SOME REMARKS ABOUT THE SENSITIVITY OF A PARTIALLY COHERENT ARRAY

VLB ARRAY MEMO No. 5

I. Some simplifying assumptions.

The array consists of N+1 antennas.
All LO's have constant phase for the coherence time Tc. They then take a flying leap.
The LO phases assume one of e possible values. (This assumption, although silly on the face of it, is probably not far wrong).

The sum of the fringe visibilities is Gaussian distributed. (This assumption, although reasonable on the face of it may be seriously in error.).

All antennas have equal sensitivity.

You, God, and the ApJ have an agreement that a result is to be believed if and only if its probability of arising from chance is less than ϵ.

Unproved assertion:
The best you can do is make all possible assumptions about instrumental phase, and pick the best looking one.

II. Point source sensitivity

A. Integration time =Tc.

There are N antennas whose phases may take one of e values. Therefore, there are

$$e^N \quad \text{possible assumptions.} \qquad 1$$

Let us make them, and see if the source is detected.
What do we mean by detected? That the sum of the visibilities has a chance of less than ϵ of arising by chance. Normalizing by the rms of a coherent array under the same conditions, the distribution of the sum of the visibilities is

$$\frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{2}} \qquad 2$$

Let us make the assymtotic approximation to the error function:

$$\int_{x_0}^{\infty} \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{2}} \simeq \frac{1}{x_0\sqrt{\pi}} e^{-x_0^2} \qquad 3$$

Then the condition for detection is

$$e^N \frac{1}{x_0\sqrt{\pi}} e^{-x_0^2} = \epsilon \qquad 4$$

where the first factor is the number of trials, the second is the probability of an excursion of the limiting amount in each trial.
For reasonably large N it is clear that

$$x_0 \simeq \sqrt{N + \log\frac{1}{\epsilon}} \qquad 5$$

That is, that the partially coherent array is $4/\pi\sqrt{N}$ times less sensitive than the coherent version of the same array.

2. Integeration time >>Tc.

Can we play the same game, making a new set of assumtions about each coherence time? First note that this is a "hard" algorithm-- it cannot be implemented in practical hardware. Second, note that if you do implement it, there are fundamental difficulties. The number of trials to be made are

$$e^{N\cdot M}$$

where M is the total integration time † divided ty Tc. The equivalent of equation 4 above is

$$e^{N\cdot M}\, e^{-X_0^2} = \epsilon$$

Which is in the assymtote

$$X_0 = \sqrt{MN}$$

However, the sensitivity in a single coherence interval is also

$$\sqrt{NM}$$

You have gained nothing by trying to use the whole integration time.

III. Some speculations about sensitivity to extended sources.

Mapping extended sources has an extraordinary property. Usually God employs the most effective possible strategy to keep you from finding out what you want to know. Not so in this case. He tries as hard as possible to keep you from making a map at all, as discussed in the section above, but once you have made a map, say of the strongest point in the field, then the rest of the field has the SNR of a coherent array. You can self-cal, and find out about all that interesting low level stuff.

The above remark applies nicely to the case of a field dominated by a strong point. What about the more usual case of a complex source with lots of strong points? Let us consider the case in which we have been given, perhaps by a fairy godfather, an almost correct map of the source, and we wish to improve it. Suppose we look at each coherence interval. Then we can calculate for each antenna in turn, the expected visilities with each other antenna. In cases of poor SNR, the obvious thing to do is to correlate the observed and expected visibilities.

$$\Delta\phi = ARG\left(\sum V_0 V_e^*\right)$$

Where V_e is the expected and V_0 the observed visibility on baselines involving the antenna being calibrated. It is this number, for the extended case, which must be of order $\sqrt{N}$ SNR antenna. This is the output of a conceptual array consisting of the antenna to be calibrated operating as an interferometer against all the others in the system, operating as a phased array, optimally tapered for the source distribution.

If we do not have a fairy Godfather, we shall be a bit worse off. I speculate that in the worst case, for detecting the thing in the first place, we need a detection on the conceptual array consisting of the antenna to be calibrated in an interferometer against the rest of the array, operating as a phased array. The practical case should fall somewhere in between. an untapered

The argument for inextensibility of integration beyond the coherence time developed above cannot be extended to the extended source case. There is a potential for a small increase in sensitivity after integrating longer. I off hand do not believe it to be of practical importance.

IV. A trivial remark about phase calibration.

Yes, it can, and is being done. Outside of the stupendous practical difficulties (mostly of software generation) of doing this on a regular basis, there is a fundamental limitation. The motion of the earth is known only to about one part in 4,000,000. Therefore the calibrator must be withing about 2,000,000 beamwidths of the unknown.