VLBA Correlator Memo No. 77

(870206)

National Radio Astronomy Observatory
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To: VLBA Correlator Memo Series

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Subject: Tests of the VLBA FX-style FFT.

In these notes I am presenting test results of the VLBA FX-style FFT. These test were conducted on the FFT (subroutine) standing alone. It was not in place in the FX correlator simulator program.

The FFT tested is the FFT subroutine that is part of the FX simulator package. It is a Radix-2 butterfly FFT, and the number of bits in the floating point mantissas and in the sine/cosine terms may be independently specified. The complex floating point words share a common exponent. The FFT is described in detail in VLBA Correlator Memo No. 74.

All of the tests reported here involved transforms of single monochromatic test signals. The test signals were generated using the Convex C-1 32-bit floating point sines and cosines, and a phasor amplitude of 15.0. Each complex floating point component was rounded to the nearest integer (-15 to +15). The FFT's were 2048 point complex transforms with uniform weighting on the input window. Comparison spectra were calculated simultaneously for each test signal using the well known Fortran floating point FFT, FOURG.

1.0 DC Tests

DC signals of amplitude 15 and various phase angles were transformed, and the spectra examined. All of the signal power appears in the zero frequency channel. No spurious signals were seen anywhere in the spectrum to a limit of 3.3e-4 percent. The FFT subroutine was set up with 5-bit mantissas in the floating point words, and 6-bit sine/cosine coefficients.

2.0 Single Frequency Test Signals (Figures 1 - 8)

The first eight figures show the FFT response to an input signal that is a sinusoid, quantized in integer values between +15 and -15. The test signal frequency is 6.0.

Figure 1 is the response of our FX-style FFT, 5-bit mantissas, 6-bit coefficients. The amplitude plotted is the square root of the sum of the squares of the complex components.

Figure 2 shows the response of the reference FFT (FOURG) to the same input signal.

Figures 3 and 4 show the absolute and relative differences between the amplitudes plotted in figures 1 and 2.

Figures 5 and 6 are the response of FX-style FFT's having 12-bit mantissas and coefficients, and 16-bit mantissas and coefficients.

Figures 7 and 8 are the absolute and relative differences for the 12-bit FX-style FFT and the Real*4 reference FFT (figure 2).

3.0 'Swept-frequency' Test Signals (Figures 9 - 15)

The 'swept-frequency' tests consists of taking the transforms of twenty-one test signals whose frequencies are at equal intervals between channels 6.0 and 7.0. The frequency step is 0.05.

Figures 9 and 10 are the results of testing the 5-bit mantissa, 6-bit coefficient FFT. Figure 9a shows the FFT amplitude in channels 6 and 7 for both the FX FFT and the reference FFT. The smooth curves are, of course, the reference FFT. Figures 9b and 9c are the relative differences between the FX FFT and the reference FFT (solid line, chan 6; dashed line, chan 7). The individual relative errors are dependent on the initial phase of the input signal. However, the mean values of the relative errors are essentially independent of the input phase angles.

Figure 10 shows the errors in determining the test signal frequency from least-squares fitting in the FFT output. The fitting routine used a sin(x)/x template, and fit for amplitude and frequency in the three channels around the channel having the largest amplitude. The relative difference between the FX and reference FFT's (figure 10a) appears more or less random, and the frequency error is at the level of a few percent of the signal frequency (6.0). Figure 10b shows the larger differences between the FX FFT output frequencies and the frequencies used to calculate the test signals. The systematic errors are due to the signal quantization.

Figures 11 through 14 show the results of the same 'swept frequency' tests as figure 9, but with different numbers of FFT bits (5/8, 6/8, 8/8, and 12/12 respectively). Note that the error scales change from one plot to the next.

Figure 15 shows the mean of the relative differences for a number of combinations of n-bit mantissas and m-bit coefficients. For each test run of n-bits and m-bits (like 5-bits, 6-bits in figure 9), the mean of the amplitude differences was taken in both channel 6.0 and 7.0 (eg., the solid and dashed lines in figure 9b). When the test frequency equaled 6.0 and 7.0, the difference values were not used. The relative differences often became very large as the FFT output values approached zero.

4.0 Results

The main purpose of the tests described above was to measure the performance of the FX FFT with 5-bit mantissas in the floating point words and 6-bit fixed point sine/cosine coefficients. The 'swept-frequency' tests show relative errors in the FFT amplitudes of about 10 %. These are unacceptably high for spectral line work. The VLBA will be expected to make high dynamic range images of spectral line emission regions. Ten percent amplitude errors that are to some degree frequency dependent cannot be corrected properly by self-calibration. Figure 15 shows that the relative errors for 8-bit mantissas and 8-bit coefficients is at the one percent level. This is probably a tolerable error level for very strong spectral lines and will disappear in the noise for weak lines. In conclusion, we find that larger word sizes should be considered for the FFT. Eight bit words would be adequate.

FX–style FFT. 5 mantissa, 6 bit coeff's. Single freq at channel 6.

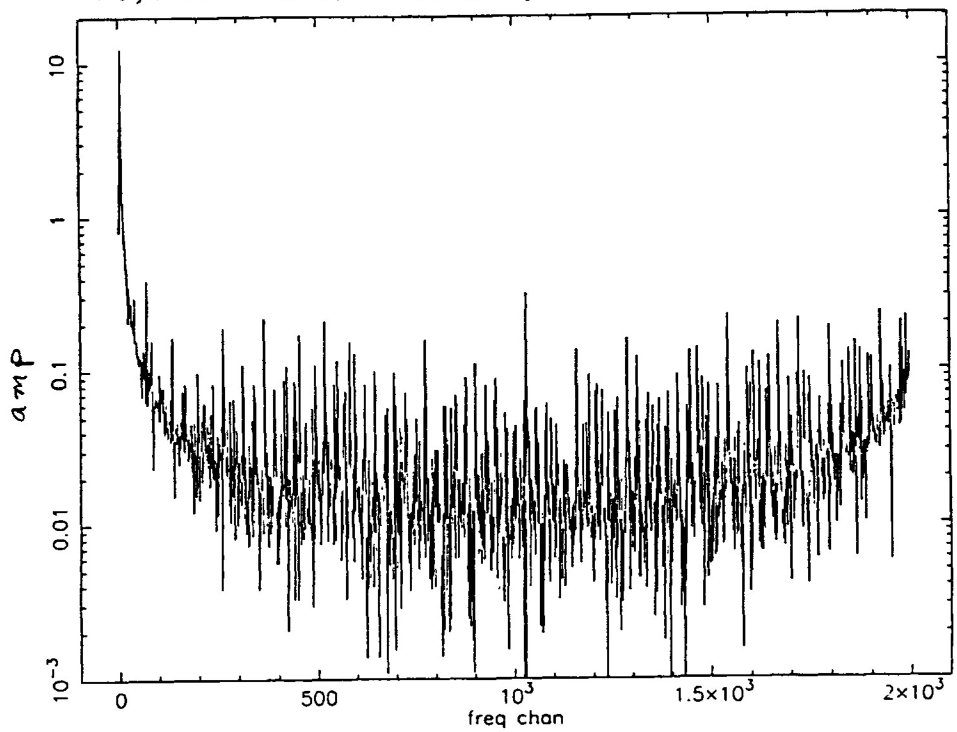


Fig 1

Real*4 FFT (FOURG). Single freq at channel 6.

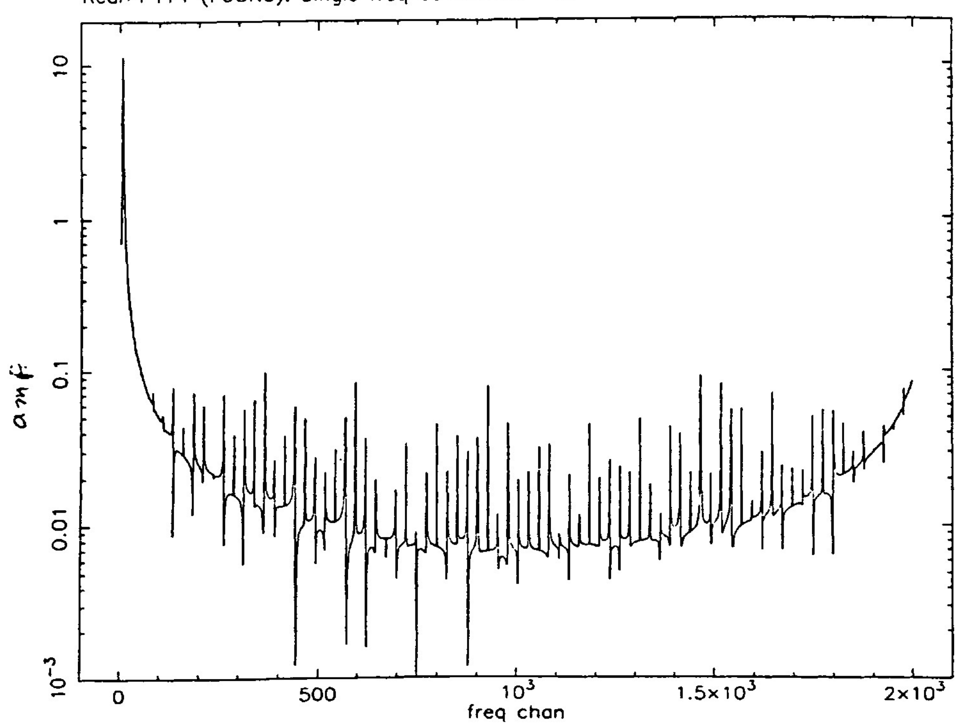


Fig. 2

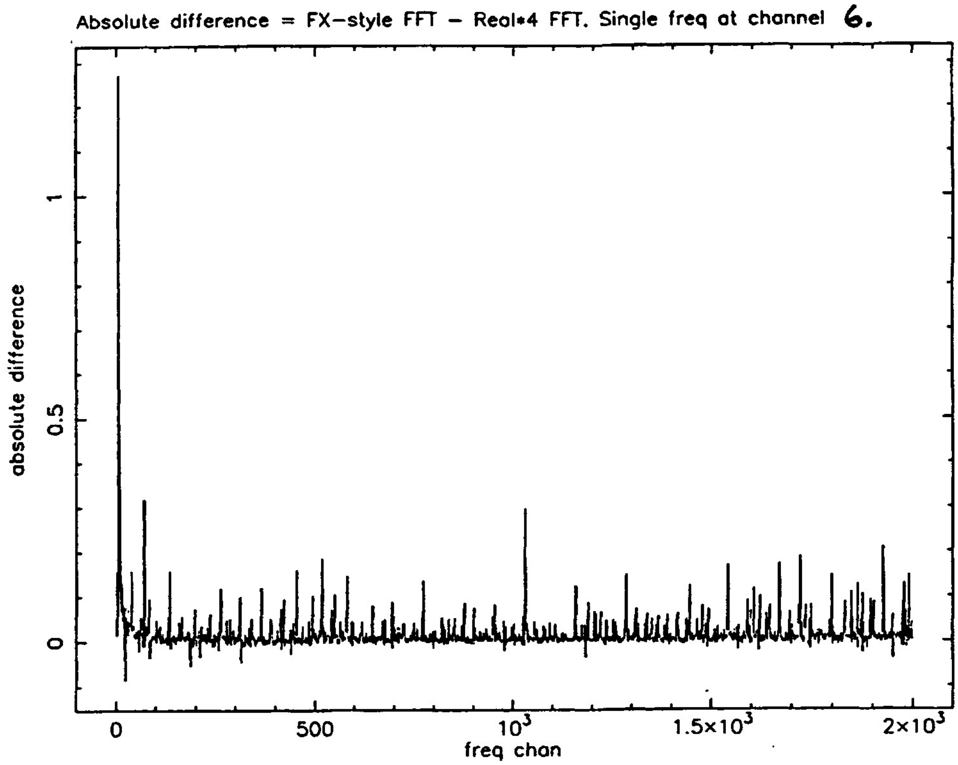

Absolute difference = FX–style FFT – Real*4 FFT. Single freq at channel 6.
absolute difference
0
0.5
1
0
500
10^3
1.5×10^3
2×10^3
freq chan

Fig. 3

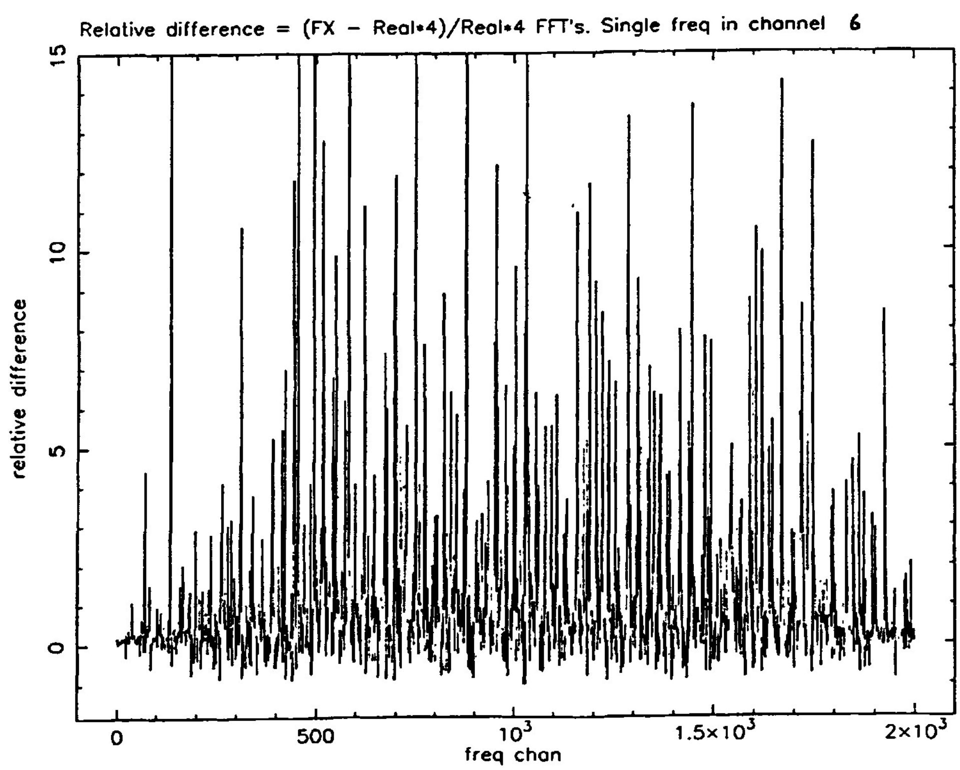

Relative difference = (FX – Real*4)/Real*4 FFT's. Single freq in channel 6
relative difference
0
5
10
15
0
500
10^3
1.5×10^3
2×10^3
freq chan

Fig. 4

FX-style FFT. 12 bit mantissa, 12 bit coeff's. Single freq at channel 16. INTEGER INPUT

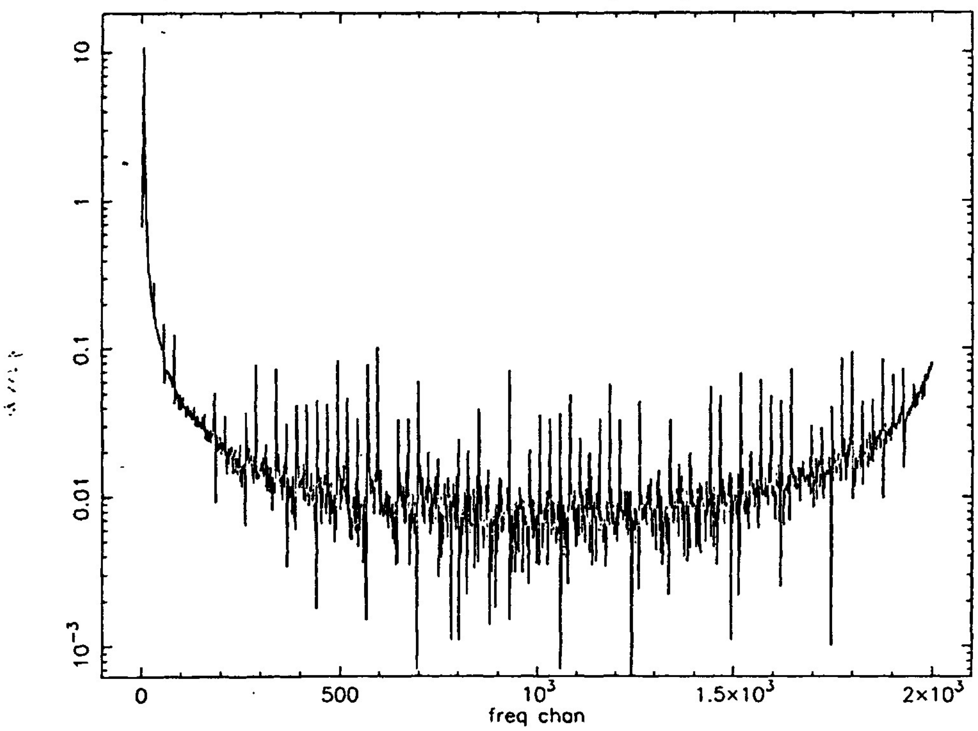


Fig. 5

FX-style FFT. 16 bit mantissa, 16 bit coeff's. Single freq at channel 6

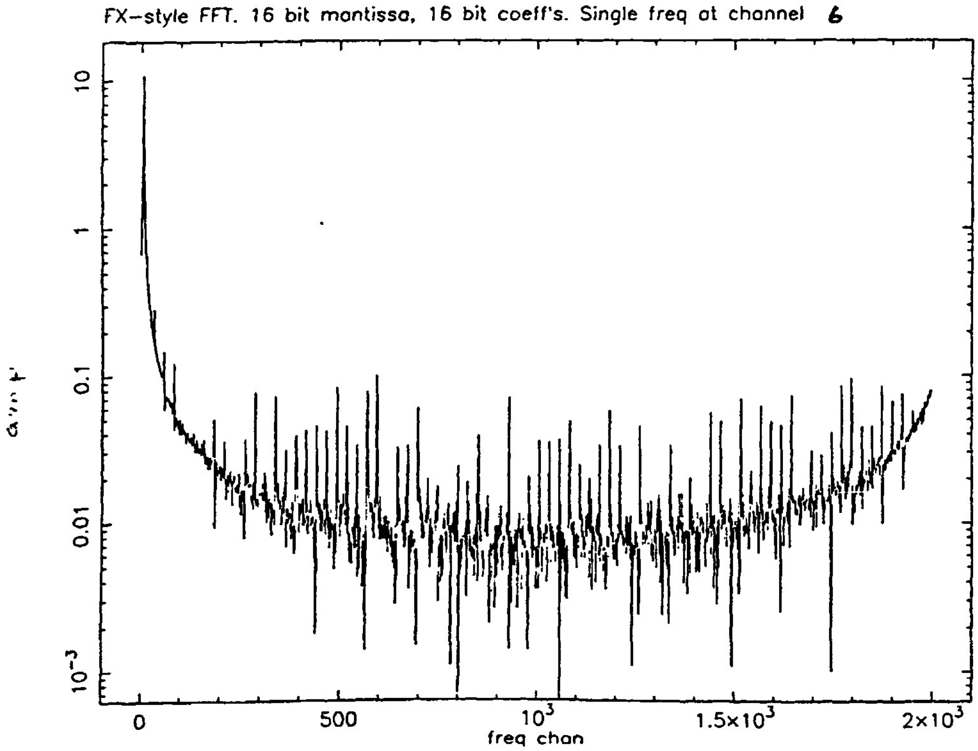


Fig. 6

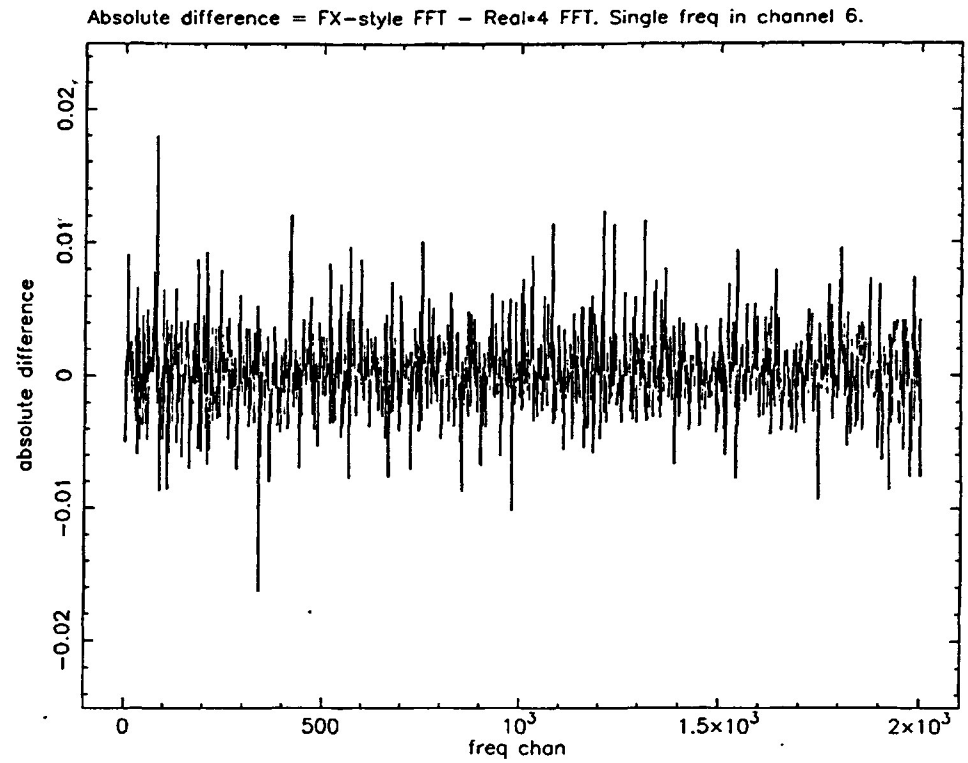

Absolute difference = FX−style FFT − Real*4 FFT. Single freq in channel 6.
absolute difference
0.02
0.01
0
−0.01
−0.02
0
500
10^3
1.5×10^3
2×10^3
freq chan

Fig 7.

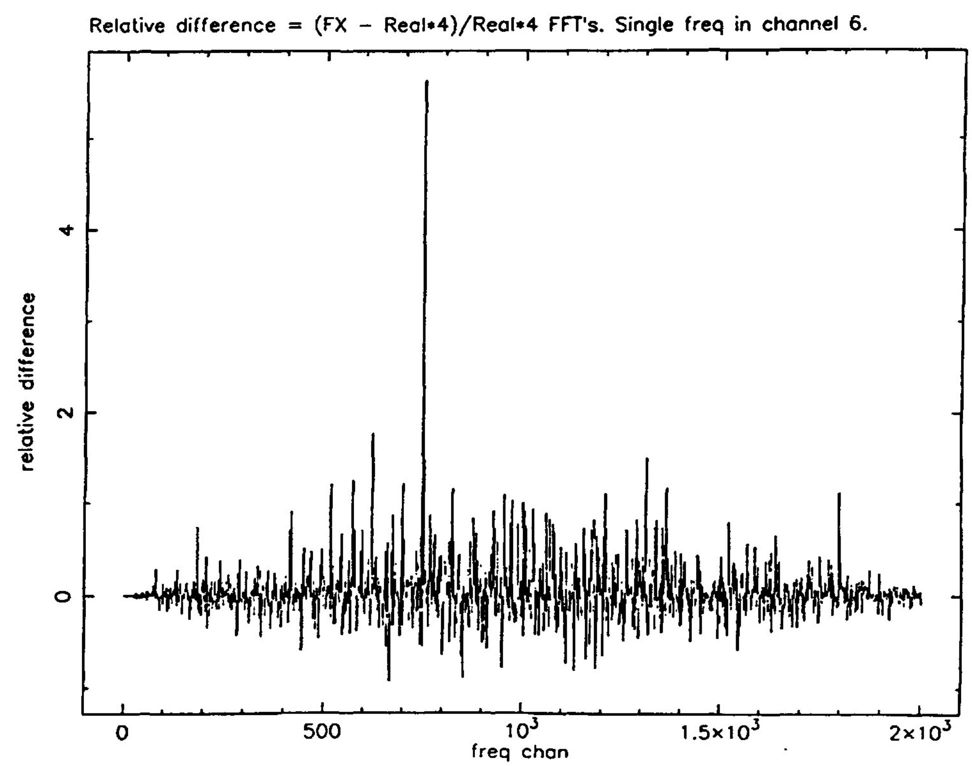
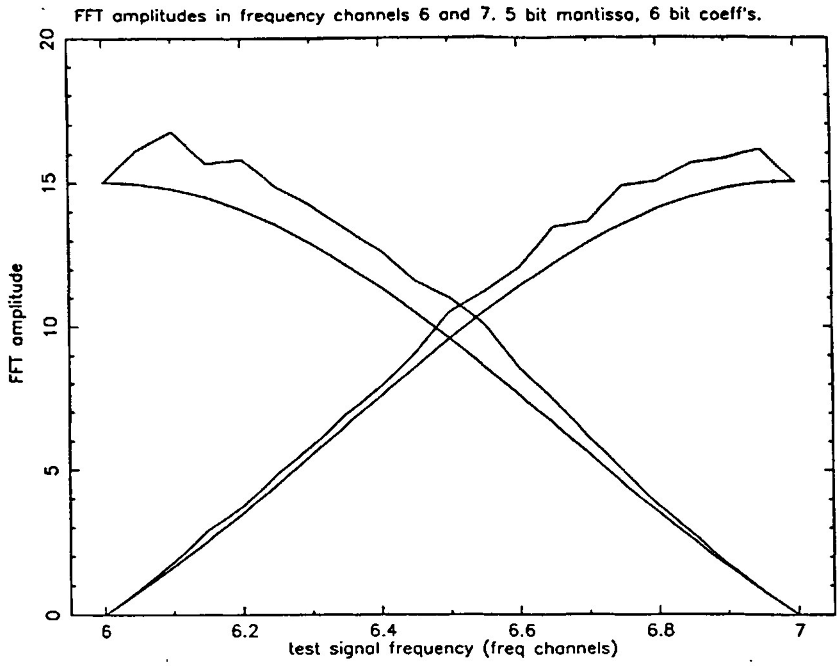
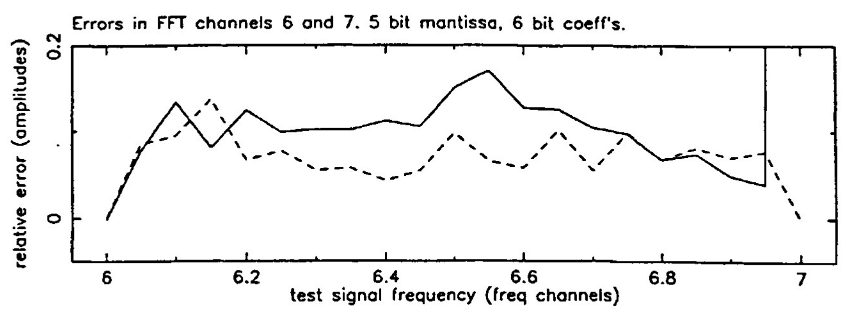

Relative difference = (FX − Real*4)/Real*4 FFT's. Single freq in channel 6.
relative difference
4
2
0
0
500
10^3
1.5×10^3
2×10^3
freq chan

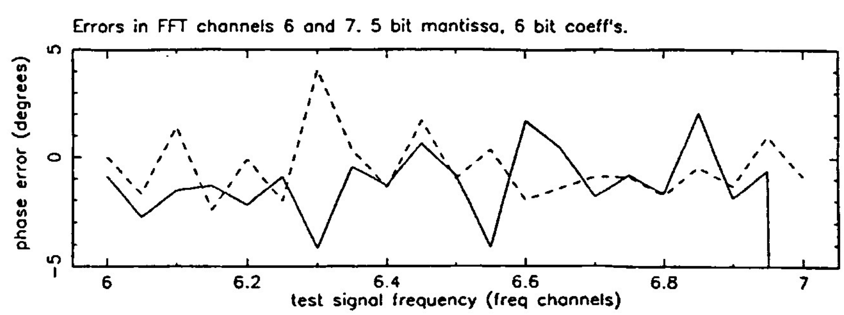
Fig. 8



9a.

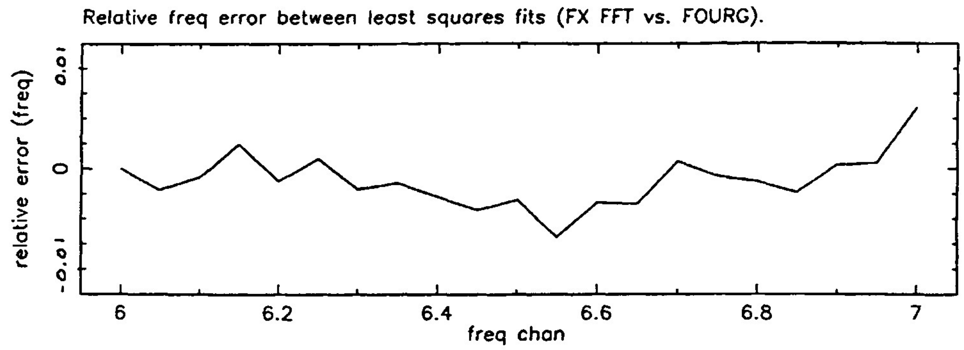


9b.

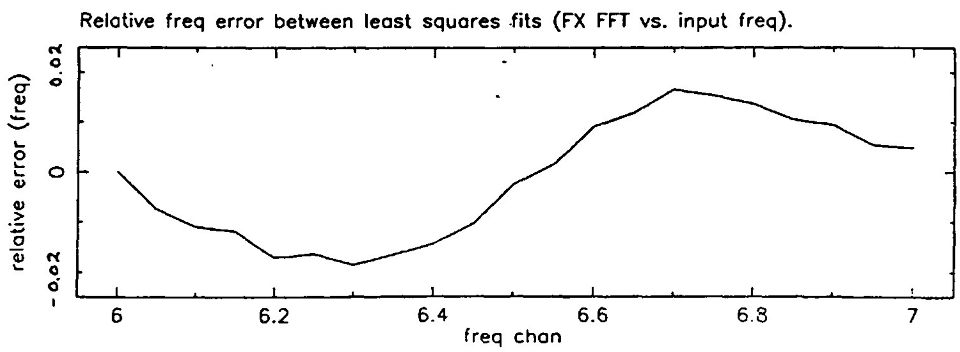


9c.

Figure 9.



10a.



10b.

Figure 10.

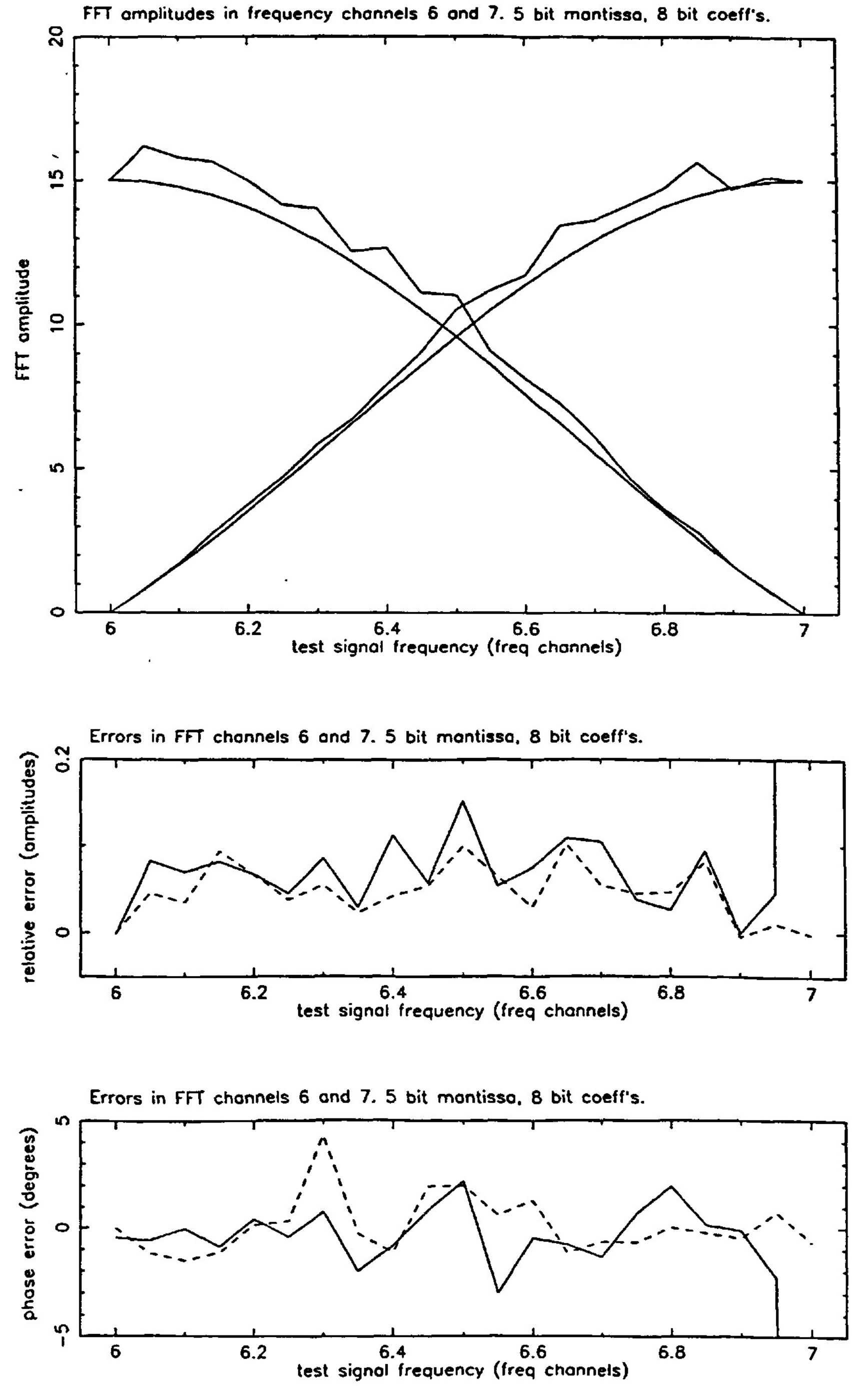

FFT amplitudes in frequency channels 6 and 7. 5 bit mantissa, 8 bit coeff's.
FFT amplitude
0
5
10
15
20
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 5 bit mantissa, 8 bit coeff's.
relative error (amplitudes)
0
0.2
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 5 bit mantissa, 8 bit coeff's.
phase error (degrees)
-5
0
5
test signal frequency (freq channels)

Figure 11.

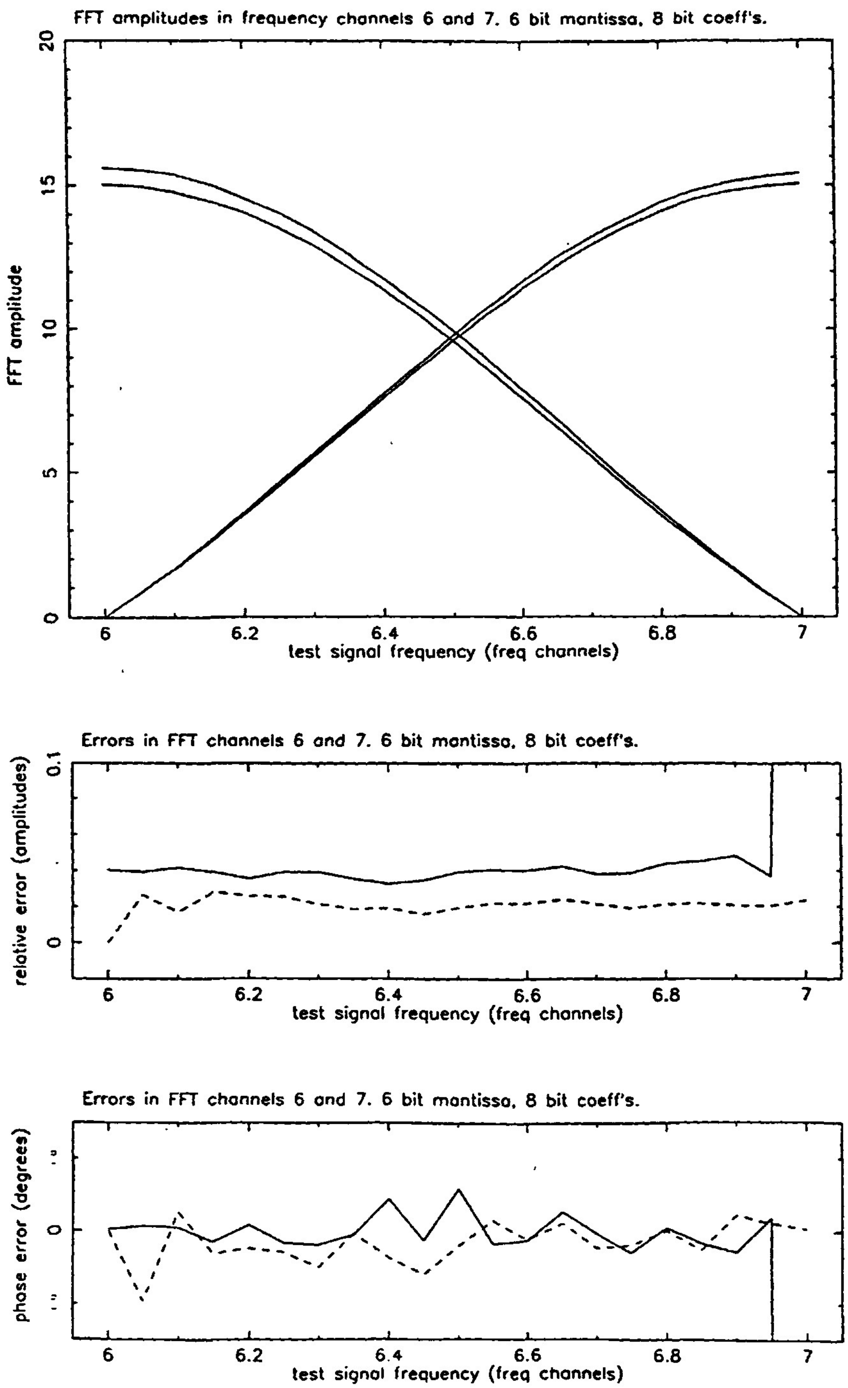

FFT amplitudes in frequency channels 6 and 7. 6 bit mantissa, 8 bit coeff's.
FFT amplitude
0
5
10
15
20
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 6 bit mantissa, 8 bit coeff's.
relative error (amplitudes)
0
0.1
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 6 bit mantissa, 8 bit coeff's.
phase error (degrees)
0
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)

Figure 12.

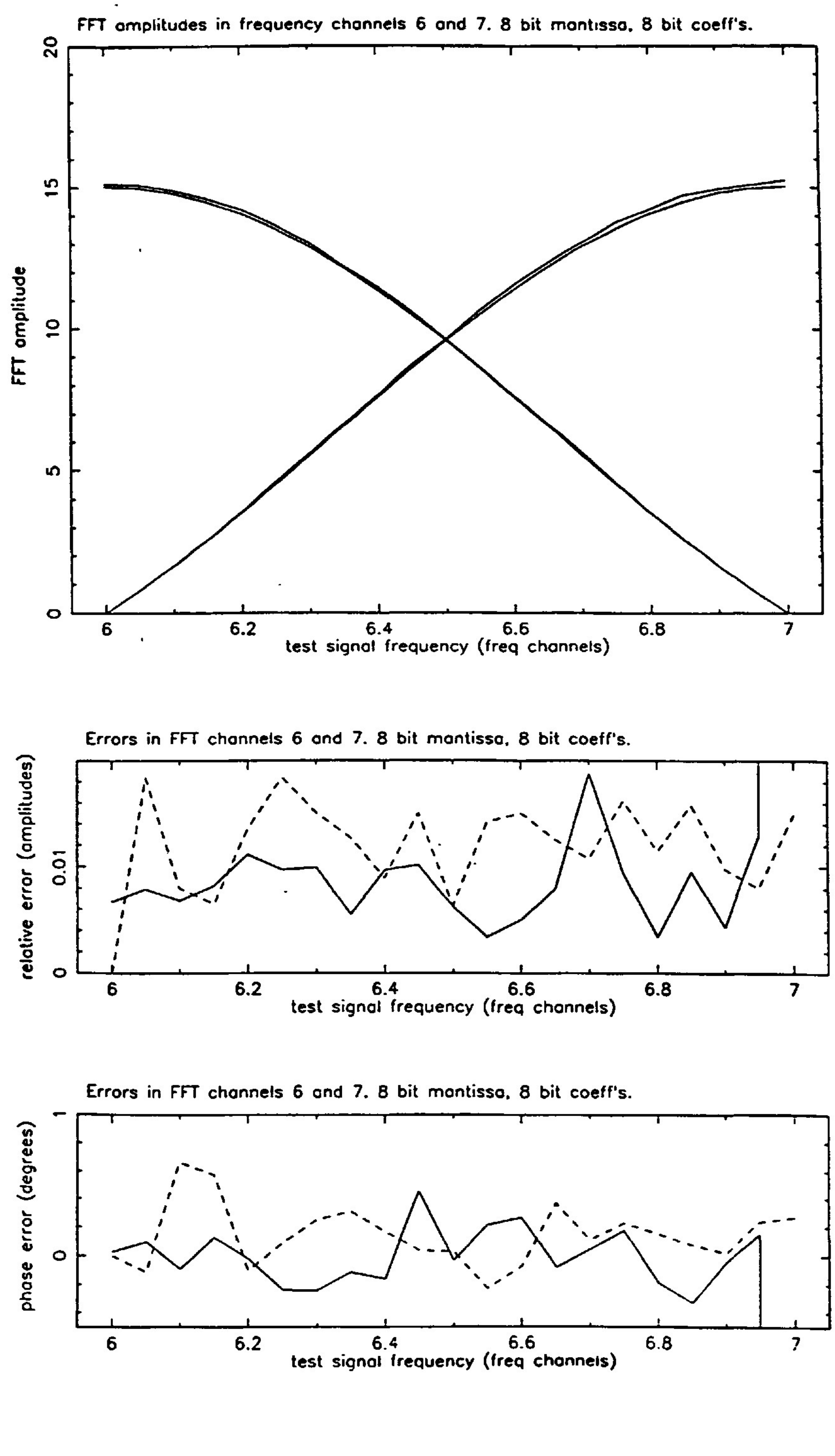

FFT amplitudes in frequency channels 6 and 7. 8 bit mantissa, 8 bit coeff's.
FFT amplitude
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 8 bit mantissa, 8 bit coeff's.
relative error (amplitudes)
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 8 bit mantissa, 8 bit coeff's.
phase error (degrees)
test signal frequency (freq channels)

Figure 13.

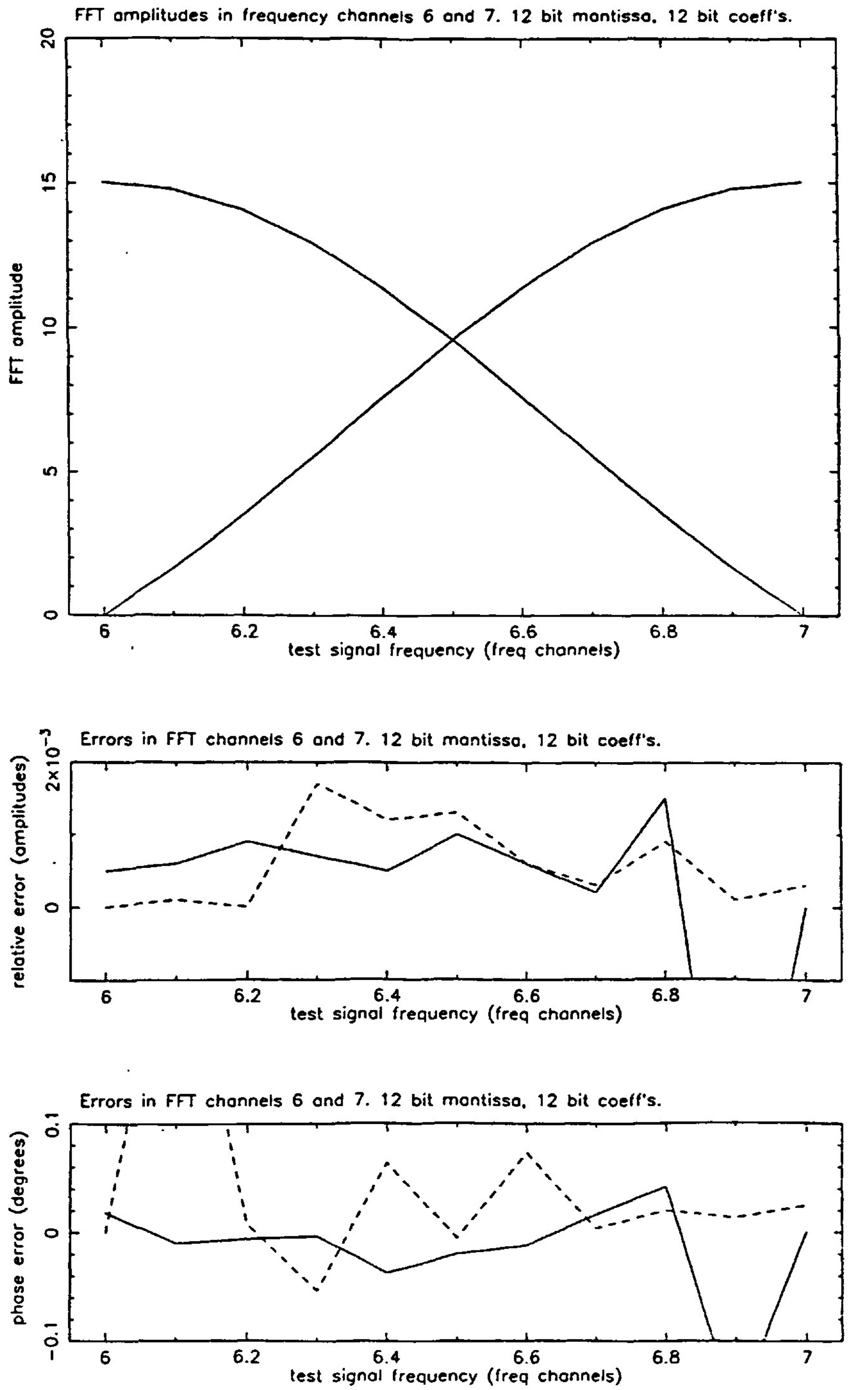

FFT amplitudes in frequency channels 6 and 7. 12 bit mantissa, 12 bit coeff's.
FFT amplitude
0
5
10
15
20
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 12 bit mantissa, 12 bit coeff's.
relative error (amplitudes)
0
2x10^-3
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)
Errors in FFT channels 6 and 7. 12 bit mantissa, 12 bit coeff's.
phase error (degrees)
-0.1
0
0.1
6
6.2
6.4
6.6
6.8
7
test signal frequency (freq channels)

Figure 14.

FX-style FFT vs. conventional FFT. Errors in monochromatic signals.

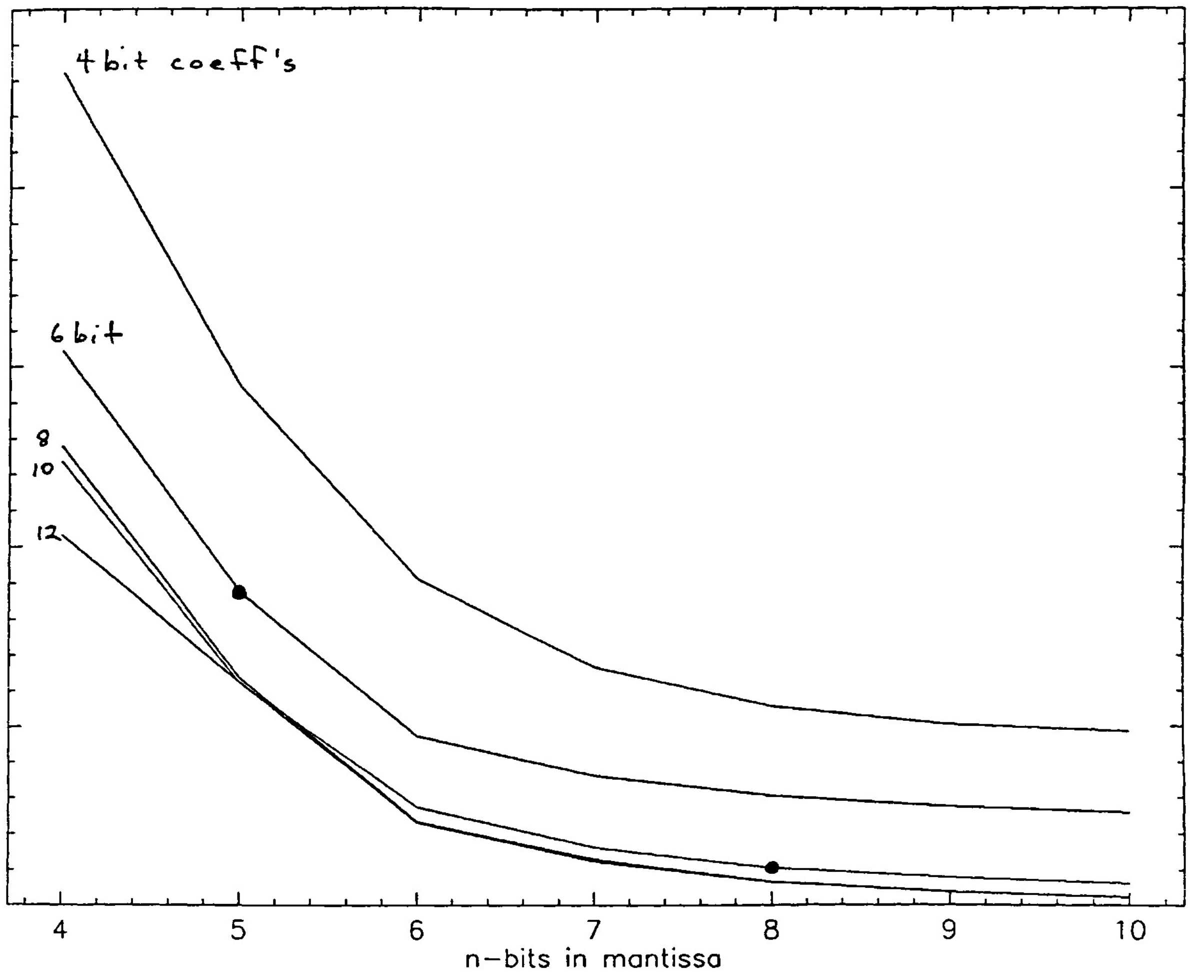


Figure 15.