NATIONAL RADIO ASTRONOMY OBSERVATORY
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PHASE ERRORS DUE TO BASELINE AND CALIBRATOR POSITION OFFSETS

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This report will show some phase error calculations and will present these errors as a component that results in an apparent source position shift, plus a component that adds random phase errors to the data.

IA. Calibrator Position Errors

The phase of the calibrator, ϕ_c, is given by

$$\phi_c = B^T \cdot \delta S_c + C$$

where δS_c = calibrator position error vector and

C = is the instrumental phase

B = baseline vector (matrix notation).

Assuming there is no source position offset, the calibrated source phase is,

$$\phi_s - \phi_c = -B^T \cdot \delta S_c = B^T \cdot \delta S_s + \phi_n$$

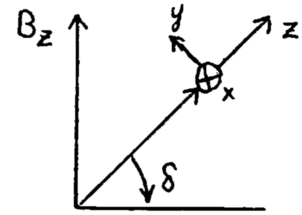
which we attempt to model as a source postion offset, δS_s plus a noise term.

The problem is that we know δS_c in its local sky frame,

$$\delta S_c = \begin{pmatrix} X_c \\ Y_c \\ 0 \end{pmatrix},$$

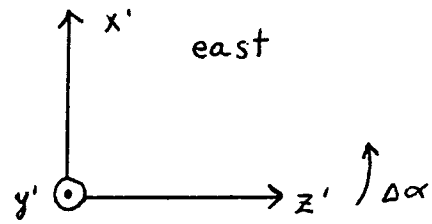
and want to express it in the tangent frame of the source. First rotate about x (sky) to align the y (sky) axis with z (earth) axis

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{pmatrix}$$



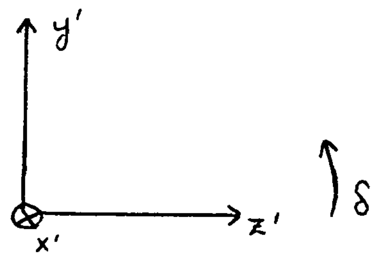
where δ is the declination of the calibrator. Then rotate about the new y' axis = z (earth) by Δα, the r.a. difference of calabrator to source.

$$R_{y'} = \begin{pmatrix} \cos\Delta\alpha & 0 & -\sin\Delta\alpha \\ 0 & 1 & 0 \\ \sin\Delta\alpha & 0 & \cos\Delta\alpha \end{pmatrix}$$



Next rotate about the x' axis to get to the sky frame of the source.

$$R_{x'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\delta' & -\sin\delta' \\ 0 & \sin\delta' & \cos\delta' \end{pmatrix}$$



where δ' is the declination of the source. The full rotation is

$$R = R_{x'}\, R_{y'}\, R_x.$$

Doing the multiplication:

$$R = \begin{pmatrix} \cos\Delta\alpha & \sin\Delta\alpha\,\sin\delta & -\sin\Delta\alpha\,\cos\delta \\ -\sin\Delta\alpha\,\sin\delta' & \cos\delta\,\cos\delta' + \cos\Delta\alpha\,\sin\delta\,\sin\delta' & \sin\delta\,\cos\delta' - \cos\Delta\alpha\,\cos\delta\,\sin\delta' \\ \sin\Delta\alpha\,\cos\delta' & \cos\delta\,\sin\delta' - \cos\Delta\alpha\,\sin\delta\,\cos\delta' & \sin\delta\,\sin\delta' + \cos\Delta\alpha\,\cos\delta\,\cos\delta' \end{pmatrix}$$

To make things a bit more pleasant, use small angle approximations for Δα and Δδ, where $\delta' = \delta + \Delta\,\delta$

$$R = \begin{pmatrix} 1 & \Delta\alpha\,\sin\delta & -\Delta\alpha\,\cos\delta \\ -\Delta\alpha\,\sin\delta & 1 & -\Delta\delta \\ \Delta\alpha\,\cos\delta & \Delta\delta & 1 \end{pmatrix}$$

to 1st order in Δα, Δδ.

So putting everything in the sky frame of the source,

$$\phi = -B^T \cdot (R\, \delta S_c)$$

$$= -(U,\ V,\ W)\ R \begin{pmatrix} X_c \\ Y_c \\ 0 \end{pmatrix}$$

$$= -(U,\ V,\ W) \begin{pmatrix} X_c + Y_c\ \Delta\alpha\ \sin\delta \\ Y_c - X_c\ \Delta\alpha\ \sin\delta \\ 0 + X_c\ \Delta\alpha\ \cos\delta + Y_c \Delta\delta \end{pmatrix} = B^T \cdot \delta S_s + \phi_n$$

The terms that multiply U and V correspond to a source translation,

$$X_s = -X_c - Y_c\ \theta$$
$$Y_s = -Y_c + X_c\ \theta$$

which is the calibrator position error rotated by $\theta = \Delta\alpha \sin\delta$ plus a W term that adds phase noise

$$\phi_n = W\ (X_c\ \Delta\alpha\ \cos\delta + Y_c\ \Delta\delta).$$

IB. Example

For: $X_c = 1'' \sim 5 \times 10^{-6}$ rad

$\Delta\delta \sim \Delta\alpha \cos\delta \sim 10^\circ \sim .2$ rad

$U \sim V \sim W \sim 35$ Km $\sim 6 \times 10^5\ \lambda$ at 6 cm.

ϕ translation $\sim X_c\ U \sim 3$ lobes

$\phi_n \sim W\ X_c\ \Delta\alpha\ \cos\delta \sim 6 \times 10^5 \cdot 5 \times 10^{-6} \cdot .2$

$\sim .6$ lobes. $\sim 100^\circ$.

IC. Note:

1) ϕ_n corresponds to a slowly varying phase error which is not removed by the calibration and is somewhat random for the whole array over a full synthesis. This phase error can be quite large for the A configuration with a lousy calibrator.

2) The rotation of the offset is zero if $\Delta\alpha = 0$ or $\delta = 0$. The magnitude of the offset is not exactly preserved; the small angle approximation is somewhat marginal.

3) The phase distortion, $\phi_n = 0$ if $\Delta\alpha = \Delta\delta = 0$, and ϕ_n will increase linearly with source-cal separation and calibrator position error.

4) The ϕ_n is antenna based and will be removed by SELCAL.

IIA. Baseline Errors

Errors in the antenna positions (baseline errors) also add phase noise and result in position errors. The calibrated phase, with no source position errors, is given by

$$\phi_s - \phi_c = \delta B^T \cdot (S_s - S_c) \text{ where}$$

$\delta B = B - B_o$ the baseline error.

S_s = the source position vector.

S_c = the calibration position vector, and

$$\delta B = (+E + a)\, B_o + F$$

which are all ways in which the baselines could be off.

$F = (F_x, F_y, F_z)$, a random additive error vector,

$$a = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \text{ a fractional scaling error,}$$

$$E = \begin{pmatrix} 0 & -t & -x \\ t & 0 & y \\ x & -y & 0 \end{pmatrix} \text{ a differential rotation matrix,}$$

where t corresponds to a clock error and x, and y are polar deviations, x lying in the plane of the local meridian.

We will model this phase error as a source position change plus a phase noise term.

$$\Delta\phi = \phi_s - \phi_c = [(E + a)B_o + F]^T (S_s - S_c) = B_o^T \cdot \delta S_s + \phi_n$$

Examine one term at a time.

IIB. The Rotation Term

$$\Delta\phi = (EB)^T (S_s - S_c) = B^T\, E^T\, (S_s - S_c)$$

If we hadn't calibrated the data, the source change δS_s would just be the apparent motion of the source due to the shift of earth coordinate system $E^T S_s$. The difference vector $(S_s - S_c)$ introduces phase errors as well as a shift.

E^T given above is expressed in the earth based coordinate system. In the local coordinate system in the direction of the source, at declination δ, hour angle H,

$$E^T_{sky} = M\,E^T\,M^{-1} \quad \text{where}$$

$$M = \begin{pmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{pmatrix}$$

IIBa. Timing Error

Consider only the clock error t, then

$$E^T_{sky} = \begin{pmatrix} 0 & t\sin\delta & -t\cos\delta \\ -t\sin\delta & 0 & 0 \\ t\cos\delta & 0 & 0 \end{pmatrix}$$

using small angle approximations for the source-calibrator separation:

$$(S_s - S_c)_{sky} = \begin{pmatrix} \Delta\alpha\cos\delta \\ \Delta\delta \\ 0 \end{pmatrix}$$

$$\Delta\phi = (U,\ V,\ W)\begin{pmatrix} \Delta\delta\ t\sin\delta \\ -\Delta\alpha\cos\delta\ t\sin\delta \\ \Delta\alpha\cos\delta\ t\cos\delta \end{pmatrix} = B^T\delta S_s + \phi_n$$

Again the first two terms look like a position shift

$$X = \Delta\delta\ t\sin\delta$$

$$Y = -\Delta\alpha\cos\delta\ t\sin\delta$$

while the third term is like a noise term that slowly varies as W varies

$$\phi_n = W\,\Delta\alpha\cos\delta\ t\cos\delta.$$

Note: A timing error causes

1) an E/W shift for a source-calibrator declination change
2) a N/S shift for a source-calibrator r.a. change
3) no shift if $\delta = 0$
4) a phase error that is zero if $\Delta\alpha = 0$

IIBb. Polar Motion

Consider the x term, the pole wandering slightly down the local meridian.

As before, go to the sky coordinate system in the direction of the source.

$$E^T_{sky} = M\ E^T_{earth}\ M^{-1}$$

$$\Delta\phi = B^T\ \delta S_s + \phi_n = B^T\ E^T_{sky}\ (S_s - S_c)$$

$$= (U,\ V,\ W) \begin{pmatrix} 0 & -x\ \sin H\ \cos\delta & -x\ \sin H\ \sin\delta \\ x\ \sin H\ \cos\delta & 0 & x\ \cos H \\ x\ \sin H\ \sin\delta & -x\ \cos H & 0 \end{pmatrix} \begin{pmatrix} \Delta\alpha\ \cos\delta \\ \Delta\delta \\ 0 \end{pmatrix}$$

As before there is an apparent source position shift

$$X = -\Delta\delta\ x\ \sin H\ \cos\delta$$

$$Y = \Delta\alpha\ \cos\delta\ x\ \sin H\ \cos\delta$$

plus a noise term

$$\phi_n = Wx\ (\Delta\alpha\ \cos\delta\ \sin H\ \sin\delta - \Delta\delta\ \cos H)$$

which varies slowly as H and W vary.

A similar analysis can be done for the polar motion term y, at HA = 6 hours.

IIBc. Example

All three polar motion terms, t, x, y are $\lesssim 0\overset{''}{.}01 \sim 5 \times 10^{-8}$ radian. This may be a bit optimistic for x and y. Again

$$\Delta\alpha,\ \Delta\delta \sim 10^\circ \sim 0.2\ \text{rad}$$

$$U \sim V \sim W \sim 6 \times 10^5\ \lambda \text{ at 6 cm}$$

apparent position shifts δS_s are $\sim 0.2 \times 0\overset{''}{.}01 \sim 0\overset{''}{.}002$ with corresponding phase shifts for $\phi_n \sim 0.006$ lobes. In effect, the small errors in time and polar motion are multiplied by the source-calibrator separation in radians.

IIC. Random Baseline Errors

The term F gives a random phase error.

$$\phi_n \sim F^T \cdot (S_s - S_c)$$

Typically F ~0.05 to 0.10 λ at 6 cm

$$\phi_n \sim .10 \times .2 \sim .02 \text{ lobes} \sim 7^\circ.$$

IID. The Expansion Term

The baseline error in the form $\delta B = a\,B$ is a scaling error, due to, for instance, a frequency error.

$$\begin{aligned}\Delta\phi = \phi_s - \phi_c &= (aB)^T(S_s - S_c) = B^T\cdot\delta S_s + \phi_n \\ &= B^T\, a^T\, (S_s - S_c) \\ &= (U, V, W)\begin{pmatrix} a_1\ \Delta\alpha\ \cos\delta \\ a_2\ \Delta\delta \\ 0 \end{pmatrix}\end{aligned}$$

This term results in only a position shift, which to 1st order is a fraction of the source-calibrator vector separation. There is no evidence that this type of error exists in our antenna positions.

III Conclusion

These results agree with intuitive estimates that one would make, but the more explicit results given here are reassuring. One item that is not generally recognized is the magnitude of the noise-like phase error that some of our worst type T calibrators may have introduced in the data. (Section IB.)