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Effect of the General Relativity Deflection on the Apparent Position of an Object

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It is pointed out that the bending of rays in the gravitational field of the sun is large enough over most of the sky to require that it be taken into account in calculating apparent source positions for the VLA. The necessary mathematical expressions are derived.

I. INTRODUCTION

According to VLA Computer Memorandum 105, the synchronous programs will make the reduction from the mean to the apparent position of a radio source with an internal accuracy of 0ʺ002. This involves the classical adjustment of the coordinates for precession, nutation, and aberration.

A further effect which is significant for the VLA, but generally ignored in optical astrometry, is the relativistic bending of rays in the sun's gravitational field. The deflection is 1ʺ75 at the sun's limb and decreases almost linearly with angular distance from the sun. According to Brandt (1974), the magnitude of the deflection, which is radially away from the sun's center, is

$$\theta = k \left(\frac{1 + \cos D}{\sin D} + 1/4 \sin 2D \right) \qquad (1)$$

where D is the angular distance from the sun, and $k = 1.9742 \times 10^{-8}$ radians = 0ʺ0040720 is the deflection at $D = 90^{\circ}$. Table 1, taken from Brandt's paper, gives the deflection for various values of D. It is evident that the deflection is as large as 0\".002 anywhere within nearly 120° of the sun. Therefore allowance must be made for it if the intended accuracy of the VLA ephemeris routines is to be achieved.

Table 1

D	θ	D	θ	D	θ
0°16'	1\".7498	10°	0\".0469	90°	0\".0041
1°	0\".4666	15°	0\".0314	105°	0\".0026
2°	0\".2334	30°	0\".0161	120°	0\".0015
3°	0\".1556	45°	0\".0108	135°	0.0007
4°	0\".1167	60°	0\".0079	150°	0\".0002
5°	0\".0934	75°	0\".0058	165°	0\".0000

II. THE RELATIVISTIC DEFLECTION IN DIRECTION COSINE FORM

The problem is treated most directly in terms of three unit vectors, which are coplanar since the relativistic shift is radially away from the sun. Two of the vectors are known *a priori*; these are the position of the sun

$$\vec{\odot} = \begin{vmatrix} \cos\delta_\odot \cos\alpha_\odot \\ \cos\delta_\odot \sin\alpha_\odot \\ \sin\delta_\odot \end{vmatrix} = \begin{vmatrix} \odot_x \\ \odot_y \\ \odot_z \end{vmatrix}$$

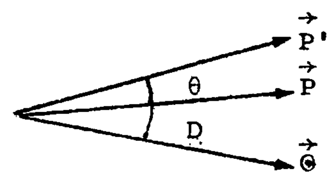
and the undeflected position of the source

$$\vec{P} = \begin{vmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{vmatrix} = \begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} .$$

The unknown vector is the deflected position of the source,

$$\vec{P}' = \begin{vmatrix} \cos\delta' \cos\alpha' \\ \cos\delta' \sin\alpha' \\ \sin\delta' \end{vmatrix} = \begin{vmatrix} P'_x \\ P'_y \\ P'_z \end{vmatrix} .$$

The relation of the vectors is shown by



Two equivalent expressions for a unit vector perpendicular to the plane of $(\vec{P}', \vec{P}', \vec{\odot})$ are

$$\frac{\vec{P}' \times \vec{P}}{|\vec{P}' \times \vec{P}|} = (\vec{P}' \times \vec{P})/\sin\theta \qquad (2)$$

and

$$\frac{\vec{P} \times \vec{\odot}}{|\vec{P} \times \vec{\odot}|} = (\vec{P} \times \vec{\odot})/\sin D. \qquad (3)$$

Equating the right-hand sides of (2) and (3) and taking the vector product with $\vec{P}$, one has

$$\vec{P} \times (\vec{P}' \times \vec{P}) = \frac{\sin\theta}{\sin D}\ \vec{P} \times (\vec{P} \times \vec{\Theta}).$$

Using the vector identify $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$, this becomes

$$\vec{P}'(\vec{P}\cdot\vec{P}) - \vec{P}(\vec{P}'\cdot\vec{P}) = \frac{\sin\theta}{\sin D}\left[\vec{P}(\vec{P}\cdot\vec{\Theta}) - \vec{\Theta}\ (\vec{P}\cdot\vec{P})\right].$$

Since $\vec{P}\cdot\vec{P} = 1$, $\vec{P}'\cdot\vec{P} = \cos\theta$, $\vec{P}\cdot\vec{\Theta} = \cos D$, one has finally

$$\vec{P}' = \vec{P}\cos\theta + \frac{\sin\theta}{\sin D}(\vec{P}\cos D - \vec{\Theta}). \qquad (4)$$

Because θ is always a very small angle, one can set $\cos\theta = 1$ and $\sin\theta = \theta$, whereupon (4) becomes

$$\vec{P}' = \vec{P} + \mu(\vec{P}\cos D - \vec{\Theta}), \qquad (5)$$

where

$$\mu \equiv \frac{\theta}{\sin D} = k\left(\frac{1}{1 - \cos D} + 1/2\cos D\right).$$

Obviously, the effect of the shift can now be expressed in direction cosine form as

$$\Delta\vec{P} = \begin{vmatrix} P'_x - P_x \\ P'_y - P_y \\ P'_z - P_z \end{vmatrix} = \mu \begin{vmatrix} P_x\cos D - \Theta_x \\ P_y\cos D - \Theta_y \\ P_z\cos D - \Theta_z \end{vmatrix} \qquad (6)$$

III. THE EFFECT ON APPARENT RIGHT ASCENSION AND DECLINATION

Differentiating the direction cosines of P, one has

$$\Delta\vec{P} = \begin{vmatrix} -\Delta\delta \sin\delta \cos\alpha - \Delta\alpha \cos\delta \sin\alpha \\ -\Delta\delta \sin\delta \sin\alpha + \Delta\alpha \cos\delta \cos\alpha \\ \Delta\delta \cos\delta \end{vmatrix} \quad (7)$$

Equating the right-hand sides of (6) and (7), and solving for $\Delta\alpha$ and $\Delta\delta$, one finds

$$\Delta\alpha = \mu \sec\delta \cos\delta_\odot \sin(\alpha - \alpha_\odot)$$

$$\Delta\delta = \mu \left[\sin\delta \cos\delta_\odot \cos(\alpha - \alpha_\odot) - \sin\delta_\odot \cos\delta\right]$$

These are equivalent to eqs. (18) - (20) of Brandt's paper.

References

Brandt, V. E. 1974, Astronomicheskii Zhurnal 51, 1100 (English translation in Soviet Astronomy 18, 649, 1975).