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VLA CONFUSION AND SENSITIVITY LIMITS FOR WIDE FIELDS AS A FUNCTION OF BANDWIDTH

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Introduction

There is a variety of experiments in which it will be desirable to have the VLA map large fields with high sensitivity. These include extending the counts of extragalactic sources to lower flux densities, and studies of clusters of galaxies and star clusters. For high sensitivity it is usually desirable to use as much bandwidth as possible; but for points in the field not near the delay tracking center, both the sensitivity and resolution are reduced as the bandwidth is increased. Thus there appears to be a tradeoff between sensitivity and useful field of view. In this memo, I consider the magnitude of the effect for the VLA.

The effect can be overcome by using multichannel correlators (spectral line system) to achieve large total bandwidth with small channel bandwidth. In that case, it is important to know how many channels are needed.

Calculations

With linear mapping algorithms (e.g., either direct or gridded Fourier transform methods), the response at map position (x,y) to a point source at (x',y') is given very nearly by

$$P(x,y,x',y') = A(x',y') \int_{-\infty}^{\infty} P_o\left(x-\frac{f}{f_o}x', y-\frac{f}{f_o}y'\right)G(f)\, df \qquad (1)$$

where $P_o(x,y)$ is the monochromatic point source response for frequency f_o ; G(f) is the instrumental bandpass function of frequency; A(x',y') is the single antenna gain; and (x,y), (x',y') are direction cosines with respect to the delay trading direction (Thompson 1973, D'Addario 1974). It should be apparent from (1) that the effect of non-zero bandwidth is to produce a smearing of the point source response which is purely radial in the xy-plane, and which is proportional to the distance from the origin.[1]

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[1] As is now fairly well known, the obsolete concept of a "delay beam", introduced to describe this effect for a single interferometer, is awkward and misleading when multi-baseline synthesis instruments are involved. The description used here holds quite generally.

We make the following approximate calculations based on (1). For a rectanglar passband

$$G(f) = \begin{cases} \frac{1}{\Delta f}, & |f-f_o| \le \frac{\Delta f}{2} \\ 0, & \text{otherwise} \end{cases} \qquad (2)$$

the radial FWHM of P(x,y, x',y') is

$$\delta = r\,\frac{\Delta f}{f_o} + \delta_o \qquad (3)$$

where $r = ||(x',y')||$ is the distance from the origin and δ_o is the FWHM of $P_o(x,y)$. [P_o is assumed sharply peaked and roughly circular.] Furthermore, since

$$\iint_{\text{main lobe}} P(x,y,x',y')\,dxdy \simeq \iint_{\text{main lobe}} P_o(x,y)\,dxdy,$$

we see that the peak response is reduced by the factor

$$\frac{P_{max}}{P_{o\,max}} = \frac{A(r)}{A(o)}\,\frac{\delta_o}{\delta} = \frac{A(r)}{A(o)}\,\frac{1}{1+\frac{r}{\delta_o}\frac{\Delta f}{f_o}} \qquad (4)$$

so that the sensitivity in detecting a point source is reduced by the same factor, compared with the sensitivity at the field center r = 0.

The point source sensitivity depends on the noise in the map; the latter depends on receiver noise, calibration errors, atmospheric effects, confusion, and the mapping algorithm. Assume a direct Fourier transform algorithm with uniform weighting (for maximum point source sensitivity). Then the rms map noise from receiver noise alone is, in units of the r = 0 peak response to a unit-flux point source,

$$\sigma_o = \frac{\sqrt{2}k\ Tsys}{A_{eff}}\ \frac{1}{\sqrt{\Delta f\tau\ N_{BL}}} \qquad (5)$$

where T_{sys} and A_{eff} are the system temperature and effective area of a single antenna, τ is the observing time, and N_{gL} is the number of baselines (both polarizations observed). Combining (4) and (5) we see that the point source flux needed to give a 5σ detection at any r is

$$S_{min} = 5\,\frac{2k\ T_{sys}}{A_{eff}}\ \frac{1}{\sqrt{\tau N_{BL}}}\ \frac{A(r)}{A(o)}\left(1+\frac{r}{\delta_o}\frac{\Delta f}{f_o}\right)(\Delta f)^{-1/2}. \qquad (6)$$

Results For the VLA

From (3) and (6) it is clear that the effects are worst at long wavelengths, where both the fractional bandwidth $\Delta f/f_o$ and radius r (limited by single antenna beam) can be largest. Therefore, we consider VLA operation at λ = 21 cm, f_o = 1400 MHz.

Let $r = \lambda/2D$, where D is the dish diameter, be called the edge of the field (approximately the single dish half power point). Approximate the central beamwidth by $\delta_o = \lambda/L$, where L is the array arm length. Then taking D = 25m, T_{sys} = 50 K, τ = 12 hours, $A_{eff} = .5\,\pi\,D^2/4$, and N_{BL} = 350, (3) and (6) become

$$\delta_e = 2.102'' \cdot (\frac{21\text{km}}{L})(1 + 3 \times 10^{-7}\,\frac{L}{21\text{km}}\,\frac{\Delta f}{1\text{Hz}}) \qquad (7)$$

and

$$S_{min,e} = .360 \text{ Jy } (1 + 3 \times 10^{-7}\,\frac{L}{21\text{km}}\,\frac{\Delta f}{1\text{Hz}})\,(\frac{\Delta f}{1\text{Hz}})^{-1/2} \qquad (8)$$

when evaluated at $r = \lambda/2D$ (indicated by subscript e).

δ_e and $S_{min,e}$ are plotted in Figs. 1 and 2 as functions of Δf for the A,B, and C configurations of the VLA (L = 21 km, 7 km, 2.1 km).

To determine whether confusion will be significant, take the following model for the integrated source count:

$$N_E = \begin{cases} N_o(S/S_o)^{-1.5}, & S \geq S_o \\ N_o(S/S_o)^{-1}, & S < S_o\ . \end{cases} \qquad (9)$$

(The weak source exponent is probably closer to −0.8 than −1.0, but we prefer to overestimate rather than underestimate the weak source count.) Here N_E is the number of sources per sterrod with flux greater than S. At 1400 MHz, N_o = 6300 and S_o = 0.1 Jy (Fomalont, Bridle, and Davis 1974).

Based on (9), Fig. 3 shows the expected number of sources per beam area stronger than 5σ as a function of bandwidth, at the edge of the field. For each configuration, this measure of confusion is nearly independent of r because of the canceling effects of the broadened beam and decreased sensitivity when $N_E \propto S^{-1}$. Confusion is slightly less near the edge because of the fall-off of single-antenna gain. In this model, configurations A and B are seen to be free of confusion up to at least Δf = 50 MHz.

This discussion has not considered the confusion arising from the sidelobes of strong sources within the field. An early study by Fomalont (1972) indicates that this may be the dominant mechanism limiting the detectability of weak sources. It may be possible to eliminate such sidelobes by carefull data processing, but probably not by simple source subtraction ("cleaning") techniques because the sidelobes will result mainly from instrumental and atmospheric phase errors rather than from the array geometry.

Although the B configuration is not confusion limited, the sensitivity at the edge of the field is worse by a factor of 12 than it is at the center at 50 MHz bandwidth. Nearly the full sensitivity could be achieved over the whole field by using a multi-spectral channel correlator and processing each channel separately. Table 1 gives the number of channels required to keep the sensitivity loss to a factor of 3 at 50 MHz (note that a factor of 2 is due to the single dish beam).

TABLE 1

CONFIGURATION	CHANNEL BANDWIDTH FOR BEAM BROADENING BY 50% AT FIELD EDGE	NO. OF CHANNELS FOR 50 MHz TOTAL BANDWIDTH
A	1.7 MHz	30
B	5.0	10
C	16.7	3
D	50.	1

Conclusions

Using the B configuration at 21 cm with the continuum system and 50 MHz bandwidth, it is possible to extend the source counts to ~500 μJy without significant confusion in the synthesized beam. A 10-channel correlator will allow the flux limit to be pushed to ~50 μJy over the entire single-dish beam (29 arcmin), and will also allow mapping of extended sources without serious distortion. Similar performance with the A configuration requires a 30-channel correlator.

References

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Fomalont, E., Bridle, A. H. and Davis, M. M. 1974, "Improved count of radio sources at 1400 MHz", Astron. Ap., **36**, 273.

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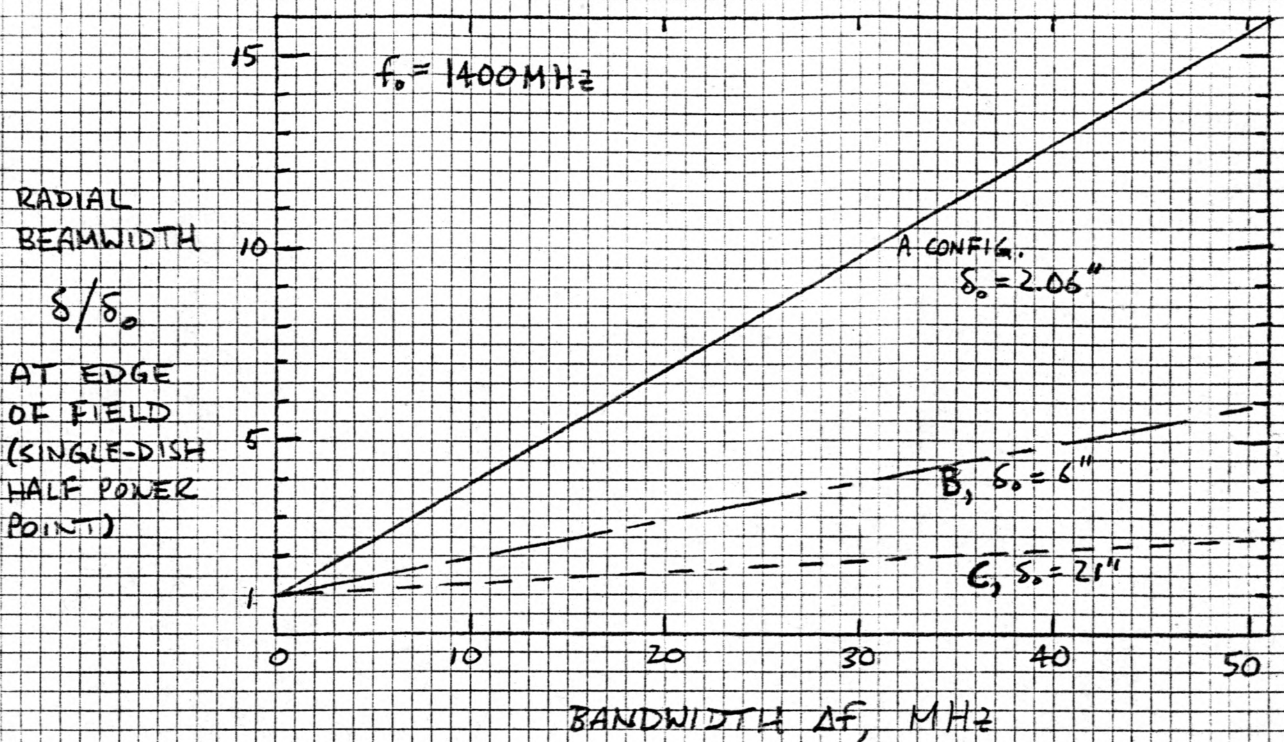

FIGURE 1: RADIAL BEAMWIDTH VS BANDWIDTH. δ_0 IS THE MONOCHROMATIC BEAMWIDTH, WHICH IS ALSO THE BEAMWIDTH AT THE FIELD CENTER AND THE AZIMUTHAL BEAMWIDTH FOR ALL Δf.

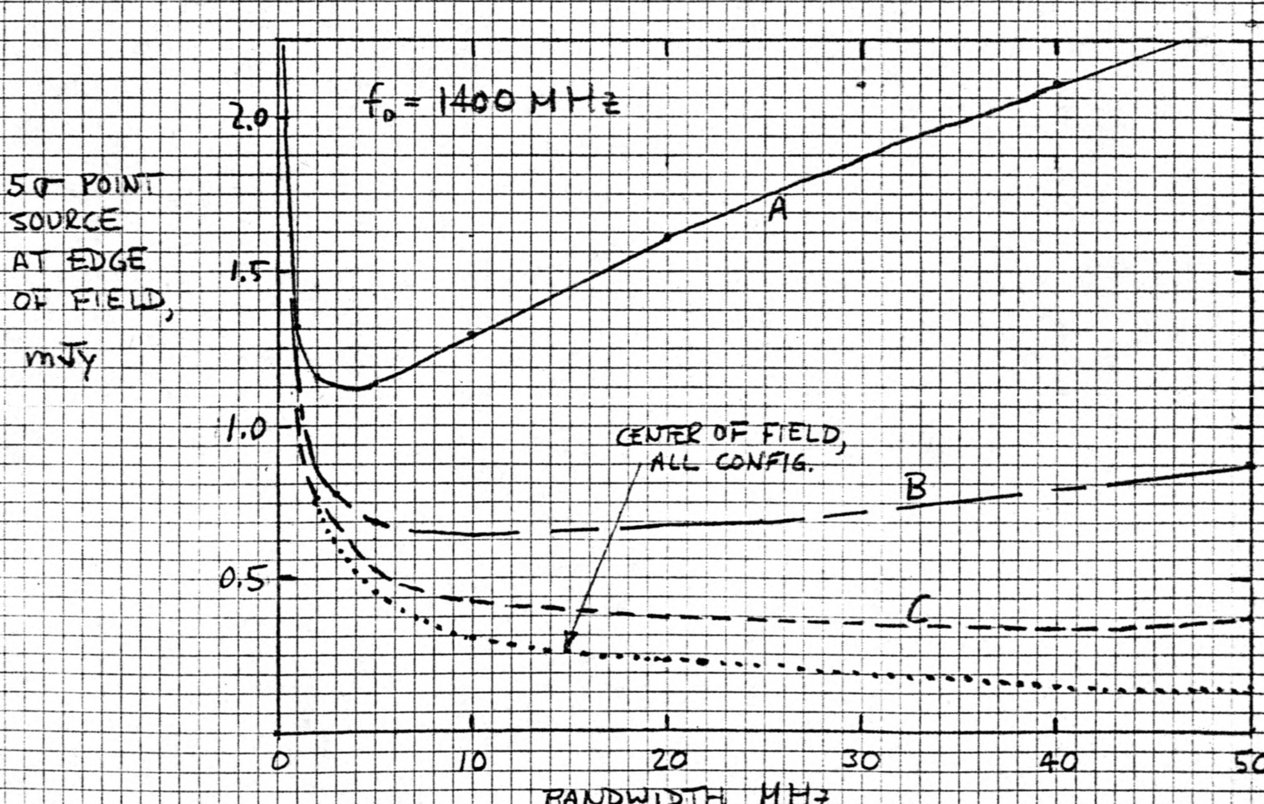

FIGURE 2: POINT SOURCE SENSITIVITY VS. BANDWIDTH.

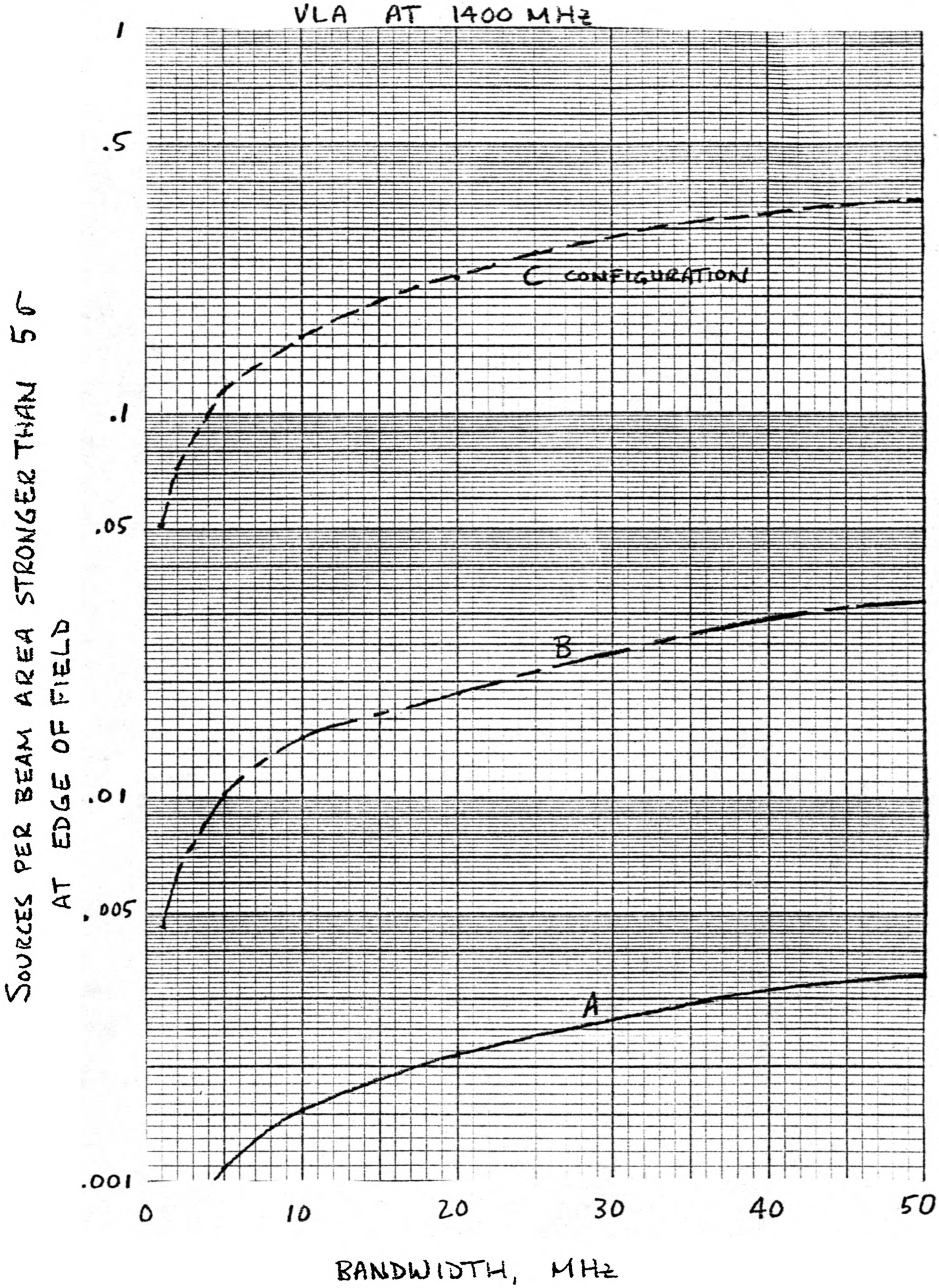

FIGURE 3: CONFUSION VS. BANDWIDTH BASED ON SOURCE COUNT MODEL $N_E \propto S^{-1}$ FOR $S < 0.1$ Jy.