VLA SCIENTIFIC REPORT 111

PERFORMANCE PARAMETERS VERSUS SYSTEM PARAMETERS FOR THE VLA

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The influence of various parameters of the VLA system on beamwidth, minimum detectable flux density, and minimum detectable brightness temperature is described below in a "cookbook" manner. Only continuum observations are considered. The report concludes with a brief remark about sidelobe levels.

I. Notation, Units, and Numerical Values.

The quantities used in the equations are defined in the table below. The system parameters are considered in more detail after the table. Each quantity is expressed in "practical" units, in other words the units which seem to me to be the least inconvenient. The first five entries in the table are performance parameters, while the remainder are system parameters or constants or items to be chosen by the observer.

Quantity	Units	Definition
β_s	arc seconds	half-power width of the synthesized beam
β_d	arc minutes	half-power width of the delay beam
β_p	arc minutes	half-power beamwidth of a single antenna
S_m	flux units	minimum detectable flux density
T_m	^oK	minimum detectable brightness temperature
λ	cm	operating wavelength
B	MHz	effective bandwidth
T_R	^oK	system noise temperature
L	km	length of a single array arm
d	m	diameter of a single antenna
ε	--	aperture efficiency of an antenna
E	--	"array aperture efficiency"
M	--	number of antennas
H	hours	duration of an observation
q_n	--	signal-to-noise ratio required for detection
k	W deg^{-1} Hz^{-1}	Boltzmann's constant $= 1.38 \times 10^{-23}$ in MKS units

(a) Wavelength (λ) and system temperature (T_R): Present plans call for the following:

Frequency	Nominal λ	T_R
1.35 - 1.72 GHz	20 cm	35 ^oK
4.5 - 5.0 "	6.3 "	40 ^oK
14.4 - 15.4 "	2.0 "	200 ^oK
22 - 24 "	1.3 "	250 ^oK

(b) Array arm length (L): The original plan called for four array sizes, differing successively by factors of three in physical extent. In a recent memorandum, Barry Clark has argued that a factor of 40/11 = 3.636...would be better. The arm lengths are as follows:

Configuration	L	
	Original	Clark
A	21 km	21 km
B	7 "	5.775 "
C	2.333 "	1.588 "
D	0.778 "	0.437 "

We shall use both sets in the graphs of β_s, β_d, and T_m.

(c) Bandwidth (B): There will be two approximately rectangular pass-bands. Each will have a nominal width of 45 MHz, and each will have its own set of delay lines. In calculating the delay beamwidth, the single-channel bandwidth should be used. In evaluating S_m and T_m, however, one should use the sum of the bandwidths, weighted by a factor required by the two-bit sampling. The effective bandwidth in this case is

$$B = 0.8 \times 2 \times 45 = 72 \text{ MHz} .$$

(d) <u>Array aperture efficiency (E)</u>: This depends on the weighting function applied to the Fourier components; it is defined by

$$E^2 = \bar{w}^2 / \overline{w^2} .$$

We use two values:

$$E = \begin{cases} 1 & \text{(uniform weighting; for } S_m) \\ 0.74 & \text{(15 db gaussian taper; for } T_m) \end{cases}$$

(e) <u>Quantities treated as constants</u>:

$$d = 25 \text{ m}$$
$$\varepsilon = 0.5$$
$$q_n = 5$$

II. <u>Beamwidths</u>.

There are three half-power beamwidths which are relevant. Ignoring foreshortening effects, these are the widths of:

(a) <u>The synthesized beam</u>: For a 15-db gaussian taper,

$$\beta_s = 2''\!.1 \, \lambda \, L^{-1} .$$

This is shown in Fig. 1 for the four values of λ and the VLA range of L.

(b) <u>The delay beam</u>: This is

$$\beta_d = 1250' \, B^{-1} \, L^{-1}$$

where B is the single-sideband bandwidth. Since B = 45 MHz in this case, we have

$$\beta_d = 27\overset{\prime}{.}67\ L^{-1} .$$

This is graphed in Fig. 2. Note that β_d is independent of wavelength.

(c) <u>The primary beam</u>: The primary beam is the power pattern of a single array element. Its half-power width is

$$\beta_p = 42\overset{\prime}{.}7\ \lambda\ d^{-1} .$$

For d = 25 m,

$$\beta_p = 1\overset{\prime}{.}707\ \lambda .$$

At the standard VLA wavelengths, we will have;

λ	β_p
20 cm	34'
6.3 "	10'.8
2.0 "	3'.4
1.3 "	2'.2

The net effective half-power beamwidth of the system is

$$\beta = (\beta_s^{-2} + \beta_d^{-2} + \beta_p^{-2})^{-1/2}$$

where all of the β's are in the same units.

III. <u>Minimum Detectable Flux Density and Minimum Detectable Brightness Temperature</u>.

In MKS units, the general expressions for these quantities are

$$S_m = \frac{2^{7/2}k}{\pi} \frac{q_n T_R}{\varepsilon E d^2 \sqrt{B t M (M-1)}}$$

$$T_m = \frac{2^{5/2}k}{\pi} \frac{q_n T_R}{\varepsilon E \sqrt{B t M (M-1)}} \left(\frac{L}{d}\right)^2$$

where t is the observing time in seconds. These expressions follow from Chapter 7 of the first volume of the VLA Proposal.

Using the practical units and numerical values from §I., we have

$$S_m = 1.56 \times 10^{-4} \frac{T_R}{\sqrt{H M (M-1)}}$$

$$T_m = 0.0765 \frac{T_R L^2}{\sqrt{H M (M-1)}}$$

For the full VLA, M = 27 and therefore

$$S_m = 5.90 \times 10^{-6} T_R H^{-1/2}$$

$$T_m = 2.85 \times 10^{-3} T_R H^{-1/2} L^2$$

Figure 3 shows S_m as a function of H for each standard wavelength (for M = 27). Figure 4 is more complicated (or rather, cluttered). It shows (again for M = 27) T_m as a function of L at each wavelength, for two different observing times:

$$H = \begin{cases} 1 \text{ hour} & \text{(dashed lines)} \\ 10 \text{ hours} & \text{(full lines)} \end{cases}$$

IV. Sidelobes.

The sidelobe levels depend on source declination, the distribution of the antennas along the arms of the array, and the weighting of the Fourier components. Other things being equal, the sidelobe level tends to vary inversely as the square root of the observing time and inversely as the first power of the number of antennas:

$$\text{sidelobe level} \propto H^{-1/2}\, M^{-1} \; .$$

Fig. 1

Synthesized Beamwidth as a Function of Arm Length and Wavelength (15 db gaussian taper)

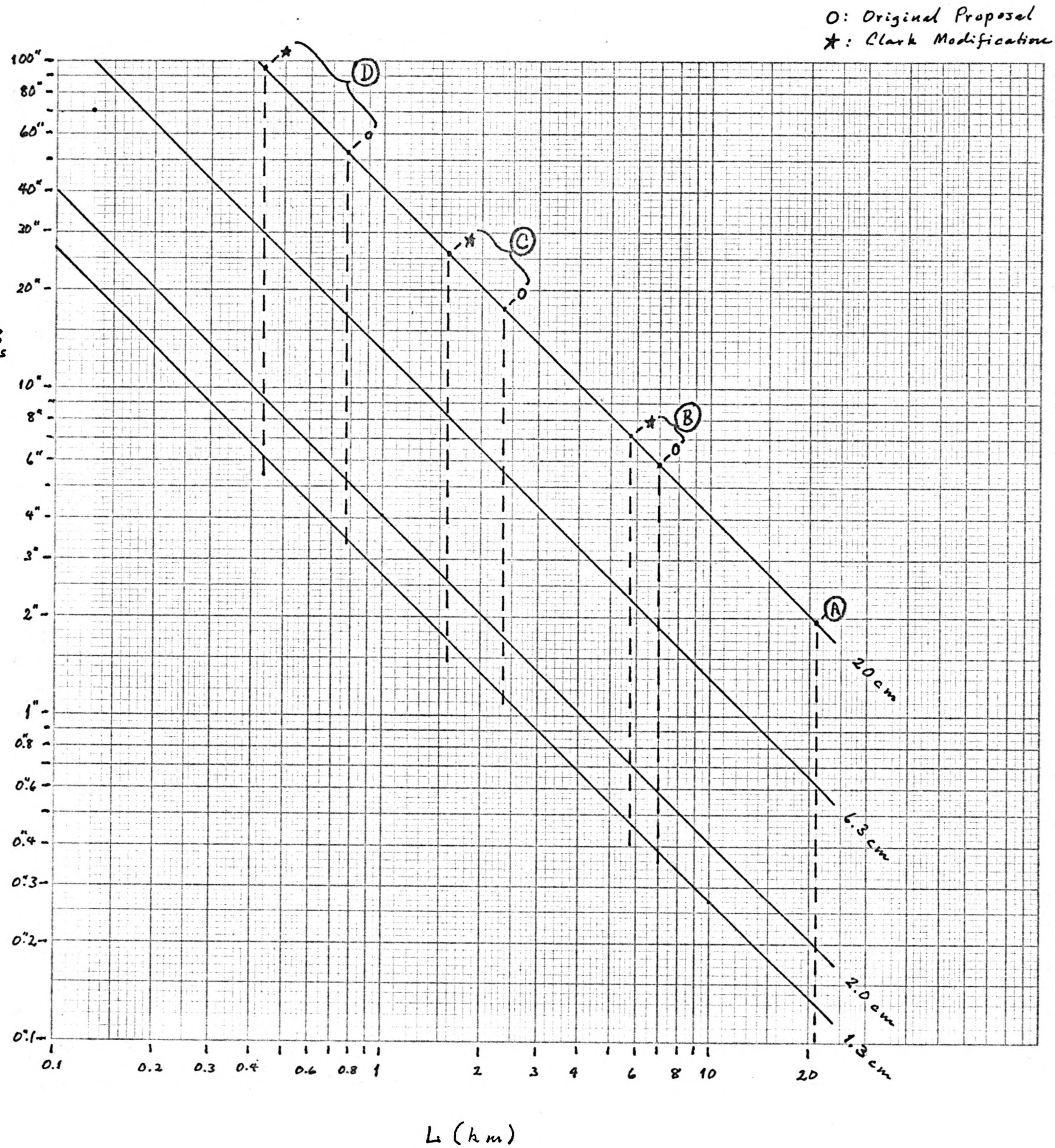
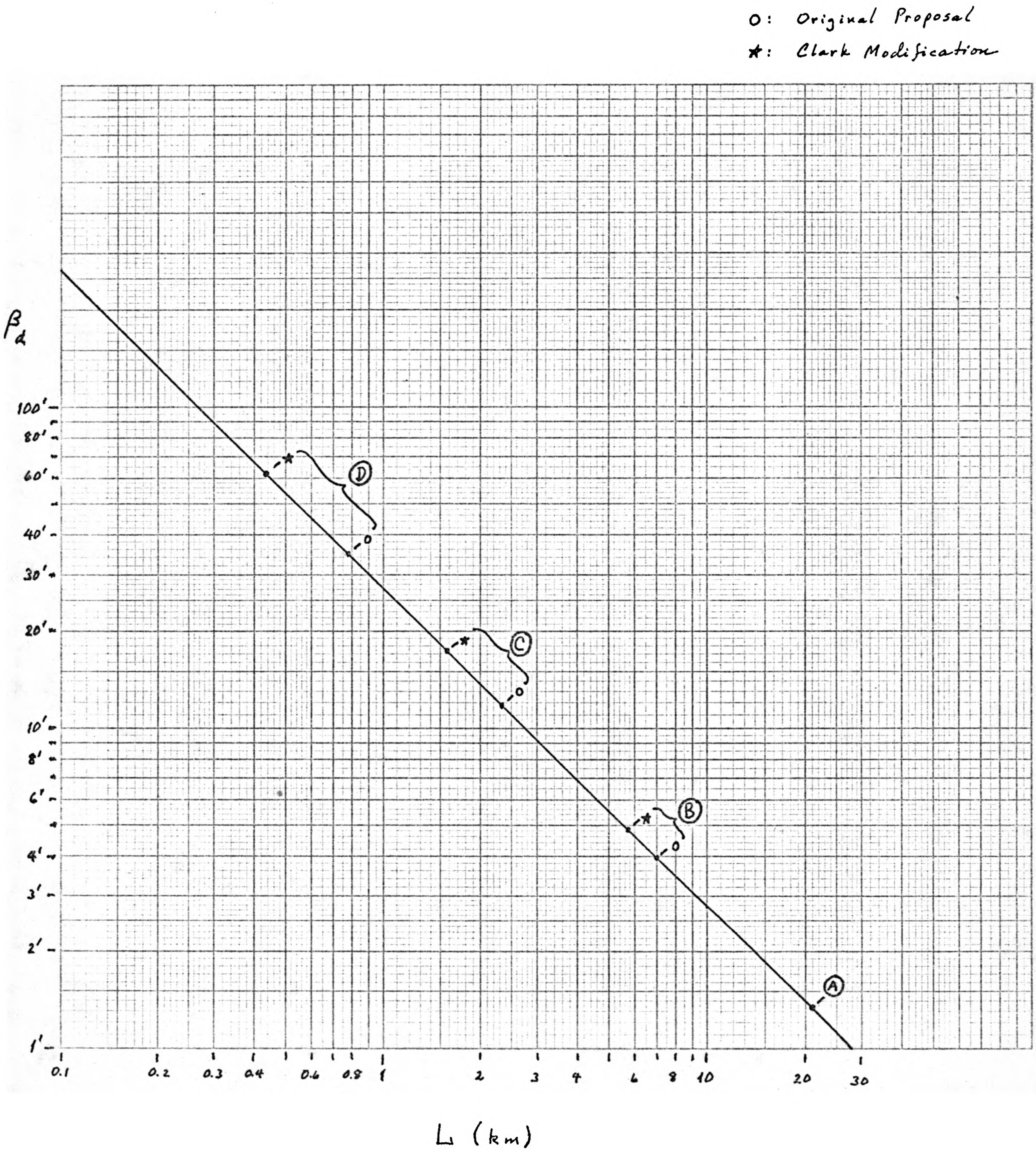


Fig. 2

Half-power Width of the Delay Beam



Minimum Detectable Flux Density as a Function of Observing Time and Wavelength

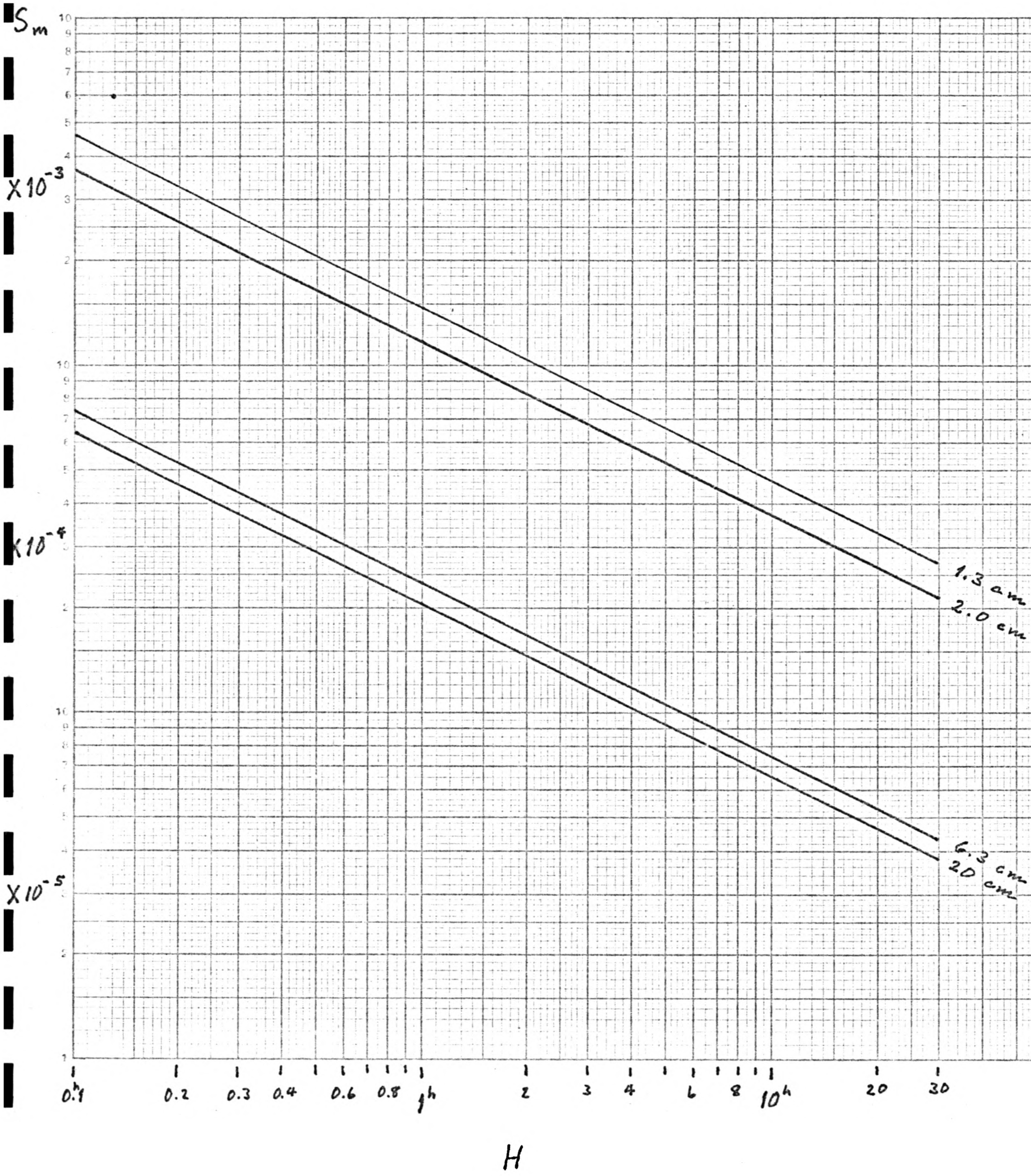


Fig. 4

Minimum Detectable Brightness Temperature as a Function of Arm Length, Wavelength, and Observing Time

O: Original Proposal

*: Clark Modification