VLBA ACQUISITION MEMO #129

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13 April 1989

Area Code 508
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To: VLBA Data Recording Group

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Subject: Model for the tracking offset dependence on tape defects

1] Temperature

The polyester tape medium has a thermal expansion coefficient (in the plane of the tape) of

$$2 \times 10^{-5} / {}^\circ C$$

for which a 1 °C differential temperature across the tape would produce a 4000' radius of curvature and a corresponding shift to remain normal to the capstan axis of

$$S = L^2/2R \approx 10 \text{ microns/differential temp. } {}^\circ C$$

where L = distance to heads and capstan (≈ 5")

R = radius of curvature of the tape

Inducing a temperature gradient of 1 °C/inch would be very hard and thus differential temperature effects are not expected to be significant.

The polyester film however has an isotropy which makes the thermal expansion coefficient about 30 times larger (and Young's modulus correspondingly smaller) in the thickness direction (from E.F. Cuddihy, JPL). Also the direction of alignment of the long chain molecules is systematically different from the tape direction (only the web center has perfect alignment) and this produces some anisotropy of thermal expansion and Young's modulus in the plane of the tape. [The large thermal expansion coefficient in the thickness direction can be easily observed by cooling a reel of tape and observing a dramatic loosening of the pack - which tightens back when the tape is returned to room temperature.]

2] Humidity

The coefficient of expansion with humidity is reported to be about

$$5 \times 10^{-6}/\% \text{ RH}$$

and hence using the same model as for temperature differences the sensitivity is expected to be

$$\approx 2 \text{ microns/\% difference in RH across tape}$$

Again the effect of humidity differences across the tape are expected to be small.

3] Differential tape stretch

Stretch a piece of tape and the tension will not be uniform across the tape. For example, a tape with a "stretched" edge will have to be put under considerable tension to even straighten out the damaged edge. With an operating vacuum of 10" the total tape tension is 0.5 lb and the expected extension for the length L between vacuum column and capstan

$$\frac{F\,L}{Ywt} \approx 100 \text{ microns}$$

where

	F	=	total force (≈ 0.5 lb)
	Y	=	Young's modulus (≈ $7\text{x}10^5$ lbs/sq")
	w	=	tape width (≈ 1")
	t	=	tape thickness (≈ 0.001")
	L	=	distance from vacuum column to capstan (≈ 6")

and if there is a 1% variation in the tension then this can be made up by a 1 micron variation in extension across the tape under tension. Differential tension on the capstan will produce a torque which will in turn be matched by a shift in the tape position on the capstan to produce a balancing torque. Equating the torques

$$F\alpha L = F\gamma\ W/6$$

where

α	=	tape shift angle
γ	=	fractional tension variation

from which

$$\alpha\ L \approx 40\ \mu\text{m}/\%\ \text{tension variation}$$

A recorder with capstan taper has additional sensitivity to differential tension across the tape. A capstan with 8 arcsec taper that matches the 1 micron extension variation will produce zero shift and a shift of 16 microns in tape position (see Memo #122) if there is differential tension. The sensitivity coefficient is

0.2 microns/arcsec/% tension variation

or about 20 microns/% tension variation with a 100 arc second taper on the capstan.

4] Tape non-uniformity

If the tape is wedge-shaped in cross-section it will form an arc under the action of the vacuum column with a radius of curvature

$$b = \frac{Y\,t\,w}{RKP}$$

where R = radius of vacuum loop (≈ 1.3")

P = pressure (≈10" H_2O)

K = fractional non-uniformity

and the shift in tracking is

$$S = L^2/2b \approx 3\ \mu m/\%\ \text{non-uniformity.}$$

Note however that the shift is proportional to vacuum pressure and the distance to the vacuum column squared and hence even constant non-uniformity will produce tracking signatures.

5] Tape slitting imperfections

Most tape slitting imperfections will produce relatively short A.C. signatures since the tape is edge guided. Long wavelength imperfections might be characterized with a radius of curvature and since the tape remains normal to the capstan the shift is

$$S = L^2/2b$$

where L = distance from capstan to the edge guiding region

b = radius of curvature

This assumes that the pull of the vacuum doesn't straighten out the tape which is a good approximation for small curvature. It is important to note that reducing L will very rapidly reduced the effect of long wavelength slitting signatures. Slitting signatures will be directly observed when a tape recorded with one value of L is reproduced with a different value of L.

6] Misalignment of the polymer strings with the tape direction

When tape is manufactured, the long chain molecules are stretched in the "machine" direction (Cuddihy, JPL - private communication), but after the large tape web is slip into strips of tape only the tape which was close to the center of the web has the long chain strings well aligned. Typical misalignment can be many tens of degrees. Young's modulus is highest along the string direction and drops by about 20% in a direction perpendicular to the string (see Jorgenson, page 279). When a piece of tape is under tension this misalignment of the direction of maximum modulus produces a force which alters the position of the tape.

Consider a semi-circular element of tape (see Figure 3) in which there is a uniform strain in tape direction. Now the strain components in a sub-element with orientation Θ are:

$S \cos^2 \Theta$ along the element

$S \sin \Theta \cos \Theta$ in shear.

Now calculate the stress assuming that the modulus of elasticity is slightly anisotropic so that Young's modulus

$$Y(\Theta) = Y_o \left(1 + \left(\frac{\varepsilon}{2}\right) \cos^2 (\Theta - \phi)\right)$$

$$= Y_o \left(1 + \left(\frac{\varepsilon}{2}\right) \cos^2 \Theta + \varepsilon \phi \sin 2 \Theta\right)$$

where

ε	=	fractional anisotropy (peak to peak)
ϕ	=	angle of polymer strings

so that the stress components are

$$T \cos^2 \Theta \ (1 + \varepsilon \cos 2 \Theta + 2 \varepsilon \phi \sin^2 \Theta)$$

and $\quad T \sin \Theta \cos \Theta \ [T = \text{tension} = SY_o]$

from which the average tension in the α direction is

$$\left(\frac{T}{2}\right) \int_{-\Pi/2}^{+\Pi/2} \left(\cos^3 \Theta + \left(\frac{\varepsilon}{2}\right) \cos^3 \Theta \cos^2 \Theta + \varepsilon \phi \cos^3 \Theta + \sin^2 \Theta + \sin^2 \Theta \cos \Theta\right) d\Theta$$

$$= T + \varepsilon T/3$$

and the average tension in the y direction is

$$\left(\frac{T}{2}\right) \int_{-\Pi/2}^{+\Pi/2} \left(\cos^2 \Theta \sin \Theta + \left(\frac{\varepsilon}{2}\right) \cos^2 \Theta \cos^2 \Theta \sin \Theta + \varepsilon \phi \cos^2 \Theta \sin^2 \Theta \sin \Theta + \sin \Theta \cos^2 \Theta\right)$$

$$d\Theta = 4\varepsilon\phi T/15$$

equating this component to the y component of force produced by a shift in tape angle

$$T\alpha = 4\varepsilon\phi T/15$$

and hence the tape shift is

$$S = \alpha L = 4\varepsilon\phi L/15 = 7\ \mu m/\%\ \text{anisotropy/degree}$$

since the polymer angle changes sign from the input to the output side of the capstan the tape shift will change sign with tape direction and produce a tape related forward-reverse offset. Variations in anisotropy and polymer angle with footage will produce a systematic forward-reverse signature along the tape and make it essential to have an adequate guard band between forward and reverse passes. The above analysis is only valid for small angles and anisotropy. A more complete analysis shows that the sensitivity coefficient saturates at an angle of about 10 degrees to a value of 30 um/% anisotropy and for large anisotropy and large angles saturates to a value of about 150 μm.

7] Tape relaxation and tape packing

Rough measurements of tape stretching on the bench clearly shows a dynamic hysteresis with relaxation time of about 5 seconds. See Figure 1 for a proposed tape rheology. A very likely method for significantly straining the tape is via a temperature cycle of the pack. Consider the tangential and radial pressures in a tightly rolled reel of tape as a function of temperature.
An element within the pack is subject to forces of radial and tangential pressure which must balance

$$P_T = -r \frac{dP_R}{dr}$$

where

r	=	radius of a point within the pack (assumed circular)
P_T	=	tangential pressure trying to stretch the tape
P_R	=	radial pressure trying to compress the tape in its thickness direction

If a(r) is the radial displacement of an element of tape

$$\frac{da}{dr} = \text{radial strain} = -\frac{P_R}{\epsilon_R} - \left(\frac{a}{r}\right)\sigma_T + \theta e_R$$

$$\frac{d\phi}{dr} + \frac{da}{dr} = \text{tangential strain} = \frac{P_T}{\epsilon_T} + \left(\frac{da}{dr}\right)\sigma_R + \theta e_T$$

where

ϵ_T	=	Young's modulus in tangential direction
σ_T	=	Poisson's ratio in tangential direction
e_T	=	thermal expansion coefficient in tangential direction
$\epsilon_R, \sigma_R, e_R$	=	same in radial direction
θ	=	temperature increase
ϕ	=	rotational displacement of the tape

There are 4 unknowns, a, ϕ, P_T, P_R, and only 3 equations so some additional information is needed for a solution. If the tape is perfectly slippery P_T a constant and a simple solution exists. If the tape doesn't slip then there is no rotational displacement of the tape and again solutions can be found - but these are coupled differential equations and are hard to solve so we resort to numerical solutions like the one shown in Figure 2. This figure illustrates the non-uniformity of longitudinal strain and amplified build up of thickness strain that occurs because of the difference in Young's modulus for stretching (≈ $7x10^5$ lbs/sq") along the tape and for compressing (≈ $9x10^4$ lbs/sq") the thickness. Taking the strain curves (for a modulus ratio of 8) and applying an increase in temperature, the strain in the thickness direction will build further by ≈ 0.06%/deg C since temperature coefficient along the tape is much smaller (by a factor of 30) and the modulus is much larger. A 16 deg C increase will bring the tape to its elastic limit (thought to be about 1%). Cooling the tape by only 7 deg C will cause the pack to loosen and gaps and ripples may develop. More tests of these implied environment limits are needed to establish what really happens in practice. [More details of the tape pack and computer model will appear in a separate note.]

8] Tests of tape related tracking signatures

A "poor" tape was recorded forward with REC #3 and the tracking offset and jitter observed upon playback for various tape speeds and angles of the tape. The jitter was judged from the amplitude variation when tracking on the half voltage point. The tape angle with respect to the precision plate was adjusted by tilting the capstan as is given in terms of the tape position in the following table of results:

Tape Position μm	Forward Jitter μm	Reverse Jitter μm	Comments
0	0	20	Tape recorded at this position.
-400	10	20	
-800	20	20	
0	10	20	Change to capstan with 100 arcsec taper

A tape position of 0 corresponds to a tape angle of about 100 arcseconds and a tape position of -800 μm to an angle of about 1400 arcseconds. Little, if any, variation was observed with tape speed until the speed was reduced to about 10 inches/sec and which point the offsets typically shifted by 15 μm presumably because at this speed the tape has time to relax the plastic strain built up on the reel pack before the tape reaches the capstan.

9] Environmental tests on the tape

Several tests were made on a good tape of the effect of refrigerating the tape. A tape was prepassed and then recorded. The tape was then refrigerated to approximately 32°F for a few hours, allowed to warm up again and replayed. Typical tracking shifts were about 20 μm, most of which relaxed after the first pass thereby demonstrating the usefulness of the prepass.

After experimenting with various things like deliberately forcing a poor tape pack, heating (with a heat gun) and cooling (with cooling spray) to try and induce a large change in the tape tracking signature a 30 μm jog did develop in a portion of the tape and again the magnitude of the shift as a function of tape position (by shimming the capstan) was measured. The results were as follows:

Tape Position μm	Tracking Shift μm	Comments
-300	27	
-300	24	
-100	16	
+100	13	Tape approx. parallel to plate
+200	13	
+300	22	
+50	15	With different capstan

During these tests it was noted that after changing the capstan from one with little taper to one with 100 arcsec taper there did appear to be about a 10 μm shift during the first pass of the tape implying that the tape pack does preserve some memory of the capstan.

10] Model for tracking variations due to imperfections in the tape

A model for tape related tracking is based on the following assumptions:

a) The tape is mostly elastic but can become plastically deformed by the capstan, reel pack and environmental changes.

b) Most of the plastic deformation is recovered with a time constant of about 5 seconds.

An approximate model for the peak to peak shift of the track recorded on machine 1 and reproduced on machine 2

$$S_{12} = 0.2\,\Delta\phi_{\epsilon} + 0.04\,\Delta\,\phi_{x} + 40e^{-t/5} \quad \mu m$$

where

$\Delta\phi_{\epsilon}$ = the difference in capstan taper in arcsec

$\Delta\phi_{x}$ = the difference in tape angle in arcsec

t = time since the tape first came off the supply reel in seconds

Notes: 1] Coefficients are for 1% tension variation intrinsic to the tape

2] The magnitude of the relaxation term is highly dependent on the environmental changes experienced by the tape since recording. No limits have yet been established, but the 40 μm is approximately the magnitude experienced.

3] Intermachine tracking signatures due to tape slitting imperfections will also be proportional to the difference in capstan tilt. [Peak to peak signatures seen with the tape turned over (supply and take-up reversed) were about 20 μm.] Since the tape is effectively guided on one edge slitting imperfections will produce only A.C. signatures.

4] Terms which do not depend on tape defects, like the sensitivity to the position of the tape in vacuum column, which are given in Memo #123 must also be added.

5] The tape pack is also influenced by the tension variations produced by a non-uniform capstan - this is a small effect ($\approx 10\mu m$ shift) but it does imply that a prepass on one machine and immediate replay on another is not as good as a prepass on the same machine.

6] The largest high frequency A.C. term for S_{12} for a given tape is given by the jitter observed when playing a forward recording in reverse. High frequencies being those which have a wavelength smaller than the distance between tape edge guiding region in the vacuum columns and the capstan.

7] Anisotropy effects should be largely repeatable. Note however that the forward-reverse offset is tape dependent and that forward and reverse passes must be well separated with adequate guard band.

8] Additional intermachine signature contributions will be present if the machines are operating in different modes with differences in the effective distance between capstan and edge guiding region.

11] Summary

The following table summarizes the maximum tracking shift expected from various tape related effects:

	Coefficient μm	Max. expected shift μm
Differential Temperature	$10/\Delta^{o}C$	1
Differential Humidity	$2/\Delta\%$	2
Stretched Tape	40/%	40 and repeatable
Anisotropy	7/%/deg	100 in for-rev offset
Non-uniformity	3/%	10 and repeatable
Relaxation		40 without prepass
Slitting		A.C. only

Maximum tracking shifts (for present recorder configuration) expected from various tape-related effects.

Much of the theory in this memo should be considered <u>preliminary</u> - more work is needed to check its validity.

Attachments: Figure 1. Deformation Model for Magnetic Tape
Figure 2. Strains on Tape Within a Pack
Figure 3. Effects of Alignment of tape Anisotropy

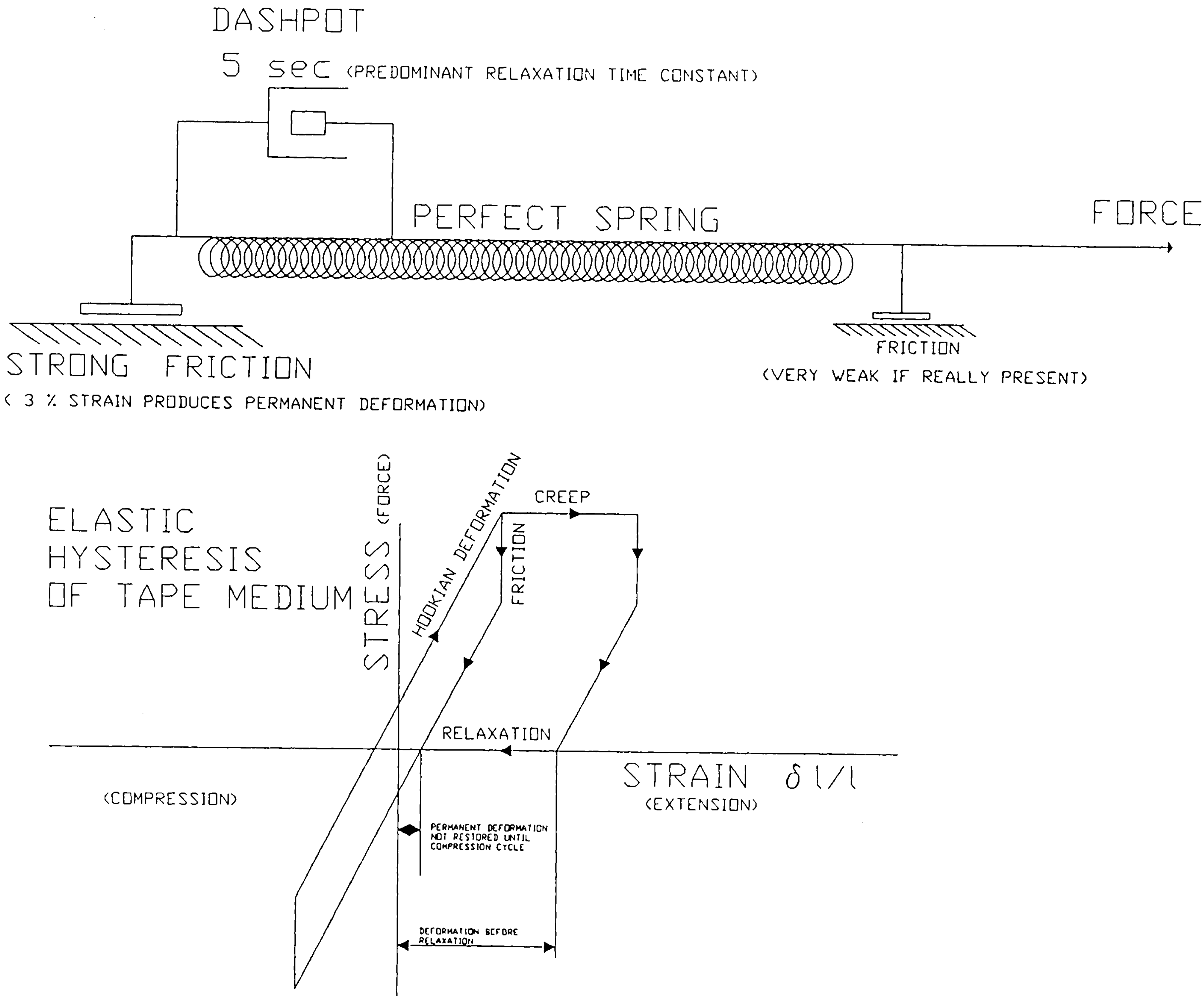


FIG [redacted] DEFORMATION MODEL FOR [redacted]G[redacted]TI[redacted] T[redacted]PE

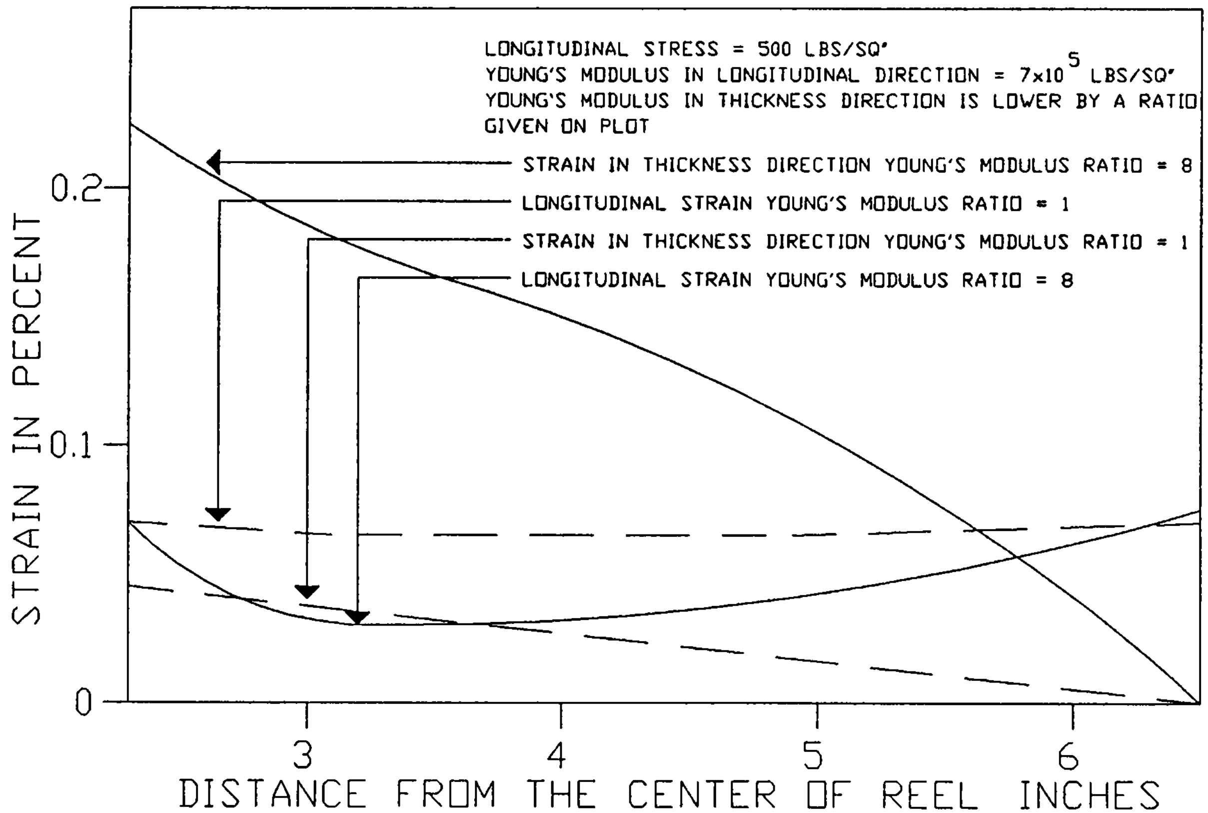
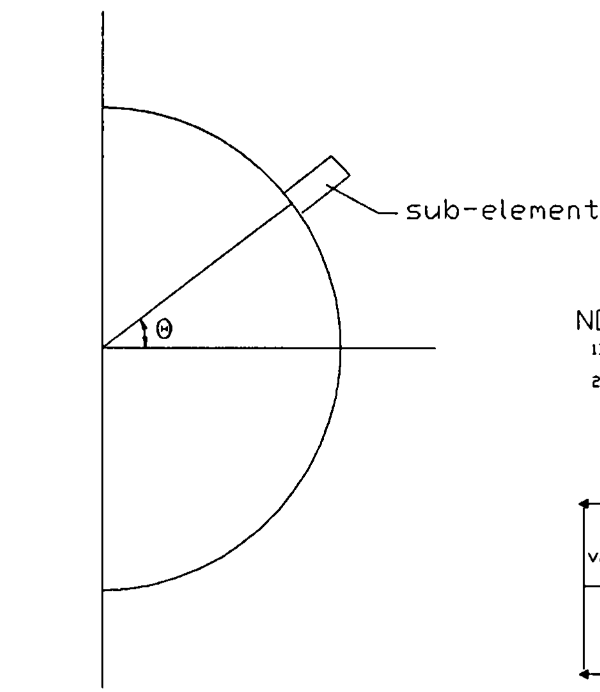
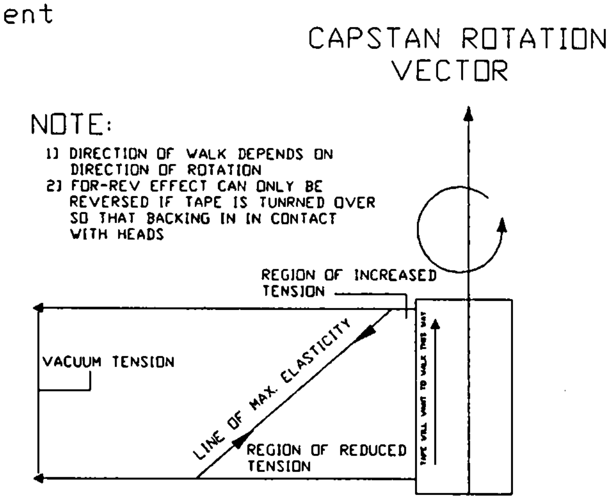


FIG 2 STRAINS ON TAPE WITHIN A PACK



GEOMETRY OF SUB-ELEMENT



SIMPLE VIEW OF THE EFFECT OF ANISOTROPY

FIG. 3. EFFECTS OF ALIGNMENT OF TAPE ANISOTROPY