VLB ARRAY MEMO No. 178

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RADIO ASTRONOMY LABORATORY BERKELEY, CALIFORNIA 94720

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MEMO TO: K. I. Kellermann

TOPIC: Phased arrays as VLBA elements

FROM: J. M. Moran

The idea of using phased arrays for VLBA stations is interesting and could be attractive in certain circumstances. (To avoid confusion I refer to VLBA "stations" and phased array "elements"). In principle, phase arrays have many advantages besides potential cost-savings over single parabolic antennas:

1. larger primary beam angle
   a. pointing easier, less gain variation
   b. larger probability of having reference source in beam
2. more uniform maintenance load and less chance of station being off the air
3. amenable to incremental funding and upgrade
4. easier to make combined mm/cm system (?)

Some obvious disadvantages are:

1. system is inherently more complex (and is probably not worthwhile unless $N > 5$)
2. receivers become obsolete more quickly than antennas and retrofitting would be more expensive
3. antennas must be spaced more than ~100 feet to avoid shadowing and there may be significant phase noise due to tropospheric irregularities at high frequencies

A simple cost model is given by the equation

$$C = NF_1 D^{\beta} + F_2 N \qquad (1)$$

where the first term is the cost of the antennas and the second is the cost of the electronics and feeds and

C = cost
N = number of antennas

β = power law for the cost of antennas
F_1= constant for antenna cost
F_2= cost of electronics and feed per antenna

This model is rather naive for several reasons:

(1) there should be another constant to cover cost of the phasing network;

(2) the single power law model for antenna cost is not very accurate and ignores threshold effects;

(3) there are economies of large numbers so F_1 and F_2 are really decreasing functions of N, which would reduce the cost of a phased array.

If we maximize the collecting area, holding the cost fixed, or minimize the cost, holding the collecting area fixed, we obtain the relations:

$$N = \frac{A}{a}\left[\frac{F_1}{F_2}\left(\frac{\beta}{2} - 1\right)\right]^{\frac{2}{\beta}} = \frac{C}{F_2}\left(1 - \frac{2}{\beta}\right) \tag{2}$$

$$D = \left[\frac{F_1}{F_2}\left(\frac{\beta}{2} - 1\right)\right]^{-\frac{1}{\beta}} \tag{3}$$

$$C = \frac{A}{a} F_1^{\frac{2}{\beta}} F_2^{1-\frac{2}{\beta}} \frac{\beta}{2}\left(\frac{\beta}{2} - 1\right)^{\frac{2}{\beta}-1} \tag{4}$$

where:

A = total collecting area = aD^2N
$a = \frac{\pi}{4}\eta \sim 0.4$
η = antenna efficiency

hence $(A/a)^{\frac{1}{2}}$ = effective antenna diameter, D_e

Several interesting results are:

(1) The optimum size of the antennas depends only on F_1, F_2 and β and is independent of the size of the budget, C, and the desired collecting area, A.

(2) The cost of the array is proportional to A rather than $\sim A^{\frac{\beta}{2}}$. Hence, arrays only make sense if $\beta > 2$ and becomes more attractive the larger β is.

Accurate cost estimates using the above formulas depend critically on the assumed value of β. The traditional value for β is 3; JPL studies give 2.7; and Von Hoerner scaled the NRAO 65-m giving a polynomial with $\beta \leqslant 2$ (Von Hoerner, A and A <u>41</u>, 301, 1975). I adopt $\beta = 2.7$. Hence,

$$N = 0.5\,\frac{A}{a}\left[\frac{F_1}{F_2}\right]^{0.74} = 0.26\,\frac{C}{F_2} \tag{5}$$

$$D = 1.5 \left[\frac{F_2}{F_1}\right]^{0.37} \tag{6}$$

$$C = 1.8\left(\frac{A}{a}\right) F_1^{0.74}\ F_2^{0.26} \tag{7}$$

The constant F_1 can be estimated from the NRAO VLBA proposal. The 25 m antenna with a short wavelength limit of ~7 mm would cost $1.717M (1982). Hence F_1 = 290($\$m^{-2.7}$) and has the approximate value

$$F_1 \sim 290\left(\frac{7}{\lambda_m}\right)^{1.5} \tag{8}$$

where λ_m is the short wavelength limit. (Estimates of the wavelength exponent range from 1 to 2). With F_1 and β determined, D depends only on the electronics cost constant F_2. Values of D versus F_2 are given in Table 1. The proposed VLBA receiver package is estimated to cost $500,000. Using this value for F_2, neglecting delay line costs, the optimum element diameter is 23.4 m, For the VLBA budget N ~1. Hence, given the constraints on number of stations (10), receiver cost (performance) and total budget ($50M) the NRAO design appears to be nearly optimum.

There are regimes where phased arrays would be useful in a VLB Array:
(1) electronics cost <$200K per antenna
(2) budget per station (antenna + electronics) >>$2.2M.
For example, if a collecting area of 770 m^2 were required per station (1 44-m antenna) and the electronics cost were $200K/element then the optimum element diameter would be 16.6m and the cost of an array of 7 antennas would be $5.4M as opposed to $8.1M for a single large antenna. As shown in the figure the minimum is very broad and depends only weakly on N. A 10 station array with this collecting area would cost about $82M.

If a millimeter VLBA is desired with λ_m = 3 mm then F_1 ~ 1030. For F_2= $600k the optimum size is 15.8 m and the element cost is $2.4M. The high element cost precludes the idea of arraying very many of them.

I conclude that, given the constraints for the VLBA design of:(1) about 10 stations for good uv coverage (2) a total budget of ~$50M; (3) state-of-the-art receivers; the single antenna per station is best.

Table 1

OPTIMUM ANTENNA DIAMETER IN A PHASED ARRAY WITH $\lambda_m = 7$ mm VERSUS ELECTRONICS COST

F_2(1) k$	D(2) meters	C_E(3) M$
700	26.4	2.70
600	25.0	2.32
500	23.4	1.94
400	21.5	1.54
300	19.3	1.15
200	16.6	.77
150	15.0	.58
100	12.9	.39
50	10.0	.20
30	8.2	.12
10	5.5	.039

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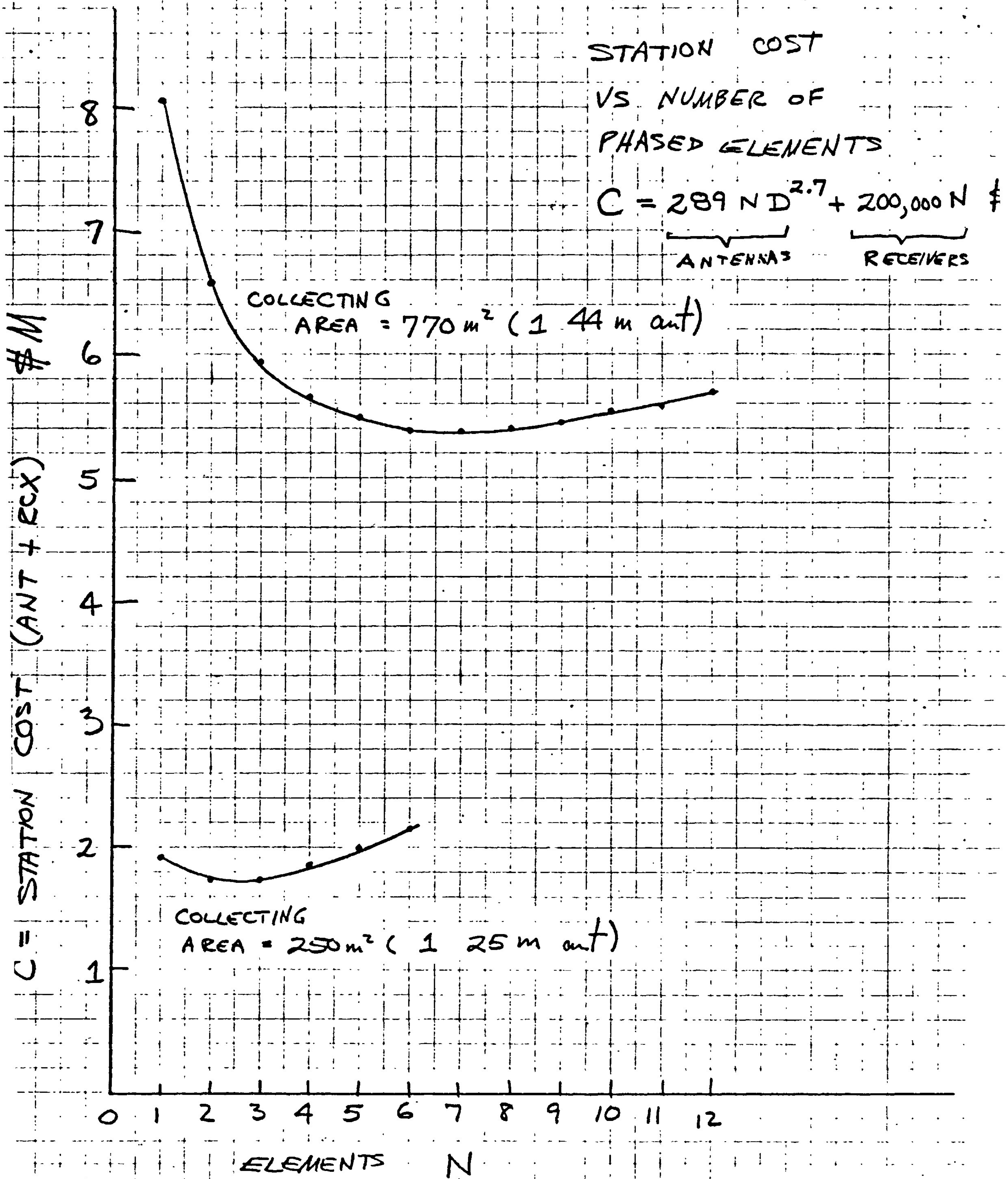
(1) electronics cost (feed, receivers, phasing hardware)
(2) element diameter
(3) element cost (antenna + electronics) = $289D^{2.7} + F_2$


STATION COST
VS NUMBER OF
PHASED ELEMENTS
C = 289 N D^2.7 + 200,000 N $
ANTENNAS
RECEIVERS
COLLECTING
AREA = 770 m² (1 44 m ant)
COLLECTING
AREA = 250 m² (1 25 m ant)
C = STATION COST (ANT + RCX) $M
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
ELEMENTS N