VLB ARRAY MEMO No. 192

Signal Quantization in the VLB Array

A. R. Thompson

March 4, 1983

This memorandum compares the theoretical performance of various quantization schemes, and notes some of the implementation problems. The results essentially support the conclusion of Barry Clark's memorandum of Feb. 14, 1983, but include a wider range of possibilities.

1. Sensitivity of the System.

Table 1 summarizes the parameters that determine the relative signal-to-noise ratios that can be obtained with several quantization schemes and sampling rates. The quantization efficiency η_q given in the second column is the theoretical signal-to-noise ratio relative to that for a non-quantizing (analog) system. Derivation of η_q can be found in various papers, the most important being by Cooper (1970), Hagen and Farley (1973), and Bowers and Klingler (1974). Two operating conditions should be considered; case(1) in which the performance is limited by the tape recording bandwidth, and case (2) in which the RF bandwidth sets the limit.

Case 1. Let the maximum bit rate of the recording system be f_t bps. The maximum RF bandwidth B_{rf} that can accommodated is given in column 3 of Table 1. Note that following the usual terminology Nyquist sampling denotes a sampling rate equal to twice the bandwidth of the unquantized signal. The overall sensitivity is proportional to $\eta_q \sqrt{B_{rf}}$ which is given in column 4. Two-level and three-level quantization result in an essentially equal and best performance. The RF bandwidth of $f_t/3.2$ for the three-level case involves the 5-sample/8-bit encoding scheme discussed by Barry Clark. Note that a 3-sample/5-bit scheme is also possible and simplifies the encoding at the expense of 2% loss in sensitivity. The correlator frequency in column 5 is the maximum that could be used, and it can be decreased by serial to parallel conversion.

Case 2. At the lower frequencies the RF bandwidth is likely to be limited by interference, and for spectral line observations the bandwidth of interest is limited by the characteristics of the line. The sensitivity is then simply proportional to η_q. Three-level quantization provides an increase of 1.27 in sensitivity over two-level sampling which corresponds to a decrease of 0.62 in required observing time. Four level quantization offers a larger increase of 1.4 but requires more complexity in the correlators because three different weights are required in combining the products. The four-level sampler is also more complicated than the three-level one, and for these reasons three-level sampling was chosen for the VLA. Omitting the low level products in the four-level correlator [scheme

2a of Cooper (1970)] simplifies the correlator at the expense of about 1% in sensitivity and is probably a worthwhile modification.

Notice that doubling the sampling rate provides relatively small increases in sensitivity: the factors are 1.17, 1.1, and 1.07 for two, three, and four-level quantization respectively. As the oversampling factor becomes very large η_q tends to 0.80, 0.93 and 0.97 for these three cases.

2. Implementation of Three and Four-level Systems.

The encoder for three-level samples and the greater complexity of the four-level sampler and correlator have been mentioned above. The other main consideration concerns the tolerances on the threshold levels and on possible time differences between sampling at the positive, zero and negative thresholds. Tolerances for three level sampling have been investigated in detail for the VLA, and most of these results[1] can be found in VLA Electronics Memoranda Nos. 112, 114, 136, 175, and 185 and VLA Computer Memorandum No. 150.

Errors in the threshold levels produce two effects: correctable errors in the measured correlation, and a loss in sensitivity. The loss in sensitivity is less than 1% for 10% errors in the threshold levels, and this order of accuracy is not difficult to achieve. Let $+\alpha_1$ and $-\alpha_2$ be the actual threshold levels, and $\pm\alpha_0$ the ideal ones. The combined errors have an odd part $\alpha_1-\alpha_2$ and an even part $\alpha_1+\alpha_2-2\alpha_0$. The odd part produces an offset in the correlator output which, in the VLA, is removed with high precision by the phase switching. The even part is equivalent to an error in the rms signal level and introduces a gain error in the output. In the VLA the gain error is corrected by measuring the total number of high level samples (of both signs) from each sampler, using a 'self-correlator' circuit. In the VLB array phase switching will not be included, but the fringe rotation should have a similar effect upon unwanted offsets. Tolerances in the VLA sampler have not been a critical problem, and in the VLB array should be even less severe because the narrower bandwidths, 25 MHz compared with 50 MHz for the VLA, will relax timing problems by a factor of two.

[1] These and more recent results are summarized in a paper that is currently in preparation by D'Addario et. al.

3. System Choices

3.1 Two Level System

The advantages of the two-level system are simplicity, and the ability to recover quickly from impulsive interference since no a.l.c. is required. The higher RF bandwidth is no particular advantage since the receiving system will be designed to incorporate bandwidth synthesis. Compatibility with existing two-level systems is not a factor since two-level data can equally well be correlated with three or four-level data. Doubling the sampling rate in the case 2 situation gives a sensitivity increase of 17%.

3.2 Three Level System

The three level system gives equivalent performance to the two-level one in case 1 and a factor of 1.27 improvement over two-level sampling at the Nyquist rate in case 2. The complication is the 5-sample/8-bit encoding. The 3-sample/5-bit coding gives a 2% decrease in sensitivity but should be relatively simple to implement.

3.3 Four Level System

Omitting the low-level products at the correlator is probably a worthwhile simplification. The sensitivity is 3% less than that with two-level sampling in case 1, and a factor of 1.37 better than two-level at the Nyquist rate in case 2. There is no encoding problem but the sampler is more complex and the correlator requires twice as many integrating counters as in the three-level case. The sampler combines the action, and probably most of the hardware, of both two-level and three-level samplers. If the sample rate can be doubled it is very easy to convert to two-level action if desired.

3.4 Cost Factors

Choice between the above schemes requires some estimate of the cost of the following factors:

(1) The encoding for a three-level system

(2) The more complex sampler and correlators for a four-level system.

3.5 Concluding Remarks

If we were limited to a single scheme the three level one would probably be the best choice. However the better interference response of the 2-level scheme is a desirable factor since not all antennas will be in quiet locations. Thus a 2-level/4-level combination seems ideal if the 4-level correlator is not too dificult. If the low-level products are omitted the optimum level weighting is 3 or 4 (Cooper 1970) and a value of 2 results in only a small degradation to η_q = 0.85.

References

Bowers, F.K., and Klinger, R.J., Quantization Noise of Correlation Spectrometers, Astron. Astrophys. Suppl., 15, 373-380, 1974

Cooper, B.F.C., Correlators with Two-Bit Quantization, Aust. J. Phys., 23, 521-527, 1970.

Hagen, J.B. and Farley, D.T., Digital-Correlation Techniques in Radio Science, Radio Science, 8, 775-784, 1973.

ART/bmg

1		2 η_q	3 $\frac{B_{rf}}{2f_t}$	4 $\eta_q\sqrt{\frac{B_{rf}}{2f_t}}$	5 $\frac{\left(\begin{array}{c}\text{Correlator}\\\text{frequency}\end{array}\right)}{f_t}$
Sampling at Nyquist Rate					
Two-level		0.637 $\left(\frac{2}{\pi}\right)$	1.0	0.637	1.0
Three-level	5 samp./8 bit	0.81	0.625	0.64	0.625
"	3 samp./5 bit	"	0.6	0.63	0.6
"	1 samp./2 bit	"	0.5	0.57	0.5
Four-level	all products	0.88	0.5	0.62	0.5
"	low-level omitted	0.87	0.5	0.61	0.5
Sampling at 2 x Nyquist Rate					
Two-level		0.74	0.5	0.52	1.0
Three-level	5 samp./8 bit	0.89	0.312	0.50	0.625
	3 samp./5 bit	"	0.3	0.49	0.6
	1 samp./2 bit	"	0.25	0.45	0.5
Four-level	all products	0.94	0.25	0.47	0.5

Tabel 1. Performance Factors for Various Sampling Schemes
f_t = bit rate of tape recorder.