\overline{i}_{ds_1} + \overline{x}_{mq_1}\,\overline{i}_{Q_1}\,\overline{i}_{ds_1} - \overline{x}_{d_1}\,\overline{i}_{ds_1}\,\overline{i}_{qs_1} - \overline{x}_{dm_1}\,\overline{i}_{fd_1}\,\overline{i}_{qs_1} - \overline{x}_{d_1}\,\overline{i}_{d_1}\,\overline{i}_{qs_1}\right] \tag{IF-20}$$

$$\overline{T}_{\underset{+}{e_2}} = -\frac{P}{3}\left[\overline{x}_{q2}\,\overline{i}_{qs2}\,\overline{i}_{ds2} + \overline{x}_{mq2}\,\overline{i}_{Q2}\,\overline{i}_{ds2} - \overline{x}_{d2}\,\overline{i}_{ds2}\,\overline{i}_{qs2} - \overline{x}_{dm2}\,\overline{i}_{fd2}\,\overline{i}_{qs2} - \overline{x}_{d2}\,\overline{i}_{d2}\,\overline{i}_{qs2}\right] \tag{IF-21}$$

The mechanical torque is expressible in terms of the mechanical power and speed as:

$$\underset{+}{\bar{T}_{m1}} = \frac{P}{2}\,\frac{\omega B}{\omega_1}\,\bar{P}_{m_1} \tag{IF-22}$$

$$\underset{+}{\bar{T}_{m2}} = \frac{P}{2}\,\frac{\omega B}{\omega_2}\,\bar{P}_{m2} \tag{IF-23}$$

where ω_1 and ω_2 are in electrical radians per second.

The only inputs which remain to be interfaced are the two AVR sensed voltages $\bar{V}_{T_1}'$ and $\bar{V}_{T_2}'$. Without the parallel compensation circuit for reactive power load sharing, these would both be the line to line terminal voltage peak value in per unit. The parallel compensation circuit of the voltage regulators adds a portion of phase B current to the v_{AC} voltage as:

$$\bar{V}_{T1}' = \text{Peak}\left[\bar{v}_{A1} - \bar{v}_{C1} + \bar{\alpha}_1\,\bar{i}_{\substack{B1\\IN}}\right]$$

$$\bar{V}_{T_2}' = \text{Peak}\left[\bar{v}_{A_2} - \bar{v}_{C_2} + \bar{\alpha}_2\,\bar{i}_{\substack{B_2\\IN}}\right]$$

Utilizing $\bar{v}_{abcs} = T_1^{-1}\,\bar{v}_{odqs_1}$ with $\bar{v}_{os_1} = 0$,

$$\bar{v}_{A_1} = \bar{v}_{as} = \sqrt{2/3}\,(\text{Cos}\;\theta_1\;\bar{v}_{ds_1} - \text{Sin}\;\theta_1\;\bar{v}_{qs_1})$$

$$\bar{v}_{B_1} = \bar{v}_{bs} = \sqrt{2/3}\,(\text{Cos}\;(\theta_1 - \tfrac{2\pi}{3})\;\bar{v}_{ds_1} - \text{Sin}\;(\theta_1 - \tfrac{2\pi}{3})\;\bar{v}_{qs_1})$$

$$\bar{v}_{C_1} = \bar{v}_{cs} = \sqrt{2/3}\,(\text{Cos}\;(\theta_1 + \tfrac{2\pi}{3})\;\bar{v}_{ds_1} - \text{Sin}\;(\theta_1 + 2\pi/3)\;\bar{v}_{qs_1})$$

Note that for $\theta_1 = \omega_s t + \theta_{1ss}(0)$ this is a positive sequence source. The addition of a portion of $\bar{i}_{\substack{bs_1\\IN}}$ to $\bar{v}_{as_1} - \bar{v}_{cs_1}$ will yield a sensed voltage which is directly additive when the load is inductive, directly subtractive when the load is capacitive, and nearly unchanged (additive by 90° phase shift) for unity power factor load. This new sensed voltage leads to a droop or rise in the actual terminal voltage depending on the load power factor. The parallel action allows the sharing of reactive power. The B phase currents are found from:

$$\bar{i}_{\substack{abcs_1\\IN}} = T_1^{-1}\,\bar{i}_{odqs_1} \quad \text{and} \quad \bar{i}_{\substack{abcs_2\\IN}} = T_2^{-1}\,\bar{i}_{odqs_2}$$

so

$$\bar{i}_{\substack{B_1\\IN}} = \bar{i}_{\substack{bs_1\\IN}} = \sqrt{2/3}\,(\text{Cos }(\theta_1 - \tfrac{2\pi}{3})\,\bar{i}_{ds_1} - \text{Sin }(\theta_1 - \tfrac{2\pi}{3})\,\bar{i}_{qs_1})$$

$$\bar{i}_{\substack{B_2\\IN}} = \bar{i}_{\substack{bs_2\\IN}} = \sqrt{2/3}\,(\text{Cos }(\theta_2 - \tfrac{2\pi}{3})\,\bar{i}_{ds_2} - \text{Sin }(\theta_2 - \tfrac{2\pi}{3})\,\bar{i}_{qs_2})$$

The sensed voltage by AVR #1 is thus,

$$\bar{V}_{T_1}' = \text{Peak}\left[\sqrt{2/3}\,(\frac{3}{2}\,\bar{v}_{ds_1} + \frac{\sqrt{3}}{2}\,v_{qs_1} - \frac{\bar{\alpha}_1}{2}\,\bar{i}_{ds_1} + \frac{\sqrt{3}\bar{\alpha}_1}{2}\,\bar{i}_{qs_1})\ \text{Cos }\theta_1 \right.$$

$$\left. + \sqrt{2/3}\,(-\frac{3}{2}\,\bar{v}_{qs_1} + \frac{\sqrt{3}}{2}\,\bar{v}_{ds_1} + \frac{\sqrt{3}\alpha_1}{2}\,\bar{i}_{ds_1} + \frac{\bar{\alpha}_1}{2}\,\bar{i}_{bs_1})\ \text{Sin }\theta_1\right]$$

Define the peak as:

$$\bar{V}_{T_1}' = \sqrt{2/3}\left[(\frac{3}{2}\,\bar{v}_{ds_1} + \frac{\sqrt{3}}{2}\,\bar{v}_{qs_1} - \frac{\bar{\alpha}_1}{2}\,\bar{i}_{ds_1} + \frac{\sqrt{3}\alpha_1}{2}\,\bar{i}_{qs_1})^2 + (-\frac{3}{2}\,\bar{v}_{qs_1} + \frac{\sqrt{3}}{2}\,\bar{v}_{ds_1} + \frac{\sqrt{3}\alpha_1}{2}\,\bar{i}_{ds_1} + \frac{\bar{\alpha}_1}{2}\,\bar{i}_{qs_1})^2\right]^{1/2}$$

(IF-24)

Similarly,

$$\bar{V}_{T_2}' = \sqrt{2/3}\left[(\frac{3}{2}\,\bar{v}_{ds_2} + \frac{\sqrt{3}}{2}\,\bar{v}_{qs_2} - \frac{\bar{\alpha}_2}{2}\,\bar{i}_{ds_2} + \frac{\sqrt{3}\alpha_1}{2}\,\bar{i}_{qs_2})^2 + (-\frac{3}{2}\,\bar{v}_{qs_2} + \frac{\sqrt{3}}{2}\,\bar{v}_{ds_2} + \frac{\sqrt{3}\alpha_2}{2}\,\bar{i}_{ds_2} + \frac{\bar{\alpha}_2}{2}\,\bar{i}_{qs_2})^2\right]^{1/2}$$

(IF-25)

The four quantities $\bar{v}_{ds_1}$ $\bar{v}_{qs_1}$ $\bar{v}_{ds_2}$ $\bar{v}_{qs_1}$ can be expressed as functions of $\dot{i}_{d_2}$ $\dot{i}_{q_2}$ $\bar{i}_{d_2}$ $\bar{i}_{q_2}$ $\bar{v}_{d_2}$ $\bar{v}_{q_2}$ through equations (IF-14) and (IF-15) (the original terminal interface equation). Furthermore, the quantities $\dot{i}_{d_2}$ $\dot{i}_{q_2}$ can be expressed as a function of all the states through the final equations. These expressions will not include $\overline{V}_{T_1}'$ and $\overline{V}_{T_2}'$ and thus completely close the state equations. Note that even if the expressions for $\dot{\overline{i}}_{d_2}$ and $\dot{\overline{i}}_{q_2}$ did include $\overline{V}_{T_1}'$ and $\overline{V}_{T_2}'$, these sensing voltages could be solved for analytically through a quadratic formula involving the other states. The final state equations can be written as follows:

$$\left[\begin{array}{c|c} M & 0 \\ \hline 0 & I \end{array}\right] \left[\begin{array}{c} \dot{X}_1 \\ \hline \dot{X}_2 \end{array}\right] = \left[\begin{array}{c|c} N_{11} & N_{12} \\ \hline N_{21} & N_{22} \end{array}\right] \left[\begin{array}{c} X_1 \\ \hline X_2 \end{array}\right] + \left[\begin{array}{c} U_1 \\ \hline U_2 \end{array}\right]$$

$$\left[\dot{X}_3\right] = \left[f\,(X_1, X_2, X_3)\right]$$

X_1: Includes the following states:

$\overline{i}_{ds_1}$ $\overline{i}_{qs_1}$ $\overline{i}_{fd_1}$ $\overline{i}_{D_1}$ $\overline{i}_{Q_1}$

$\overline{i}_{ds_2}$ $\overline{i}_{qs_2}$ $\overline{i}_{fd_2}$ $\overline{i}_{D_2}$ $\overline{i}_{Q_2}$

$\overline{i}_{d_2}$ $\overline{i}_{q_2}$

X_2: Includes the following states:

$\overline{v}_{fd_1}$ $\overline{v}_{fe_1}'$ $\overline{y}_{11}$ $\overline{y}_{12}$

$\overline{v}_{fd_2}$ $\overline{v}_{fe_2}'$ $\overline{y}_{21}$ $\overline{y}_{22}$

$\overline{v}_{d_2}$ $\overline{v}_{q_2}$ $\overline{i}_{d_3}$ $\overline{i}_{q_3}$ $\overline{i}_{d_4}$ $\overline{i}_{q_4}$ $\overline{v}_{d_4}$ $\overline{v}_{q_4}$

X_3: Includes the following states:

θ_1 ω_1 $\overline{P}_{m_1}$ Z_{11} Z_{12}

θ_2 ω_2 $\overline{P}_{m_2}$ Z_{21} Z_{22}

M : Includes the machine parameters and some of the network parameters plus the states θ_1 and θ_2. It is a 12 x 12 matrix whose entries vary with $(\theta_1-\theta_2)$.

I : Is the 16 x 16 identy matrix

N_{11}: Includes system parameters and the states θ_1, θ_2, ω_1 and ω_2. It is a 12 x 12 matrix whose entries vary with $(\theta_1-\theta_2)$, ω_1 and ω_2.

N_{12}: Includes system parameters and the states θ_1 and θ_2. It is a 12 x 16 matrix whose entries vary with $(\theta_1-\theta_2)$.

N_{21}: Includes two nonzero constant entries.

It is a 16 x 12 matrix whose entries are constant.

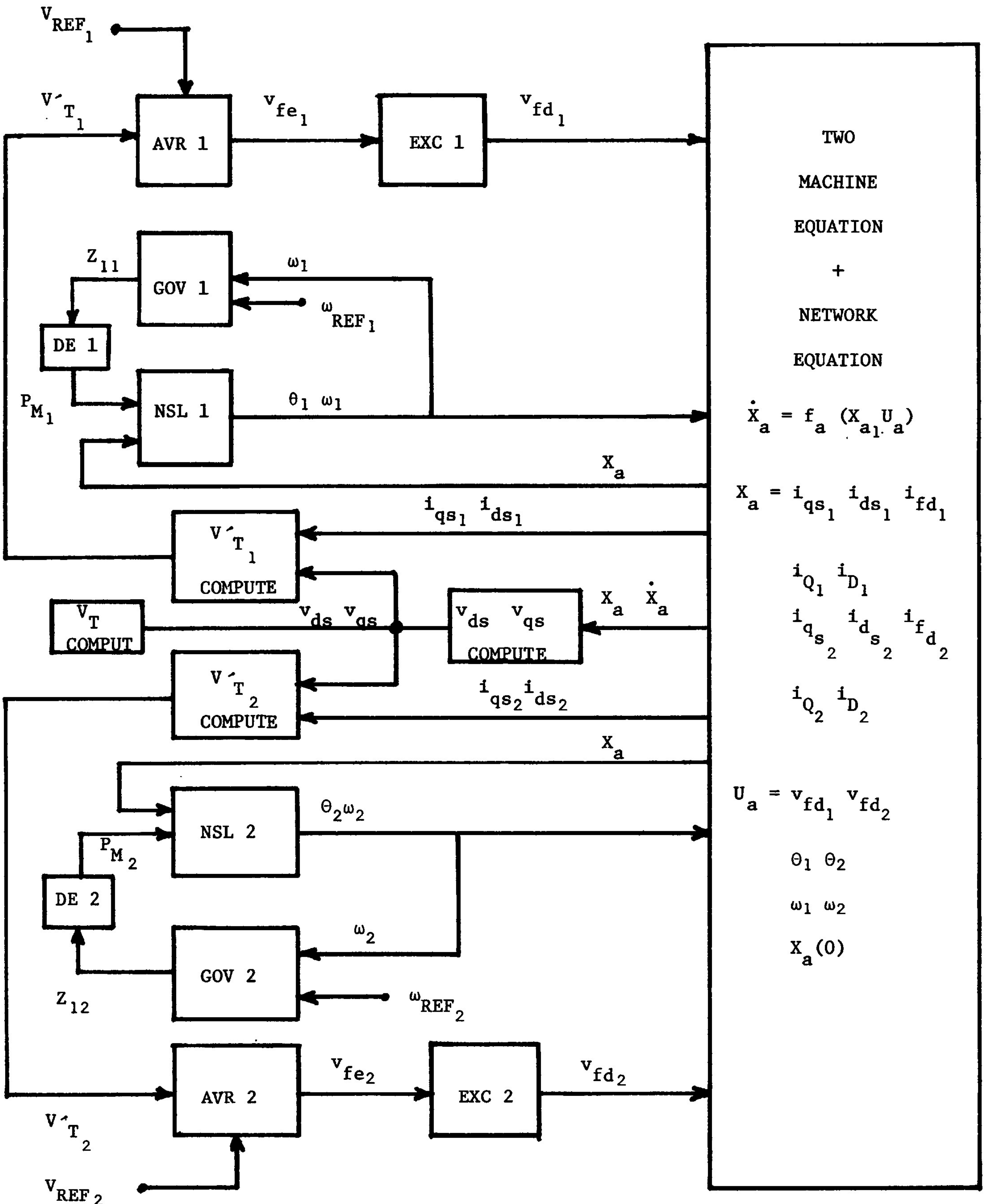
N_{22}: Includes system parameters and the state ω_1. It is a 16 x 16 matrix whose entries vary with ω_1.

U_1 : Is a 12 x 1 vector of zeros.

U_2 : Is a 16 x 1 vector with two nonzero entries including $\overline{V}_{REF_1}$, $\overline{V}_{REF_2}$, and the inputs $\overline{V}_{T_1}'$ and $\overline{V}_{T_2}'$ which can be expressed as a function of $\dot{X}_1$ and X_1 through the procedure discussed earlier in this section. Furthermore, it can be expressed explicitly as a function of X_1, X_2, N_{11}, N_{12}, and the inverse of M (also a function of θ_1 and θ_2).

f : Includes the two speed equations, the two torque (Newton's Second Law) equations, and the six governor equations. As such, it contains products and ratios of the states in X_1 and X_3.

The system state flow diagram is given in Figure 23. The values of U_a and all initial conditions are determined in Section 3.6.



V_{REF_1}
V'_{T_1}
AVR 1
v_{fe_1}
EXC 1
v_{fd_1}
TWO
MACHINE
EQUATION
+
NETWORK
EQUATION
$\dot{X}_a = f_a\ (X_{a_1} U_a)$
Z_{11}
GOV 1
ω_1
ω_{REF_1}
DE 1
P_{M_1}
NSL 1
$\theta_1\ \omega_1$
X_a
$X_a = i_{qs_1}\ i_{ds_1}\ i_{fd_1}$
$i_{qs_1}\ i_{ds_1}$
V'_{T_1}
COMPUTE
$i_{Q_1}\ i_{D_1}$
V_T
COMPUT
$v_{ds}\ v_{qs}$
$v_{ds}\ v_{qs}$
COMPUTE
$X_a\ \dot{X}_a$
$i_{q_{s_2}}\ i_{d_{s_2}}\ i_{f_{d_2}}$
V'_{T_2}
COMPUTE
$i_{qs_2}\ i_{ds_2}$
$i_{Q_2}\ i_{D_2}$
X_a
$U_a = v_{fd_1}\ v_{fd_2}$
NSL 2
$\theta_2\omega_2$
P_{M_2}
$\theta_1\ \theta_2$
DE 2
$\omega_1\ \omega_2$
ω_2
$X_a(0)$
GOV 2
Z_{12}
ω_{REF_2}
AVR 2
v_{fe_2}
EXC 2
v_{fd_2}
V'_{T_2}
V_{REF_2}

Figure 23

The following summary of per unit base quantities is given as the completion of the interfacing of the system equations.

$V_{BS} = \sqrt{2} \times 277$ v. peak ℓ-n

$V_{BR} = \sqrt{2} \times 277 \times 64$ v. D.C.

$V_{BA} = \sqrt{2} \times 277 \times 64$ v. D.C.

$V_{BF} = \sqrt{2} \times 277 \times 64 \times 10$ v. D.C.

$P_B = 625 \times 10^3$ VA 3ϕ

$I_{BS} = \sqrt{2} \times 752$ A. peak

$I_{BR} = \sqrt{2} \times 752/64$ A.

$I_{BA} = \sqrt{2} \times 752/64$ A.

$I_{BF} = \sqrt{2} \times 752/(64 \times 10)$ A.

$Z_{BS} = 0.37$ ohms

$\omega_B = 2\pi \times 60$ Electrical radians/sec.

3.6 Initial Conditions, Post Initial Conditions and Solution Method

This section presents the initial conditions and simulation method for various studies performed. At time zero it was assumed that the generator terminals were open circuited from the network with rated voltage and speed. Thus, from section 3.2,

$$\bar{i}_{qs_1}{}^{\circ} = 0. \qquad \bar{i}_{qs_2}{}^{\circ} = 0.$$

$$\bar{i}_{ds_1}{}^{\circ} = 0. \qquad \bar{i}_{ds_2}{}^{\circ} = 0.$$

$$\bar{i}_{Q_1}{}^{\circ} = 0. \qquad \bar{i}_{Q_2}{}^{\circ} = 0.$$

$$\bar{i}_{D_1}{}^{\circ} = 0. \qquad \bar{i}_{D_2}{}^{\circ} = 0.$$

$$\bar{V}'_{T_1}{}^{\circ} = 1.732. \qquad \bar{V}'_{T_2}{}^{\circ} = 1.732.$$

$$\omega_1{}^{\circ} = 377. \qquad \omega_2{}^{\circ} = 377.$$

$$\Theta_1{}^{\circ} = 0. \qquad \Theta_2{}^{\circ} = 0.$$

Note that the arbitrary selection of $\Theta_1{}^{\circ}$ and $\Theta_2{}^{\circ}$ to be zero also fixes the stator phase A voltage v_{as} to be at a zero at time t = 0. (See section 3.2.) That is, if no load is added to the terminals, and the excitation is not changed, the instantaneous terminal voltages can be computed from the equations in Section 3.2.

The machine equations were written such that the no load field current would be negative. This happened because the electrical torque angle was defined as being $\Theta_1(0)_{SS} - \gamma - \pi/2$ with $\Theta_1(0)_{SS}$ equal to zero.

Consequently, since $\Theta_1{}^{\circ}$ was arbitrarily set equal to zero, equation (G-12) yields the terminal voltages with $\gamma = -90^{\circ}$ as:

$$v^{\circ}_{as} = \sqrt{2} \times 277 \times \cos(-90^{\circ})$$

$$v^{\circ}_{bs} = \sqrt{2} \times 277 \times \cos(-210^{\circ})$$

$$v^{\circ}_{cs} = \sqrt{2} \times 277 \times \cos(+30^{\circ})$$

The terminal voltages in the dq reference frame are then:

$$v^{\circ}_{ds_1} = 0.$$

$$v^{\circ}_{qs_1} = -\sqrt{2} \times 277 \times \sqrt{3/2}$$

or in per unit,

$$\bar{v}^{\circ}_{ds_1} = 0.$$

$$\bar{v}^{\circ}_{qs_1} = -\sqrt{3/2}.$$

As a consequence of this definition of the torque angle, the field current will be negative at no load. This is clear from Equation (G-21) at open circuit,

$$\bar{i}_{fd_1}{}^{\circ} = \frac{\omega_B}{\omega_1{}^{\circ}} \bar{v}^{\circ}{}_{qs} \frac{1}{\bar{X}_{dm_1}}$$

or

$$\bar{i}_{fd_1}{}^{\circ} = -\sqrt{3/2}\,\frac{1}{1.366} = -0.897 \text{ p.u.a.}$$

and

$$\bar{i}_{fd_2}{}^{\circ} = -0.897 \text{ p.u.a.}$$

The network and loads are assumed to be de-energized at time zero so that:

$$\bar{i}_{d_2}{}^{\circ} = \bar{v}_{d_2}{}^{\circ} = \bar{i}_{d_3}{}^{\circ} = \bar{i}_{d_4}{}^{\circ} = \bar{v}_{d_4}{}^{\circ} = 0.$$

$$\bar{i}_{q_2}{}^{\circ} = \bar{v}_{q_2}{}^{\circ} = \bar{i}_{q_3}{}^{\circ} = \bar{i}_{q_4}{}^{\circ} = \bar{v}_{q_4}{}^{\circ} = 0.$$

The exciter was assumed to be in steady state. Thus, since the synchronous machine field currents are -15 amps (-.897 p.u.a.), and the synchronous machine field resistance is 3.85 ohms, the initial field voltage can be computed from Equation (G-6) as,

$$v_{fd}^{\circ} = -58 \text{ volts}$$

or

$$\bar{v}_{fd_1}^{\circ} = \bar{v}_{fd_2}^{\circ} = -0.0023 \text{ p.u.v.}$$

Utilizing the AVR/exciter transfer function, this means that the exciter field voltage must be:

$$\bar{v}_{fe}^{\circ} = -8.89 \times 10^{-5} \text{ p.u.v.}$$

and so

$$\bar{v}'_{fe_1}{}^{\circ} = \bar{v}'_{fe_2}{}^{\circ} = -0.962 \text{ p.u.v.}$$

The automatic voltage regulator was assumed to be de-energized at time zero with:

$$\bar{y}_{11}^{\circ} = \bar{y}_{12}^{\circ} = \bar{y}_{21}^{\circ} = \bar{y}_{22}^{\circ} = 0.$$

The reference voltages for the two regulators were taken as

$$\bar{V}_{REF_1} = \bar{V}_{REF_2} = 1.732.$$

Note that the value of $\bar{v}_{fe}^{\circ}$ can be checked by realizing that $\bar{v}_{fd}^{\circ} = -0.0023$ p.u.v. is also the exciter armature initial voltage. From Section 3.3 this was found to occur when the exciter excitation is $i_{fe}^{\circ} = -0.38$ amps and $v_{fe}^{\circ} = -17.5$ volts. Utilizing the voltage bases which were assigned according to turns ratios, $\bar{v}_{fe}^{\circ} = -7.0 \times 10^{-5}$ p.u.v. This is slightly lower than the value computed through the AVR/exciter transfer function. This inconsistancy is due to the N_R/N_S = 64 computation in Section 3.2. It indicates that the machine parameters supplied by the manufacturer are not exactly consistent with the machine open circuit characteristics. This is not uncommon when linear models are employed. The original values of $\bar{v}_{fe}^{\circ} = -8.89 \times 10^{-5}$ and $\bar{v}'_{fe}{}^{\circ} = -0.962$ were used. A "-1" multiplier was added to the AVR.

The diesel engines and governors were assumed to be at no load at time zero with:

$$\bar{P}_{M_1}{}^{\circ} = \bar{P}_{M_2}{}^{\circ} = 0.$$

$$Z_{11}{}^{\circ} = Z_{21}{}^{\circ} = 0.$$

$$Z_{12}{}^{\circ} = Z_{22}{}^{\circ} = 0.$$

The reference speeds for the two governors were taken as:

$$\omega_{REF_1} = \omega_{REF_2} = 377 \text{ electrical radians/second}$$

This concludes the derivation of the initial conditions. All states are zero at time zero except as follows:

$$\omega_1{}^{\circ} = \omega_2{}^{\circ} = 377$$

$$\bar{v}_{fd_1}{}^{\circ} = \bar{v}_{fd_2}{}^{\circ} = -0.0023$$

$$\bar{v}'_{fe_1} = \bar{v}'_{fe_2} = -0.962$$

The other quantities required at time zero are:

$$\bar{V}'_{T_1}{}^{\circ} = \bar{V}'_{T_2}{}^{\circ} = 1.732$$

$$\bar{V}_{REF_1} = \bar{V}_{REF_2} = 1.732 \quad \text{(for all time)}$$

$$\omega_{REF_1} = \omega_{REF_2} = 377 \quad \text{(for all time)}$$

The solution of the 38 non-linear differential equations could be done using any reliable numerical integration algorithm. A fourth order Runge-Kutta algorithm was used. This algorithm was selected because of its traditional success and simplicity. Gear's implicit algorithms were not used since computation time was forseen to be a significant factor.

Consider now the initial response of the dynamic system when the first time step is completed. If the network is maintained at the de-energized state, the voltage regulator would still experience some transient response for $t > 0$. This is because the initial conditions have the regulator de-energized, but the exciter field voltage non-zero. Thus, if these initial conditions are used, the output would appear in the feedback and force the regulator to initiate its action to bring its initial zero output up to the required v_{fe}°. Note that in steady state, the regulator input and output may be constantly changing very rapidly so that the input is never exactly zero. The network will, however, be energized automatically through the interface equations. Thus, the initial transient includes both the network plus load dynamics plus the regulator initial and final dynamics.

The value of load to be switched at different times can be controlled by changing the value of the load equivalent resistance and inductance at different times. The affect of the shunt reactor can be simulated by varying the value of the shunt inductance.

The following time steps were utilized in the Runge-Kutta algorithm to provide accurate results while requiring minimal execution time. (Note that the time step is in per unit where $t = \overline{t}/377$.)

$\bar{t}$	$\bar{t}$ step
0 to 15	0.030
15 to 45	0.075
45 to 135	0.100
135 to 380	0.120
380 to 750	0.130
750 to 3000	0.140

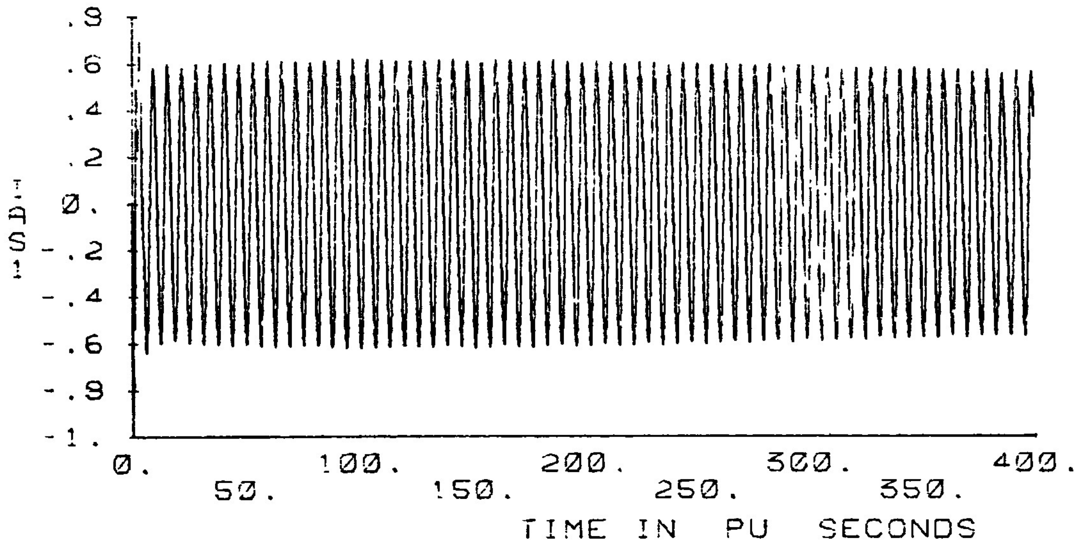
The load values were switched as follows:

$\bar{t}$	$\frac{1}{3}\bar{R}_2$	$\frac{1}{3}\bar{L}_4$	Three phase equivalent load
0 to 377	16.683	12.28	30 kW .8 pF
377 to 3000	1.6683	1.228	300 kW .8 pF

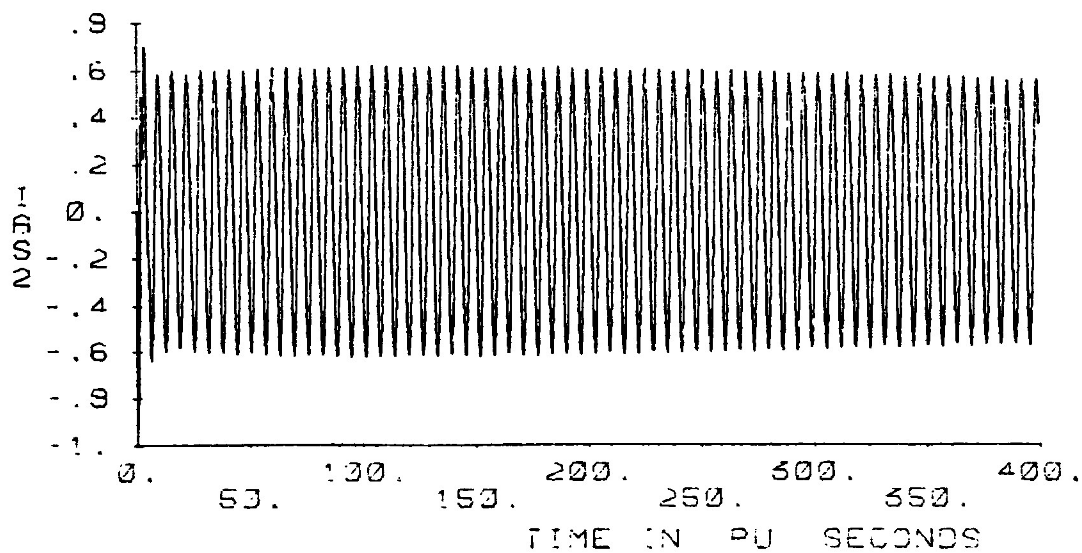
The simulation was run for shunt reactor values of equivalent 200 kVAR per phase and 0 kVAR per phase.

3.7 Simulation Results

The system was studied for various load levels and various cable lengths with and without the shunt inductor. The extreme case for generator underexcitation and pending instability was the 30 KW 0.8 power factor lag load with the full 13 mile cable system. The simulation for this case without the shunt inductor is shown in Figure 24(a) through (p). The simulation for this case with the 200 KVAR per phase shunt inductor is shown in Figure 25(a) through (p). The most significant difference is noted in Figures 24(g)-(j) and Figures 25(g)-(j). Without the shunt reactor, the field excitation is approaching zero with virtually no voltage regulator action. This is also evidenced in the high terminal voltage. With the shunt reactor, the field excitation is being adjusted correctly by the voltage regulator as evidenced by the return of field voltage in Figures 25(i) and (j). Without significant excitation, any imbalance in the parallel operation will yield unstable operation with either reverse power flow or large circulating currents. The instantaneous solution did not show any abnormal operation with either a 100 KVAR or 200 KVAR per phase shunt reactor installed. The 200 KVAR shunt reactor brings the machine stator currents well within their rated values even when only one machine is on line. The electrical transient lasts longer when the shunt reactor is installed, but this is still limited to less than one second real time.



<u>Figure 24(a)</u> Machine One stator A phase current in per unit, without inductor.



<u>Figure 24(b)</u> Machine Two stator A phase current in per unit, without inductor.

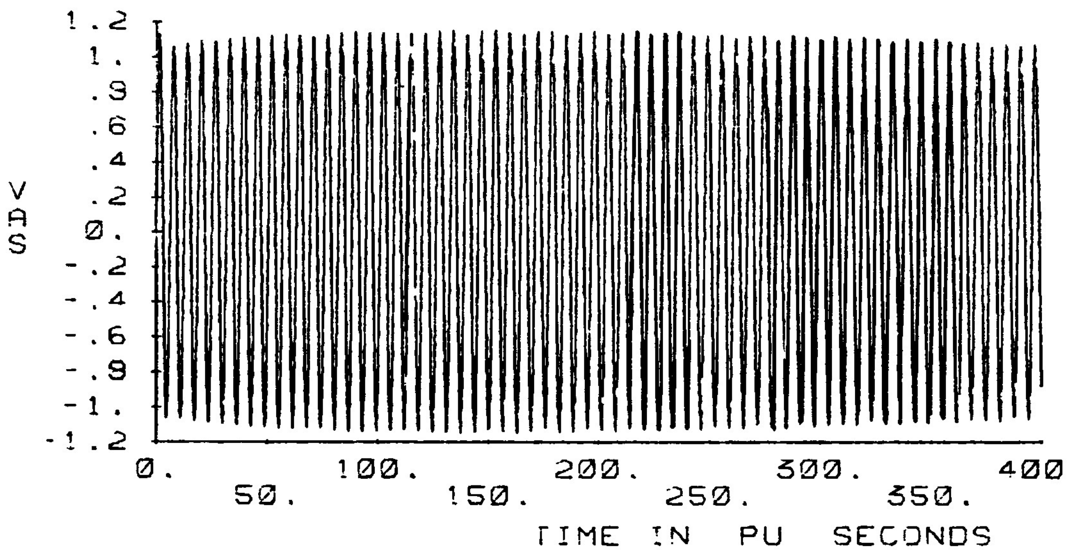


Figure 24(c) Machine terminal A phase line to neutral voltage in per unit, without inductor.

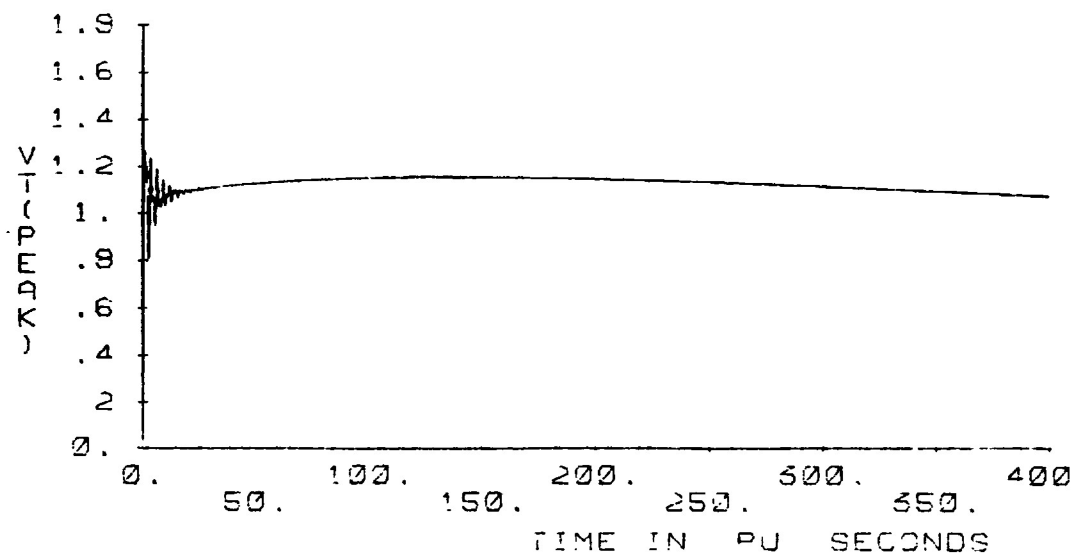


Figure 24(d) Machine average peak terminal voltage in per unit, without inductor.

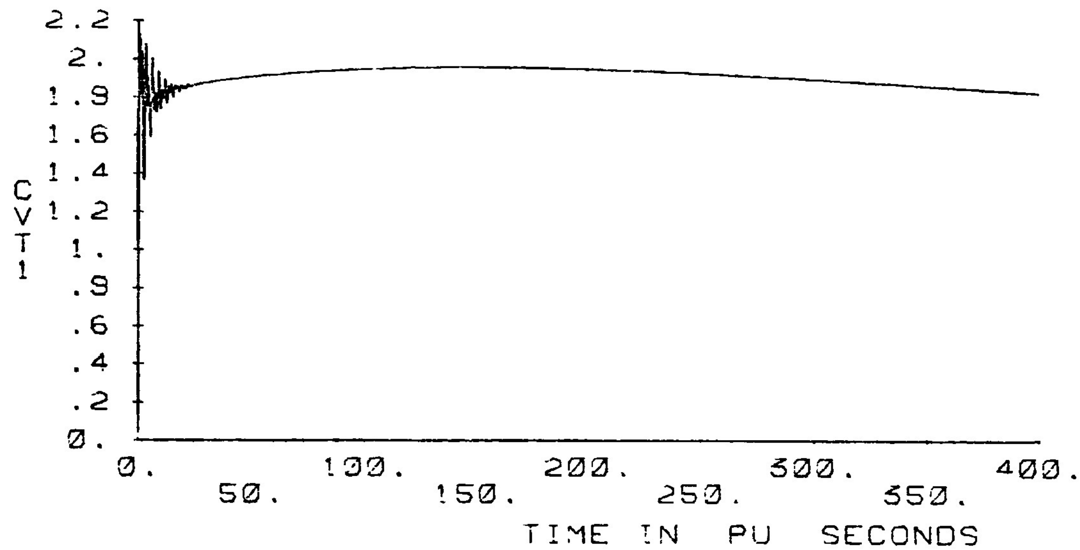


Figure 24(e) Machine One AVR control voltage in per unit, without inductor.

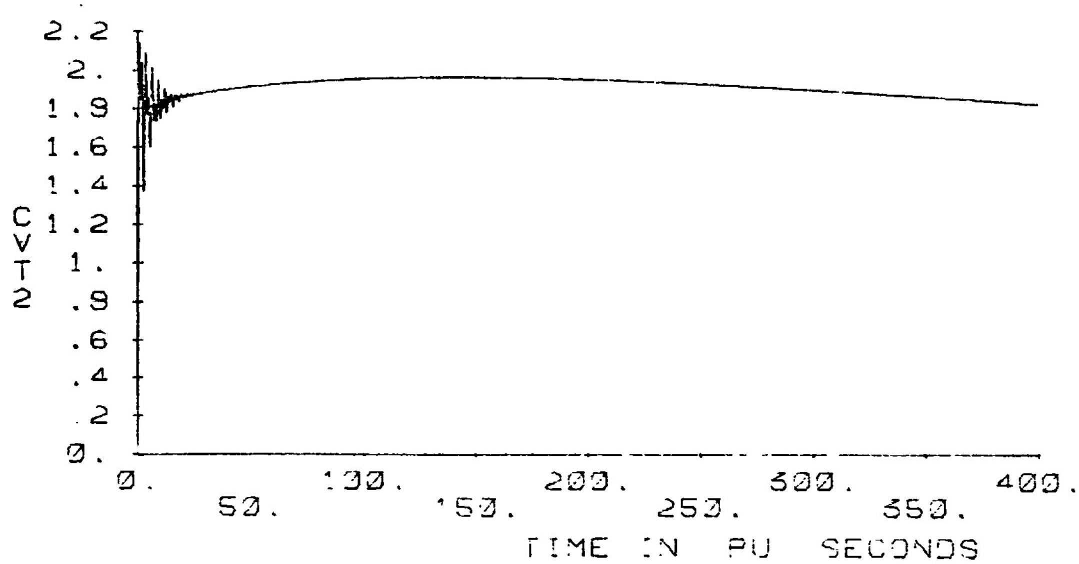


Figure 24(f) Machine Two AVR control voltage in per unit, without inductor.

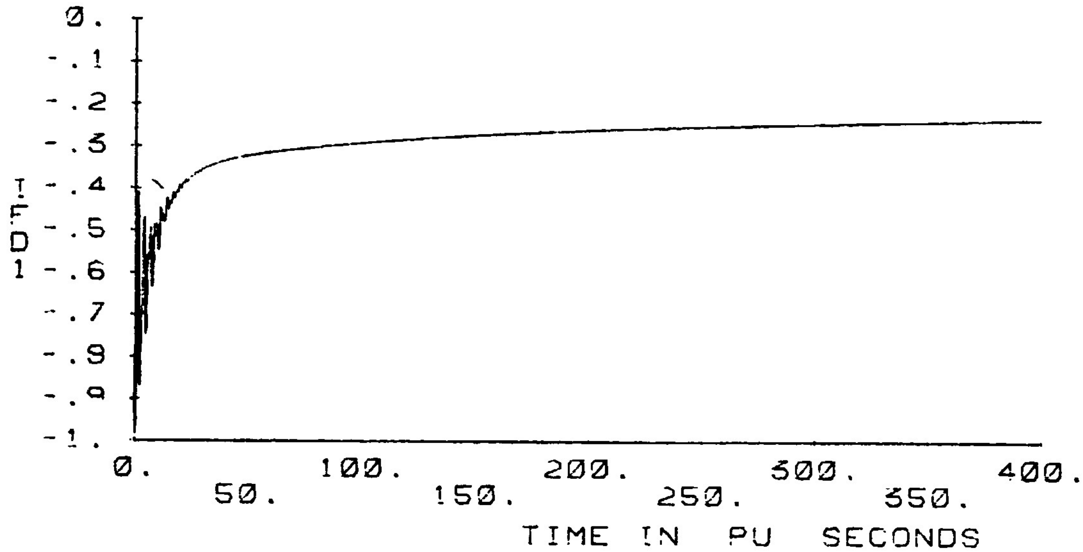


Figure 24(g) Machine One field current in per unit, without inductor.

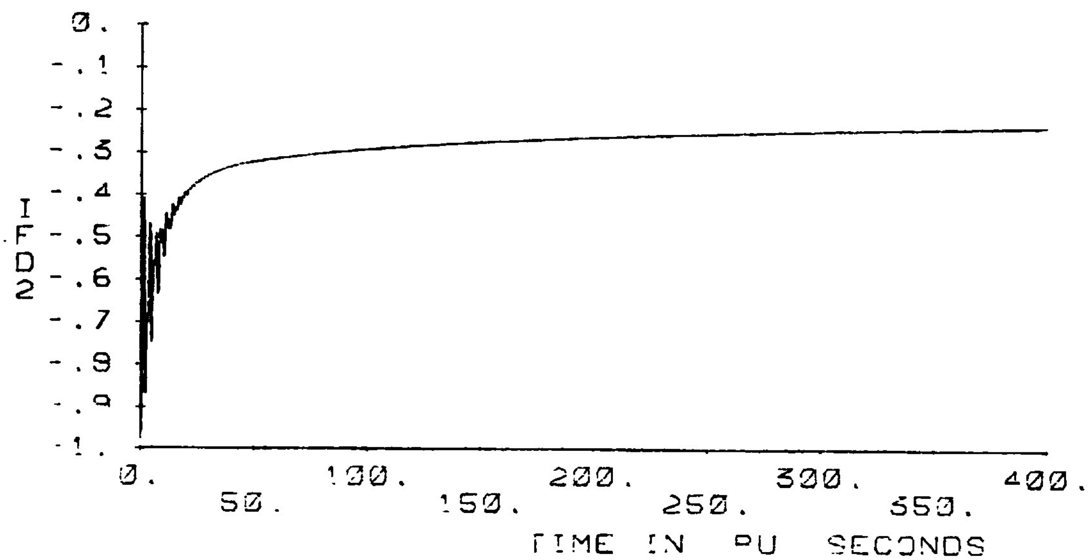


Figure 24(h) Machine Two field current in per unit, without inductor.

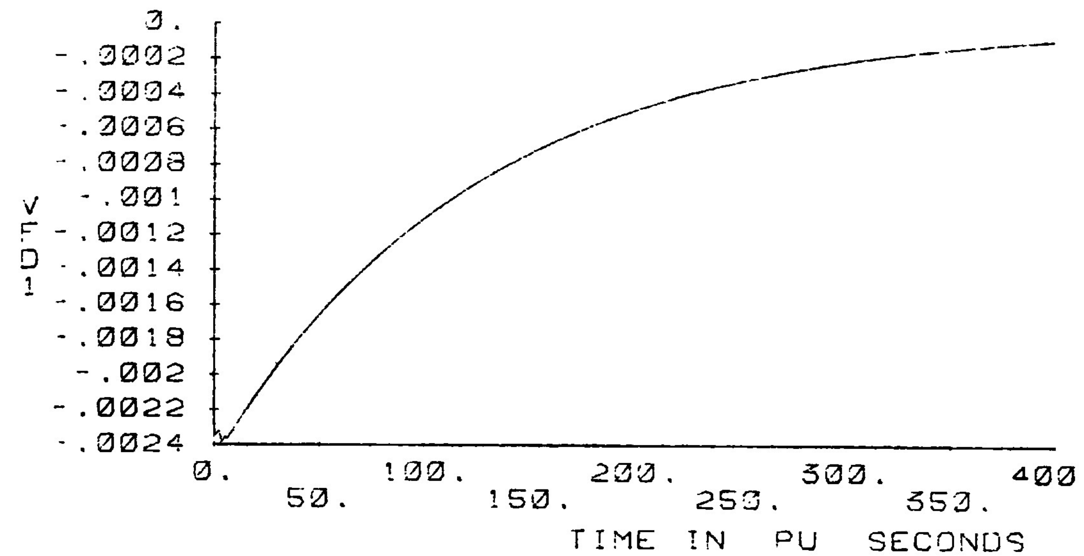


Figure 24(i) Machine One field voltage in per unit, without inductor.

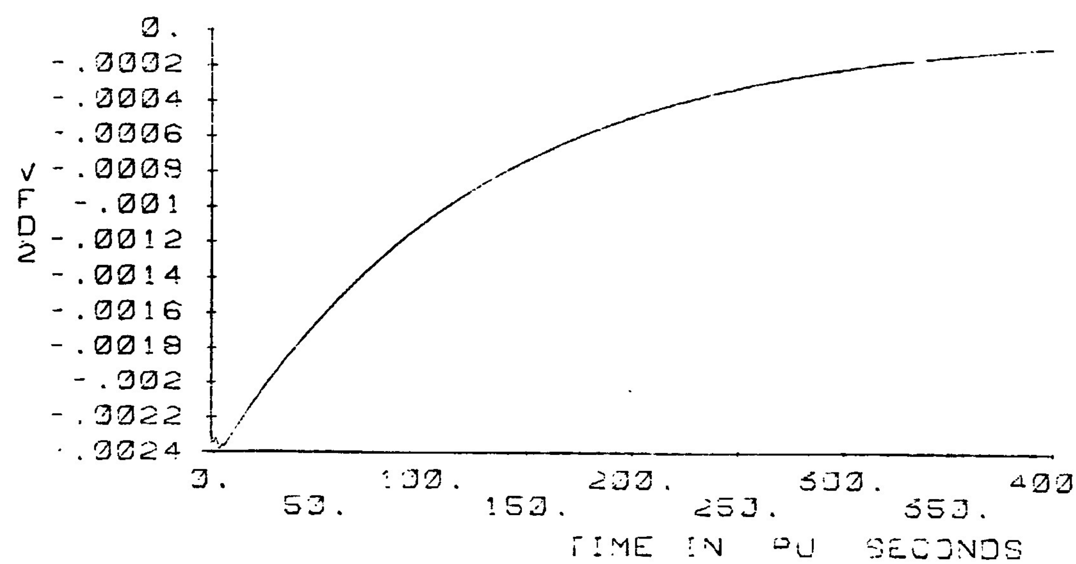


Figure 24(j) Machine Two field voltage in per unit, without inductor.

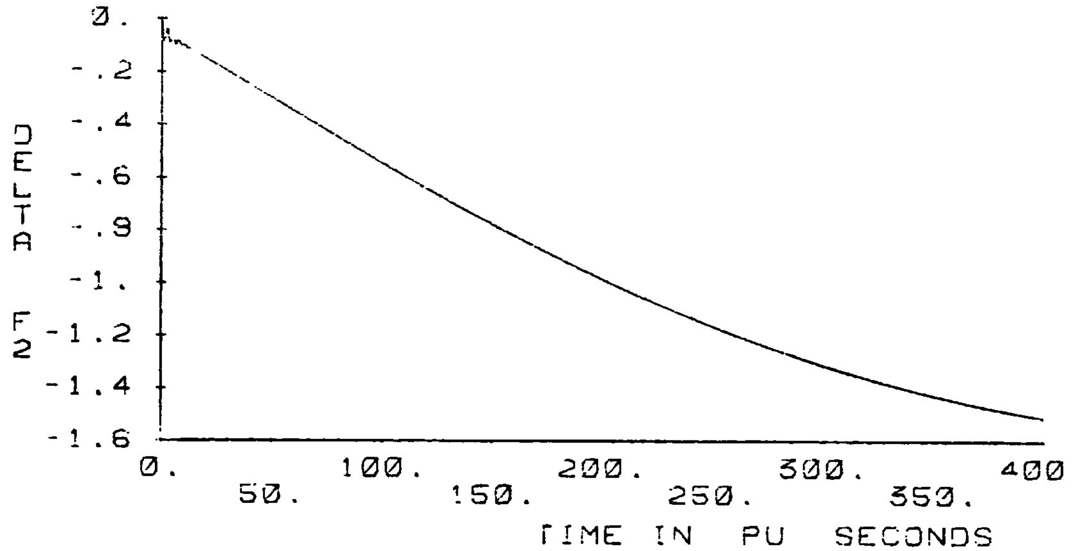


Figure 24(k) Machine One frequency change from 60 Hz, without inductor.

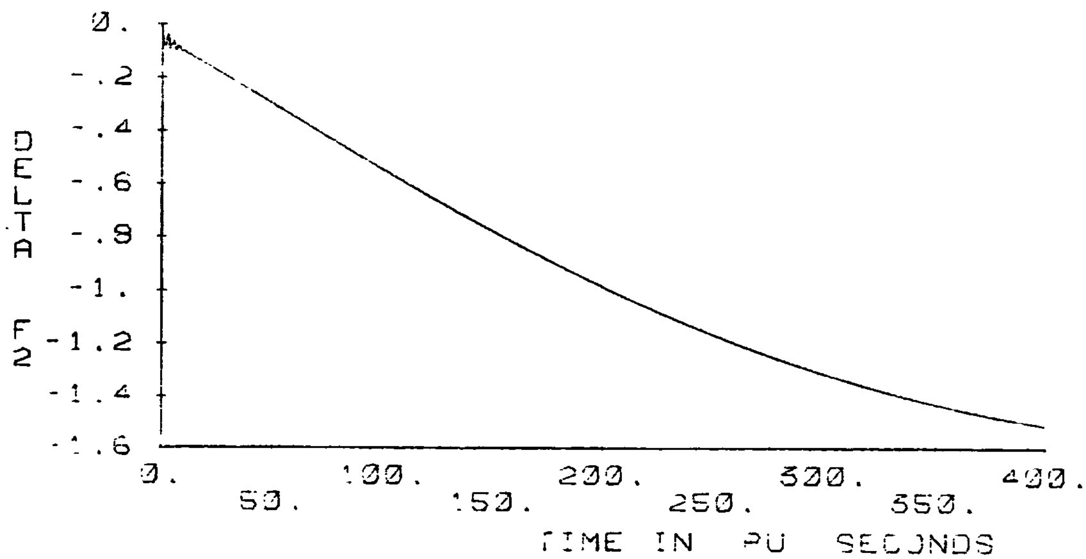


Figure 24(ℓ) Machine Two frequency change from 60 Hz, without inductor.

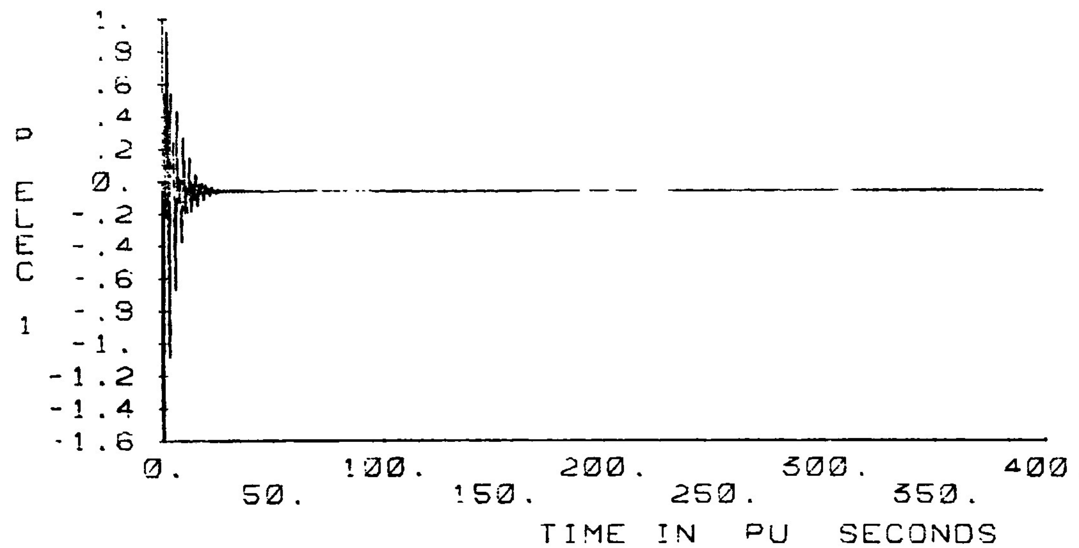


Figure 24(m) Machine One real power (in) in per unit, without inductor.

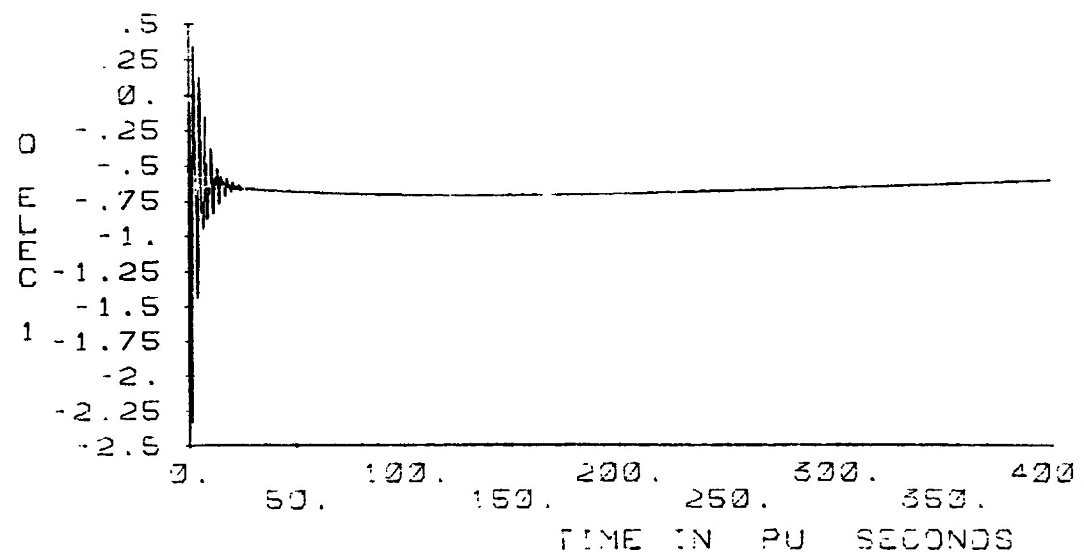


Figure 24(n) Machine One reactive power output in per unit, without inductor.

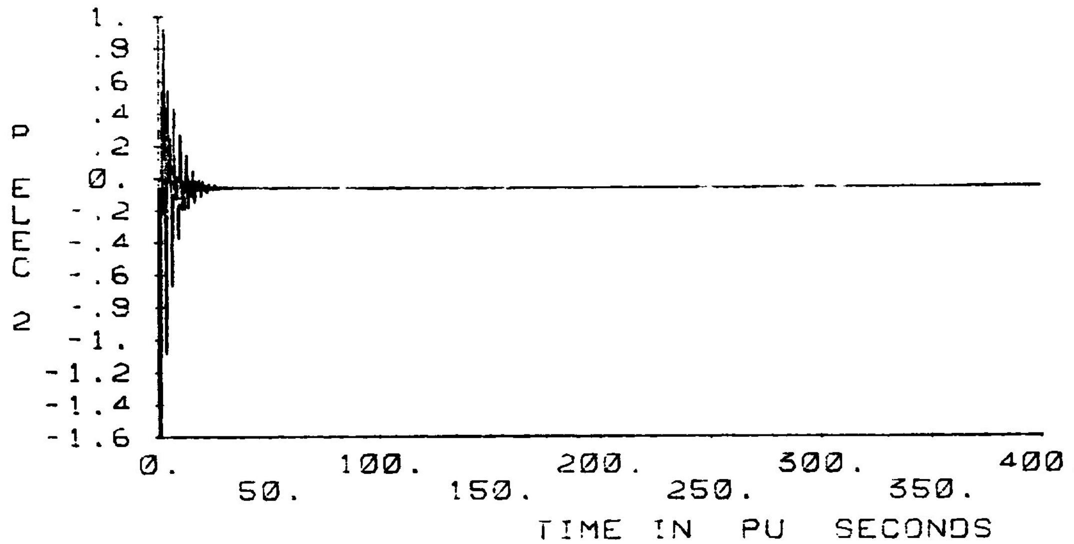


Figure 24(o) Machine Two real power (in) in per unit, without inductor.

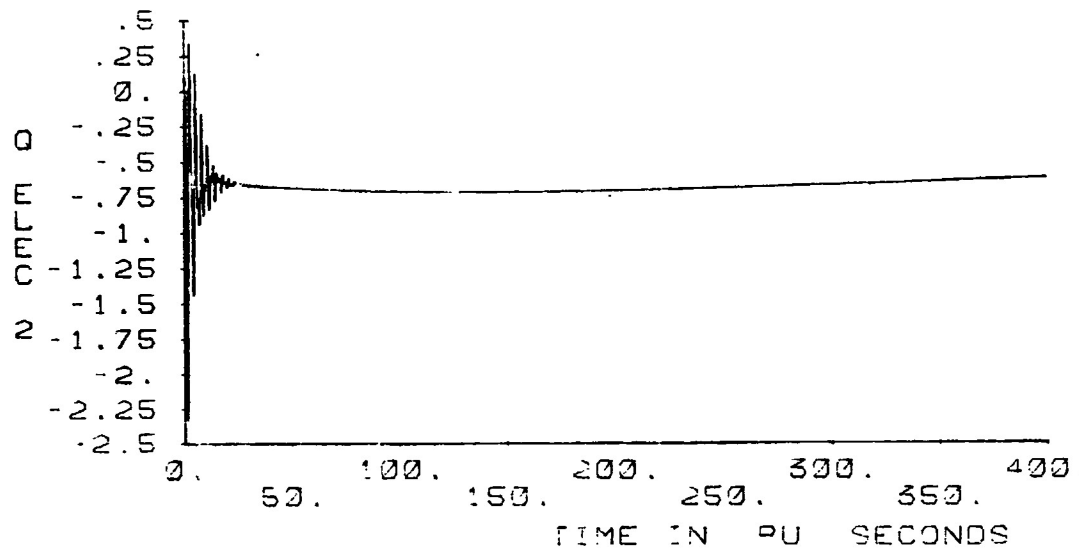


Figure 24(p) Machine Two reactive power output in per unit, without inductor.

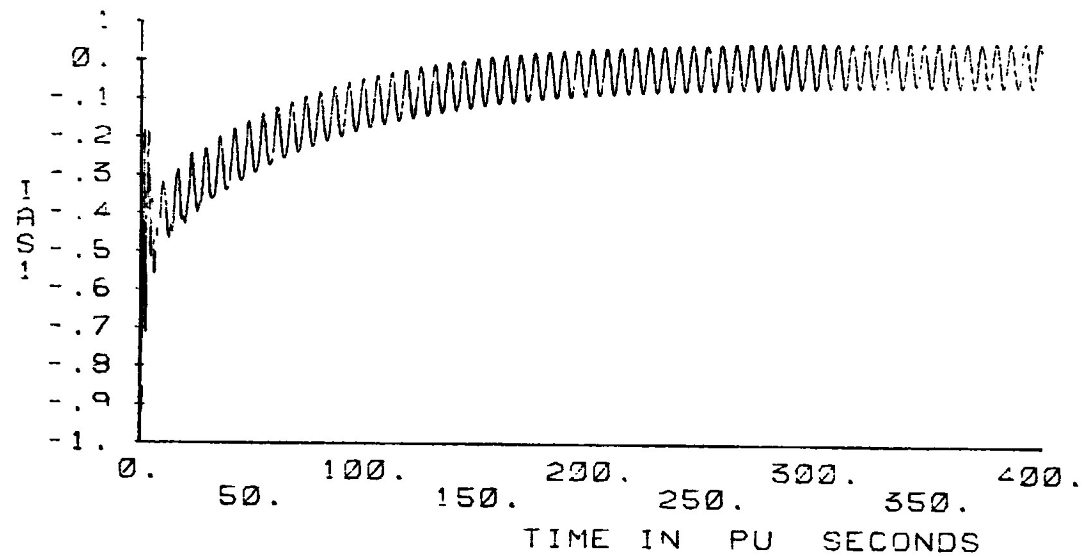


Figure 25(a) Machine One stator A phase current in per unit, with inductor.

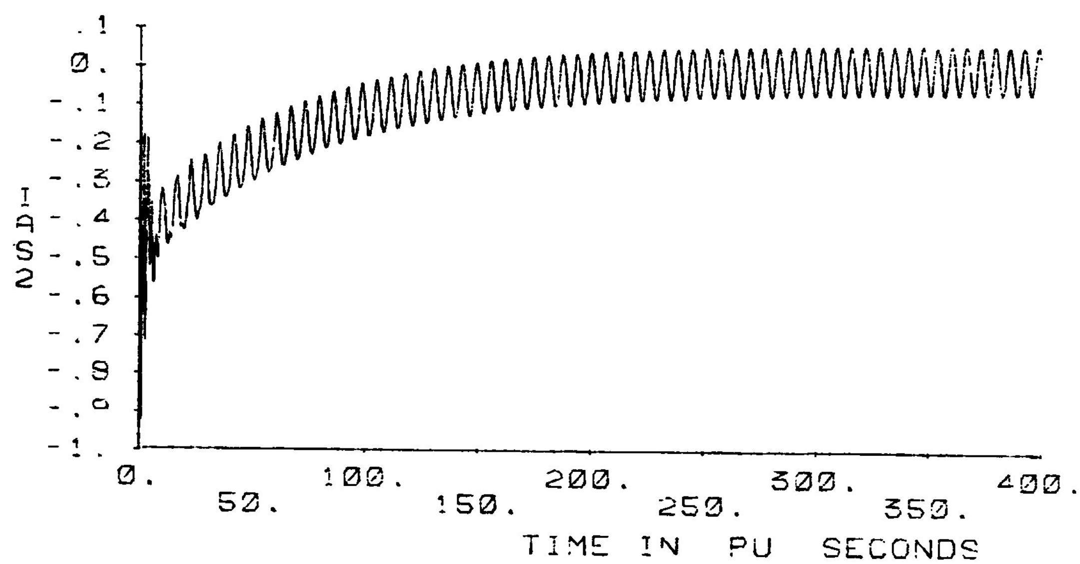


Figure 25(b) Machine Two stator A phase current in per unit, with inductor.

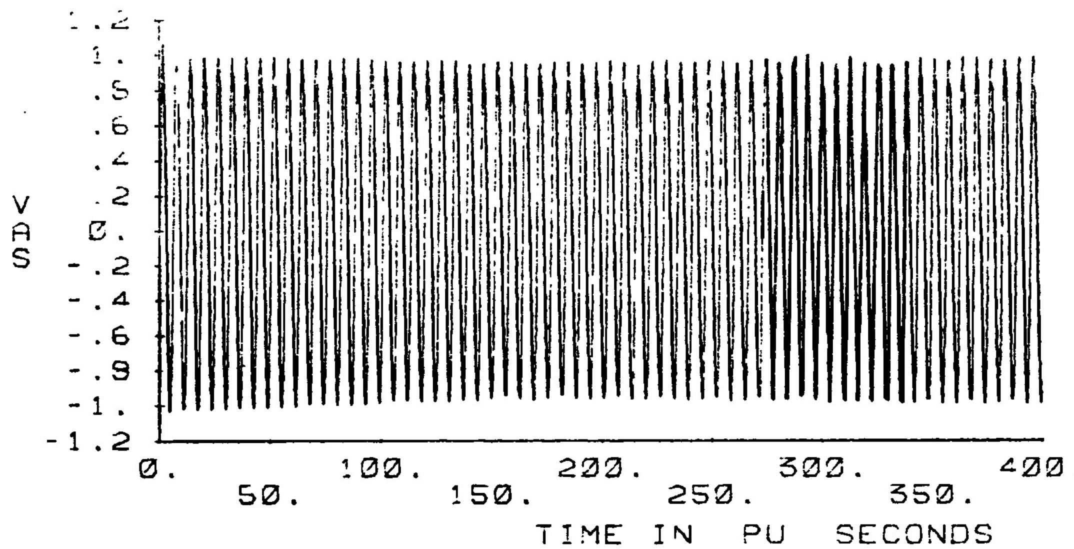


Figure 25(c) Machine terminal A phase line to neutral voltage in per unit, with inductor.

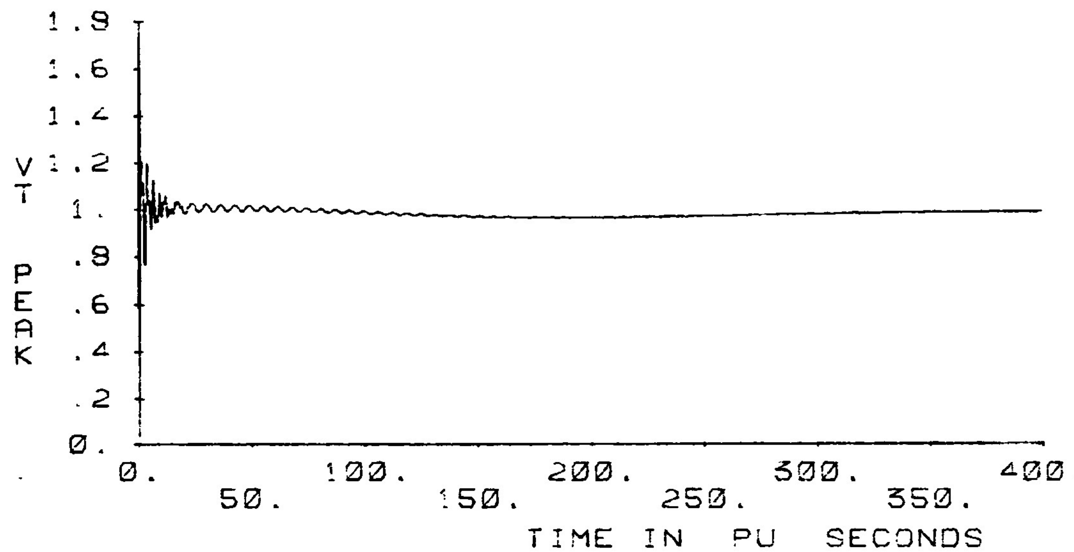


Figure 25(d) Machine average peak terminal voltage in per unit, with inductor.

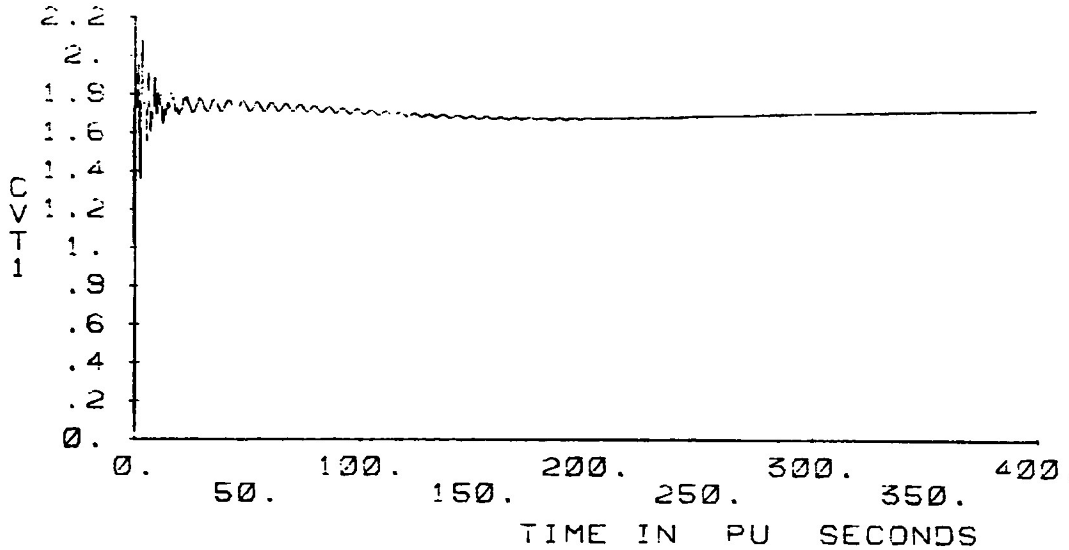


Figure 25(e) Machine One AVR control voltage in per unit, with inductor.

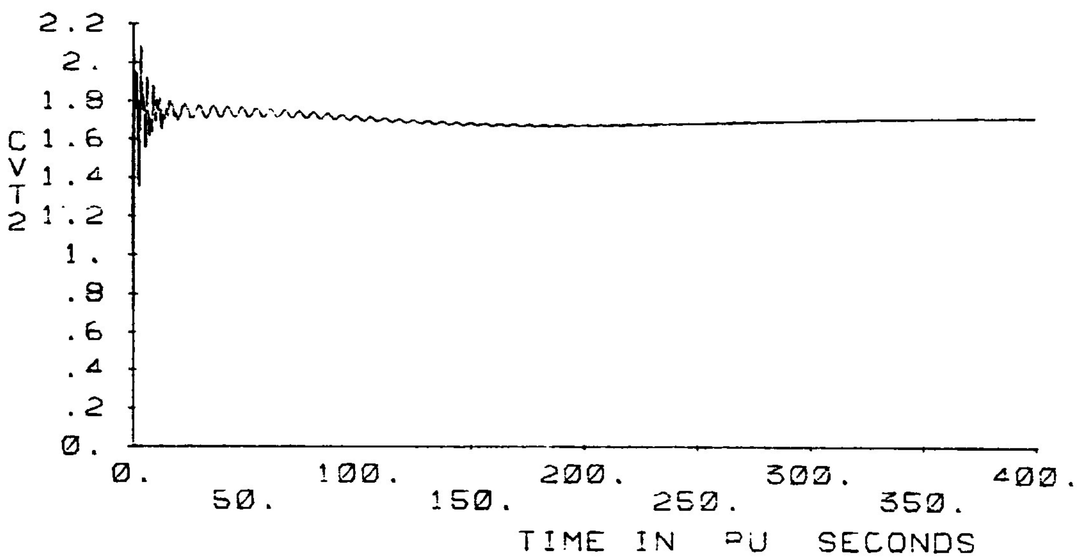
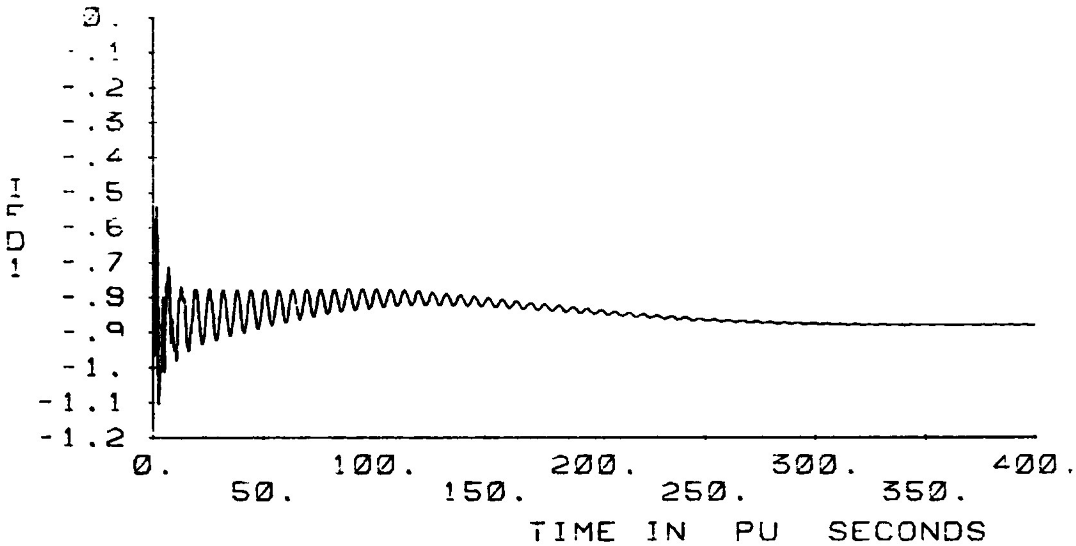
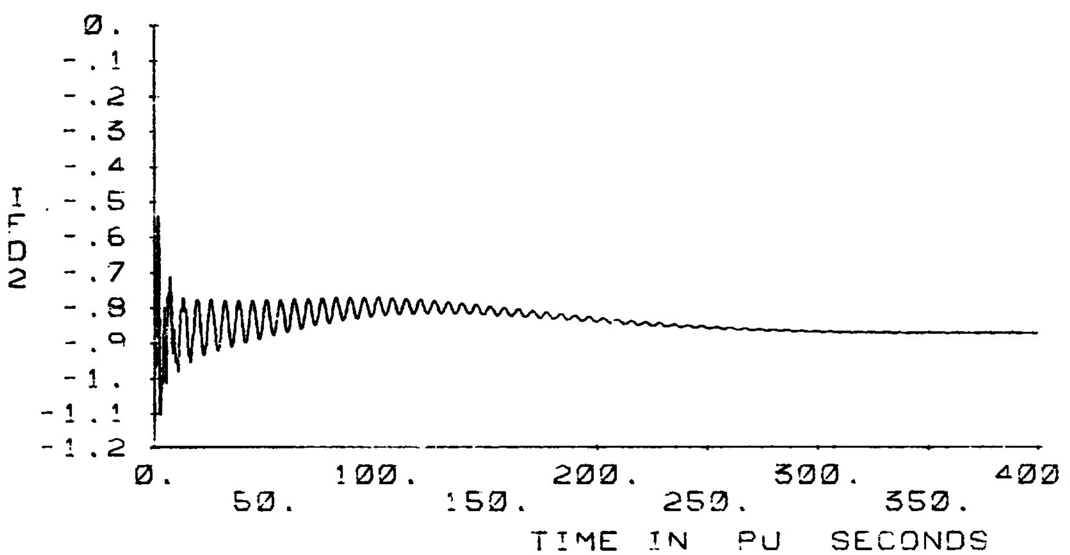


Figure 25(f) Machine Two AVR control voltage in per unit, with inductor.



<u>Figure 25(g)</u> **Machine One field current in per unit, with inductor.**



<u>Figure 25(h)</u> **Machine Two field current in per unit, with inductor.**

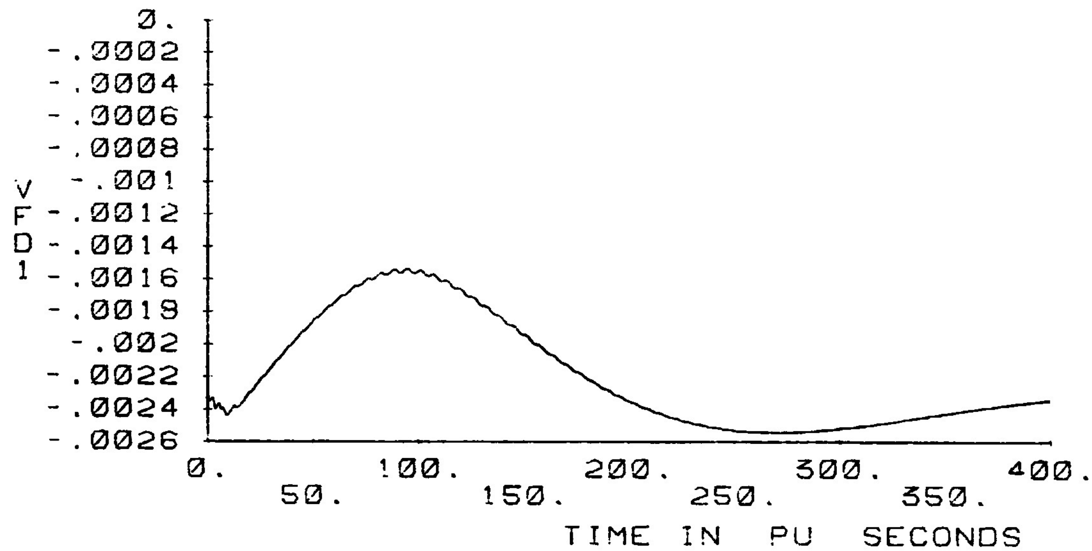


Figure 25(i) Machine One field voltage in per unit, with inductor.

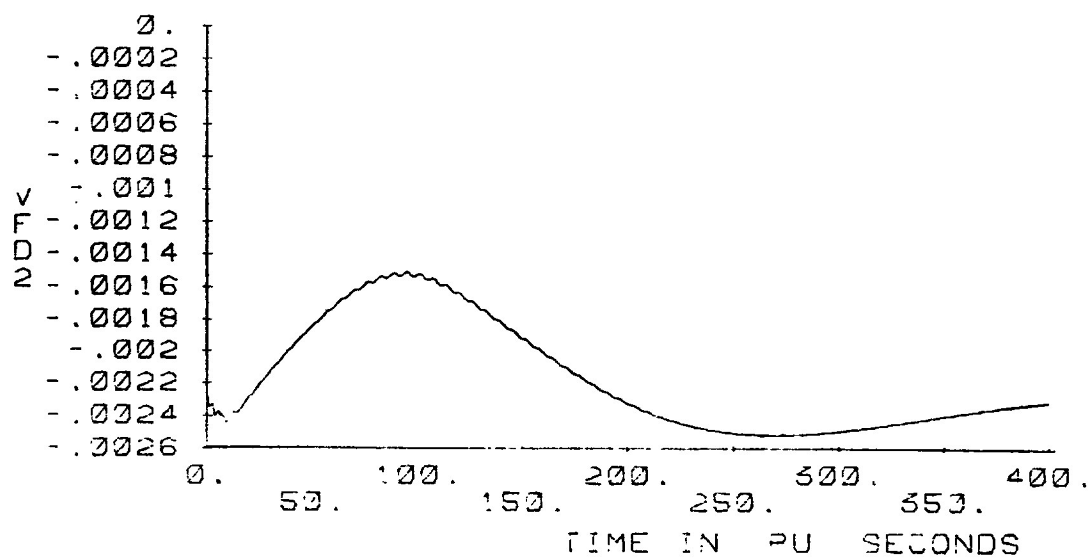


Figure 25(j) Machine Two field voltage in per unit, with inductor.

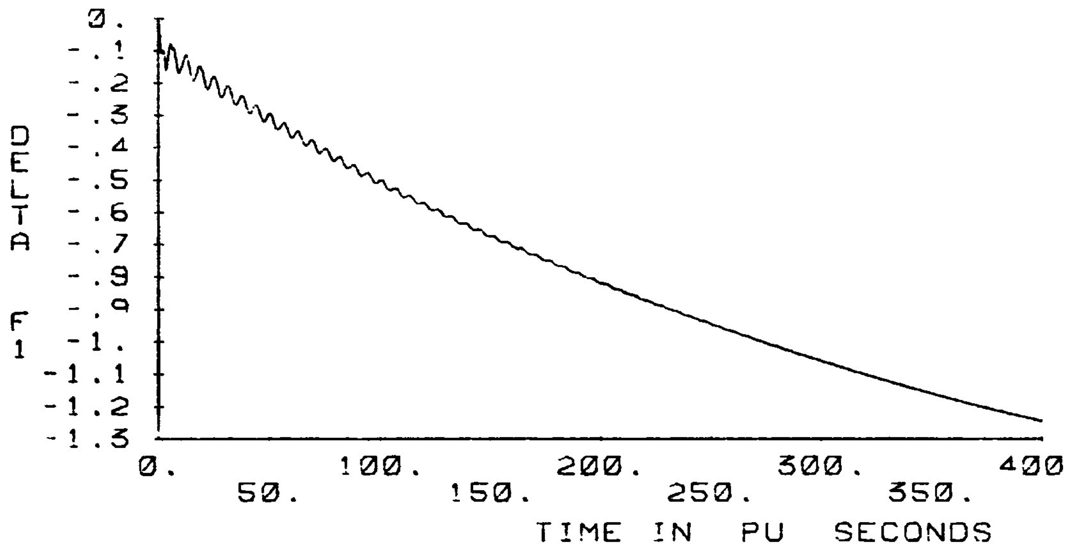


Figure 25(k) Machine One frequency change from 60 Hz, with inductor.

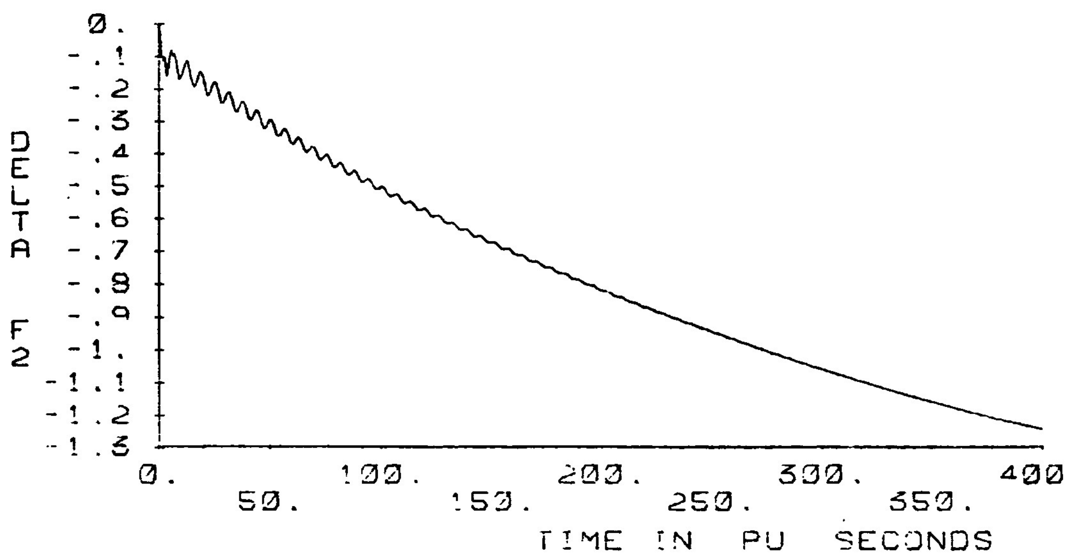


Figure 25(ℓ) Machine Two frequency change from 60 Hz, with inductor.

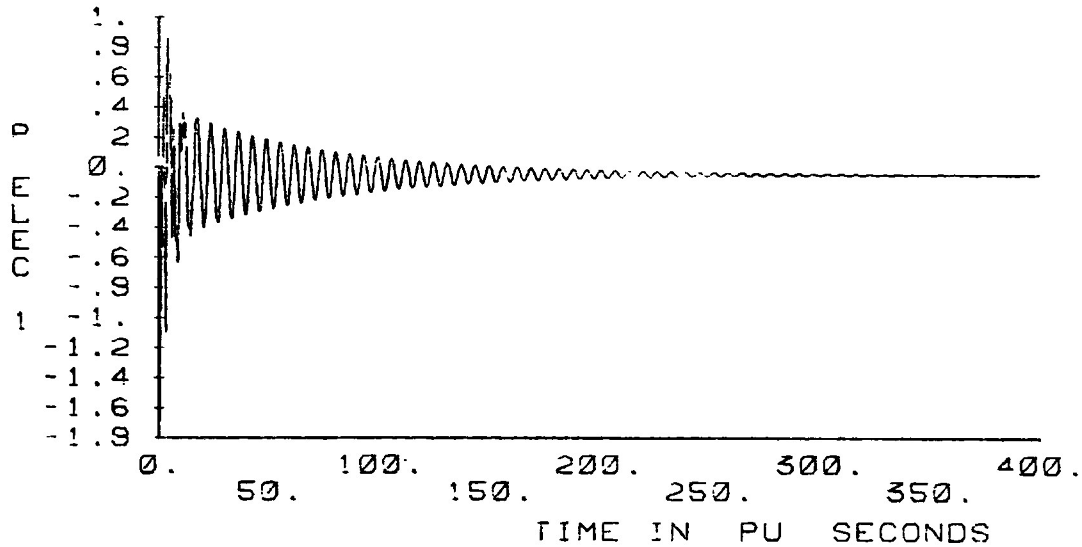


Figure 25(m) Machine One real power (in) in per unit, with inductor.

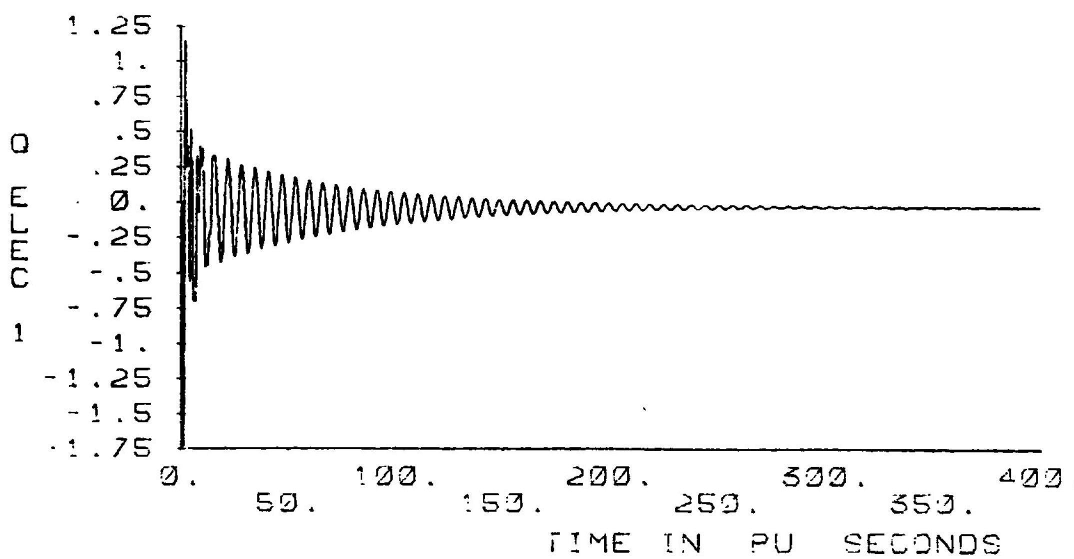


Figure 25(n) Machine One reactive power output in per unit, with inductor.

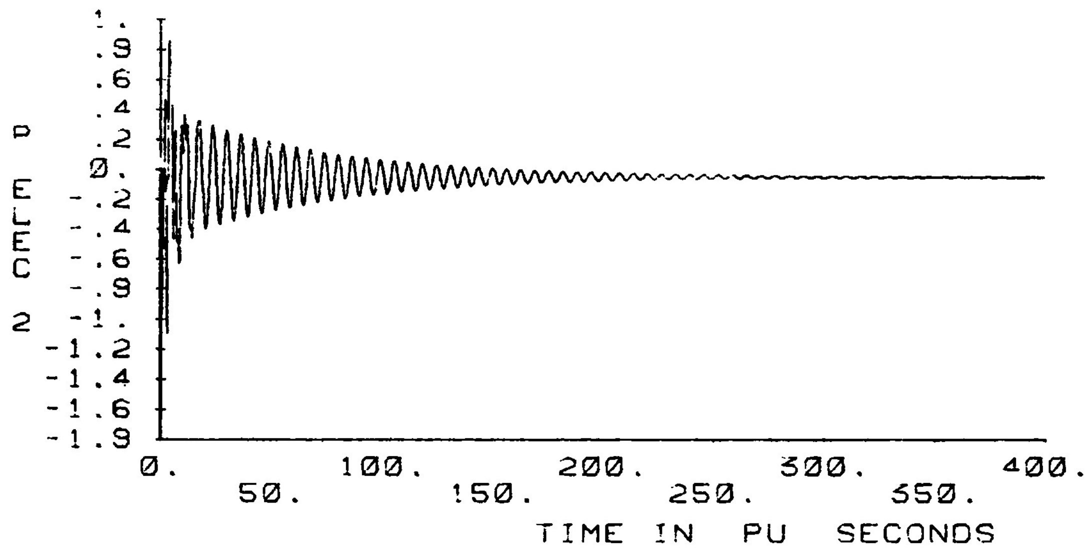


Figure 25(o) Machine Two real power (in) in per unit, with inductor.

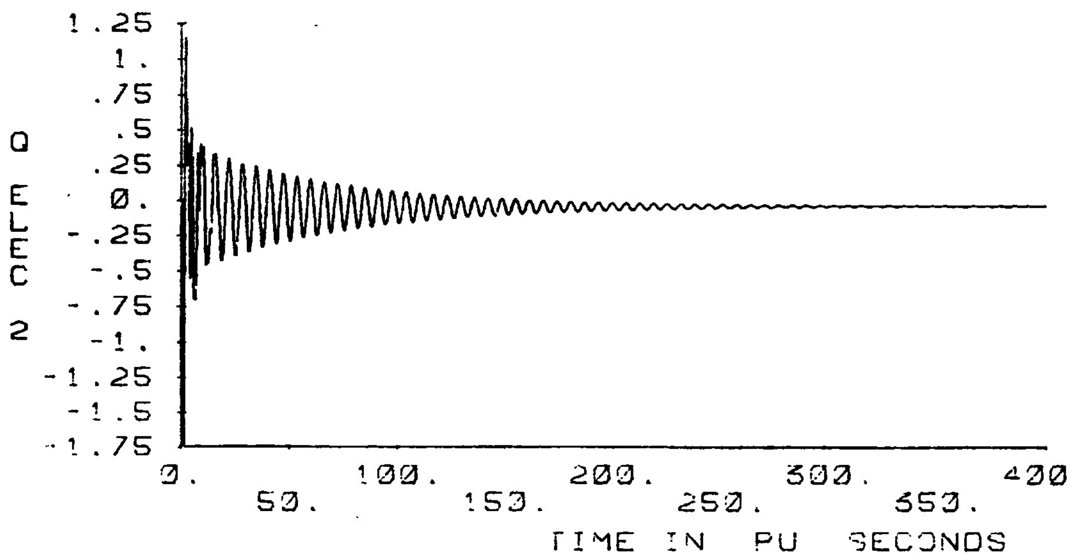


Figure 25(p) Machine Two reactive power output in per unit, with inductor.

3.8 Equilibrium Point and Eigenvalue Analysis

The instantaneous time solutions of the previous section will either tend towards a stable operating point or become unstable. If there is a stable operating point, it will occur when the derivatives of the state variables are zero. In order to determine the existence of a stable operating point, it is first necessary to solve the nonlinear algebraic equations obtained by setting all derivatives equal to zero in the system equations. If a solution is found, its stability must then be checked. This was done by linearizing the 38 algebraic equations around the equilibrium point to be tested for stability. The equilibrium points were found using Newton Raphson iteration and the linearized coefficient "A" matrix was formed. The eigenvalues of the resulting "A" matrix were computed and analyzed for stability. This was done for all combinations of 30 kw and 300 kw loads, 8 mile and 13 mile cables, with and without the 200 KVAR per phase shunt reactor. In addition, since the value of X_d is sensitive to excitation, the analysis was also done for machine parameters corresponding to $X_d = 1.75$. This is slightly higher than the manufacturers' listed 1.41 for operation at rated load.

The results indicated stable eigenvalues for all conditions when the reactor was included. Without the reactor, all 8 mile cable operation was stable. Without the reactor and with 13 miles of cable, the higher value of X_d gave an unstable operating point.

3.9 Field Tests

Based on the initial recommendations of this work, three single phase shunt reactors were ordered for installation directly on the generator common bus. The basic specifications were:

"Three (3) single phase shunt reactors rated 277 volts, 60 Hz, 200 KVAR with taps for 100 KVAR and 150 KVAR, and connected in a three phase bank for service on a 480/277 volt three phase grounded neutral system for power factor correction."

The Niagara Transformer Corporation provided iron core units for air cooling in a dust tight enclosure designed for operation outdoors at an elevation of 7000 feet above sea level. The shunt reactors were installed with relays such that when either generator activates the main generator bus, the shunt reactor would switch onto the main generator bus. When commercial power is restored, the shunt reactors are removed from the main generator bus manually before the main utility tie is reclosed.

On November 4, 1981, a field test was made on the shunt reactor installation. Reactive power meters had recently been installed on each generating unit. Strip chart recorders were set up to measure V_{AB} V_{BC} I_A I_B V_f and V_{AVR} for each machine. The utility power was disconnected, and both engines started after a 90 second delay. Both units came up to rated speed and voltage in about ten seconds. The shunt reactor did not come on line. The units synchronized and the units came on line followed by what appeared to be an immediate tripping of unit One. The strip charts show that both units were serving the load for a period of about 4 or 5 seconds. During this time, the field voltage for unit One decreased to zero and the field voltage for unit Two decreased to 40 volts. Unit One tripped at the point where its field voltage hit zero. The voltage regulator for unit Two had zero output when the tie breaker closed, but came on after about two seconds and maintained a significant output required for the 40 volts to the machine field. Generator Two carried the load for twenty minutes overloaded to around 700 amps per phase.

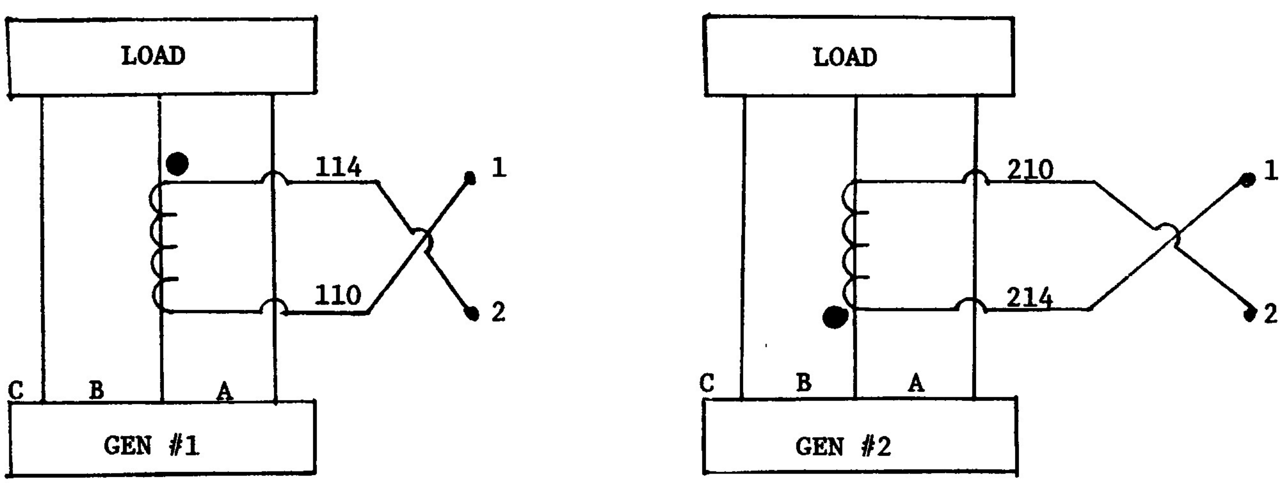
The engines were shut down, and the relay breaker trip mechanism for the shunt reactor was adjusted. The engines were started again and both units came up to rated speed and voltage in about three seconds. The shunt reactor did not come on line. The units synchronized and came on line in about fifteen seconds and served the load in parallel for about five seconds when unit One tripped out. This occurred as in test #1 only the unit tripped out with a field voltage of 10 volts compared to 60 volts on unit Two. The unstable line currents are visible growing quantities on the strip charts for unit One. Generator Two carried the load for ten minutes overloaded to around 800 amps per phase. While the two units were in operation, a severe imbalance in the reactive power output of each unit was observed on the newly installed reactive power meters. The engines were started again in order to observe each meter individually to determine if the meters were wired with the proper polarity. The meters read the same when energized individually. The units were shut down and the regulator wiring was examined and compared to the construction drawings. The parallel compensation circuits were checked and the as wired polarities are shown in Figures 26 and 27. The parallel compensation current transformer for unit #1 was looped on B phase in reverse polarity. Since the regulators had operated adequately with short lengths of load cable in the past, it was decided to test the units as wired with the reactor to make sure that the polarity needed to be reversed.

The shunt reactor was put on line manually. The engines were started and both units reached rated voltage and speed in about three seconds. The units

(a) Unit #1 (b) Unit #2

<u>Figure 26</u>

Voltage Sensing Transformer
(As wired)



(a) Unit #1 (b) Unit #2

<u>Figure 27</u>

Current Sensing Transformers
(As wired)

immediately paralleled and were automatically switched on to the load. The units stayed in parallel operation for eight minutes at which time they were shut down. During operation, unit One was producing reactive power while unit Two was absorbing reactive power with almost zero going to or coming from the load. The generator line currents were still near 700 amps each. Clearly, the reversed polarity on the parallel compensating current transformer of unit One was causing a large circulating reactive power out of One and into Two. This is verified by the strip chart recordings. During the parallel operation, the unit One field voltage was nearly 100 volts, while that of unit Two was 15 volts. The reverse polarity which caused unit One to be extremely underexcited with the cable load caused it to be extremely overexcited with the shunt reactor installed to give unity power factor.

Wire number 114 from the unit One parallel compensating current transformer polarity mark was moved from the voltage regulator terminal 2 to terminal 1. Wire number 110 was moved from terminal 1 to terminal 2. This was done since physically flipping the current transformer would have required disconnecting the main generator supply conductors. The units were started again with the reactor put on manually. They synchronized and came on line immediately. The generator line currents dropped to 200 amps each and the reactive power imbalance was considerably less, showing 100 out of unit One and 100 into unit Two and zero to the load. This is verified by the strip chart recordings which showed 65 volts on the field of unit One and 40 volts on the field of unit Two. This was considered a successful test as all of the load was served by both units in parallel in the 13 mile configuration with low generator currents.

The utility power was restored as the group attempted to determine the reason why the shunt reactor did not come on line as designed. The drive motor arm on the shunt reactor breaker was found to be inoperative and adjusted. The utility power was then disconnected and a final test was performed. The engines started and the generators came up to rated speed and voltage. The shunt reactor attempted to come on line but did not. The flag was half way down indicating the drive motor either did not have full voltage or was only energized for a small amount of time. Only unit One came on line and it could not supply the load. The control building load had not shunt tripped when the utility power was disconnected. Unit One was disconnected and allowed to synchronize with unit Two before reclosing the tie breaker manually. The units operated in parallel without the shunt reactor for ten minutes, at which time the reactor came on line without any manual activation. The units were both shut down, and utility power was restored.

IV. CONCLUSIONS AND RECOMMENDATIONS

There were two causes of the major problems with the parallel operation of the two generators at the VLA site. The basic problem of extreme leading power factor load was eliminated successfully with the installation of the shunt reactor on the generator terminals. This allows either generator One or Two to supply the total emergency load with full cable connection for extended periods of time without overload. The main generator step up transformer loading should be monitored to ensure that this loading is not excessive in future runs. Since the reactors are on the low voltage side, the step up transformer will be required to carry the same load as when the reactors were not installed. The reactors serve only to unload the generators, and not the main transformer.

The generators do not currently have under/over frequency relays or under/over voltage relays for emergencies. These should be considered for protection of the generators and load in the case when load shunt trips malfunction or other failures occur.

The second cause of the problems with the parallel operation was the incorrect wiring of the parallel compensating current transformer on generator One. Although this did not cause instabilities at light reactive power loading, it did cause excessive current circulation which was not apparent until the reactive power meters were installed. Even when the polarity was corrected, a slight imbalance was still present. This can be eliminated by adjusting the current compensating resistor R25 of the Basler regulator. It may have been possible to fine tune the regulators to allow automatic parallel operation without the shunt reactors after the polarity was corrected. This still would not have allowed single unit operation during outages and would thus have eliminated the original reason for having two generators as a reliability factor.

The engineering drawings of the control wiring have considerable errors. These should be corrected to agree with the existing wiring.

Recent conversations with site personnel indicate that the shunt reactor trip was prohibited by a defective relay that was replaced. Subsequent tests and actual outages have resulted in normal operation of all standby power equipment.

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