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VLB ARRAY MEMO No. 25

Self-Calibration with a Low SNR

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Dear Ken,

I'd like to offer a few comments on the performance of my present self-calibration scheme and to outline a more sophisticated scheme which I think would be practical for the VLBA and which ought to perform better at low S/N than the present algorithm (though I'm not sure how much better). I'm enclosing a copy of Alan Rogers' paper, which I recall your inquiring about, as well as a copy of my own paper, which I suppose you've already seen.

My present scheme seems to require S/N $\gtrsim$ 2 or 3 in order to perform reliably. I am not optimistic, as Craig seems to be, that with observations of low S/N and with a small number of antennas the algorithm would converge to something reasonable if allowed to run for many iterations. Recall that I use a least-squares method to solve, say, for a phase corruption to assign to each interferometer element. In the presence of well-behaved random noise, and given a perfect source model, the expected value of the solution for a particular array element is the correct solution, but the solution itself is a random variable with a nonzero variance which roughly goes as $1/(n-1)(qn-2)$, where n is the number of elements and q is the number of integration periods during which the phase corruptions are assumed to remain constant. Given an imperfect source model, the distribution would be offset and skewed. Going from one source model to a better one, the width of the distribution doesn't necessarily narrow, though the mean, or the expected value of the solution improves.*

I am enclosing some notes on an extension of my method which, assuming that the systematic errors are smoothly varying on an appropriate time scale, ought to perform somewhat better in the presence of low S/N. I have assumed phase errors only in the notes, but the algorithm could obviously be extended to treat amplitude errors as well. Bob Burns and Ed Fomalont are agreeable to my trying this scheme on VLA data -- it's probably practical enough to apply to short synthesis VLA observations.

Regards,

Fred Schwab

<u>Notation</u>

N = number of integration periods. $t_k \equiv$ time of k-th observation.

$\tilde{V}_{ij}(t) \equiv \tilde{V}(u_{ij}(t), v_{ij}(t), w_{ij}(t))$, visibility observed on the i-j baseline at time t. Spatial frequency (u, v, w) has been parametrized by time.

$V_{ij}^{(m)}$, best estimate, at m^{th} iteration, of true visibility

$\psi_k^{(m)}(t)$, curve, at m^{th} iteration giving best (smooth) least-squares approximation to the phase corruption associated with interferometer element k.

<u>Assumption</u>: Smoothness of the curves ψ_k defining the data corruption. That the ψ_k are adequately represented by spline curves, and that the time scale of the variations is appropriately matched to the frequency of the observations (i.e., is long enough).

<u>Set-up</u>: Choose λ, $0 \le \lambda < 1$. Minimize (m superscript omitted on $\psi_k^{(m)}$):

$$S_\lambda = (1-\lambda) \sum_{k=1}^{N} \sum_{i<j} \frac{1}{\sigma_{ij}} \left| \tilde{V}_{ij}(t_k) - e^{i[\psi_i(t_k) - \psi_j(t_k)]} V_{ij}^{(m)}(t_k) \right|^2$$

m^{th} time derivative of ψ

$$+ \lambda \sum_{l} \int_{t_1}^{t_N} \left| \psi_l^{(m)}(t) \right|^2 dt.$$

The ψ_k can be taken as spline curves with continuous derivatives through order m. Choose cubic splines, say, ($m=2$) with knots at the $\{t_k\}$. In the numerical implementation, one would use (for stability) linear combinations of orthogonal <u>B-splines</u> (de Boor):

$\psi_l(t) = \sum_{k=1}^{2N} \alpha_k B_k(t)$. The B_k form an orthogonal basis for polynomial splines of the chosen order with the given knots, and each B_k has small support ($|\mathrm{supp}(B_k)| = m+1$).

Larger $\lambda \Rightarrow$ greater smoothness of the $\Psi_k^{(m)}$. Including σ_{ij}^2 in the χ^2 term means that the smoothness is chosen (roughly) independent of S/N. When $\lambda = 0$, this is essentially my present method.

Notes:

1) This nonlinear optimization problem, in practical cases, would require an iterative method of solution involving multiple passes through the data (stored on disk), since N would be large.

2) If the χ^2 term were made linear (i.e., by taking logarithms, to fit phases directly) this would be a linear least-squares problem (since polynomials are easy to integrate) with a banded system of normal equations. But such a method would be subject to lobe ambiguities when pairwise differences of the Ψ_k exceed π or when the data are very noisy.

→ 3) But, S_λ has a sparse Hessian (the matrix $\nabla^2 S_\lambda$ of 2nd order mixed partials of S_λ, w.r.t. the parameters defining the spline curves Ψ_k) so that a rapidly convergent Newton type method could be used even when only a "small" (~ 1 AP) amount of storage is available. (I think it's a banded matrix with bandwidth a few $\cdot N \cdot 10$ for a 10 ele. array). There is a sparse matrix package for the AP; still, if there were too little storage, one could use a conjugate gradient algorithm (requiring only ∇S_λ, not ∇^2), but this would require more iterations ($\propto$ more passes through the data).

4) Similar problems, of similar scale, arise in structural mechanics and are solved by the finite element method (basically a spline method).