VLB ARRAY MEMO No. 270

NORTHEAST RADIO OBSERVATORY CORPORATION
HAYSTACK OBSERVATORY

20 September 1983

TO: Marty Ewing

FROM: Alan R. Whitney and Alan E. E. Rogers

SUBJECT: Loss Factors in VLBI Correlator

Your recent revision of Chapter VII (Correlator System) for the VLBA document contains a discussion of correlator loss factors in section VII.6.2. There seems to be some confusion over the losses for different "fractional-bit" correction approaches.

The "fractional-bit" correction approach used in the Haystack correlator is a "continuous" correction that shifts delays and rotator phases in synchronism so that the phase at mid-band is always correct and the phase error at bandedge never exceeds 45°. The average loss due to this method is easily calculated to be 3.5%, not 6-12%. The savings in correlator hardware is substantial.

Furthermore, the "medium rate" case, in which delay changes by the order of one bit per accumulation period, is easily corrected in both amplitude and phase by simple formulas (see VLA Memo 112). The stickiest case actually occurs when the fringe phase changes only by order of <u>one rotation or less</u> during an accumulation period. This case occurs only very seldomly and then only for short periods of time on VLBA-type baselines; in any case, the cure for this problem is to shut off the hardware rotators and let software do the rotation.

xc: H. Hvatum

8.0 MARK III SIGNAL-TO-NOISE RATIO

The Mark III acquisition and processing system achieves the following signal-to-noise ratio:

$$SNR=L(Ta/Ts)(2/\pi)((2BT)^{1/2})$$

where:

- Ta = geometric mean of the antenna temperatures (correlated portion)
- Ts = geometric mean of the system temperatures
- $(2/\pi)$ = clipping loss
- (2BT) = number of bits processed
- L = loss factor

SNR is defined so that one-sigma phase noise is 1/SNR radians in the strong signal case.

The loss factor is 0.93 (3% loss in filter fold over, 4% loss in the 3-level fringe rotation) using software fractional-bit correction. An additional 3.5% loss is suffered when the hardware approximation to the fractional bit correction is used. The signal to noise ratio is, with the exception of the loss factor L, the optimum achievable for a fixed number of data bits processed. At first glance it may appear that the processor lacks a quadrature channel thereby losing square root of two in SNR. The quadrature channel is formed from the information contained in the many lags of the cross-correlation function and is extracted when the complex delay function is derived from the cross-spectral function.

NORTHEAST RADIO OBSERVATORY CORPORATION
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7 March 1983

TO: VLBI Group

FROM: Alan E. E. Rogers

SUBJECT: Fractional bit shift corrections - an update

In two previous memos (10 August 81 and 10 August 82) I gave formula used for the fractional bit correction in the general case of automatic bit shifting. Unfortunately there were typographic errors in both memos. So once more FRNGE applies a frequency dependent phase correction

$$\phi = \left[FB \underset{o}{\pm} 0.5 \right] \left[1 - \left(\frac{INT\ |S|}{|S|} \right) \right] \left[\frac{(\omega - \omega_m)\pi}{2\omega_m} \right]$$

where ϕ = phase shift correction in radians

ω = video frequency

ω_m = video frequency for the band center

$S = \dot{\tau}\, AP/\tau_s$ = "Floating point" # of shifts

where $\dot{\tau}$ = delay rate

AP = accumulation period

τ_s = sample period

INT/S/ = actual number of bit shifts performed in an accumulation period
FB = fractional part of τ/τ_s = "fractional bit shift"
where τ = delay

$\underset{o}{\pm}$ means plus when INT/S/ is odd and FB<0
minus when INT/S/ is odd and FB>0
zero when INT/S/ is even

Explanation of formula:

The first two terms within the [ ] brackets can be derived as follows (see figure):

The average phase is

$$\langle\phi\rangle = \frac{\Delta_1}{2}\left(-1 + \Delta_1 S\right) + \frac{\Delta_2}{2}\left(1 - \Delta_2 S\right)$$

$$= \left(\frac{\Delta_2 - \Delta_1}{2}\right)\left(\frac{1}{2} - \frac{S}{2}\left(\Delta_1 + \Delta_2\right)\right)$$

$$\Delta_1 = \frac{1}{2} - \frac{(N-1)}{S} - \frac{1}{2S} + \Delta$$

$$\Delta_2 = \frac{1}{2} - \frac{(M-1)}{S} - \frac{1}{2S} - \Delta$$

where N is the number of bit shifts in 2nd half of AP
M is the number of bit shifts in 1st half of AP

$$\Delta = FB/S$$

$$\langle\phi\rangle = \left(-2\Delta - \left(\frac{M-N}{S}\right)\right)\left(\frac{1}{2} - \frac{S}{2}\left(1 - \left(\frac{M+N-2}{S}\right) - \frac{1}{S}\right)\right)$$

$$= \left[FB + \left(\frac{M-N}{2}\right)\right]\left[1 - \left(\frac{M+N}{S}\right)\right]$$

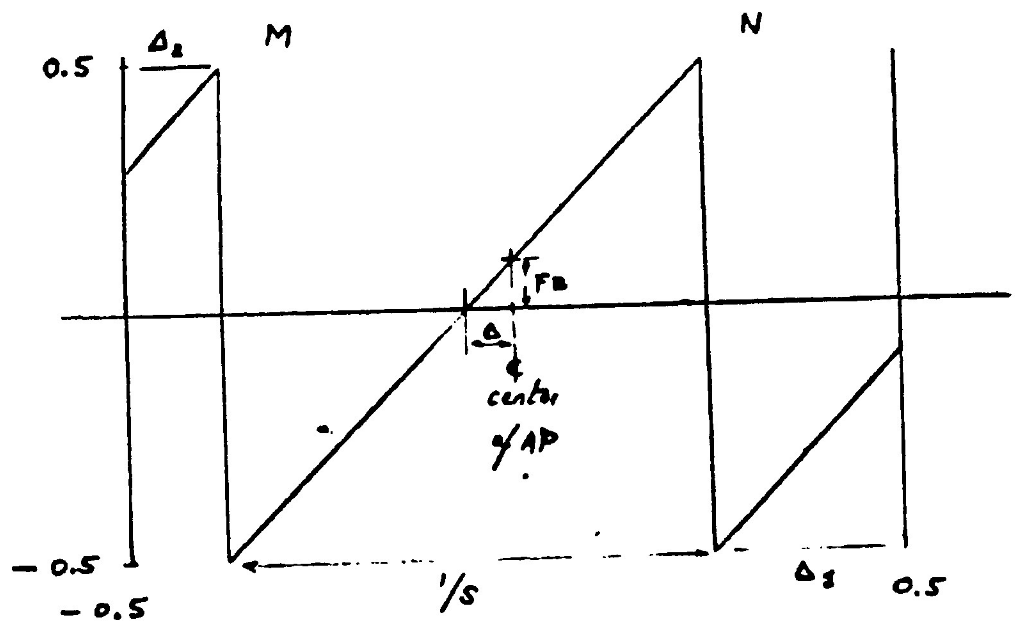
$$M + N = \mathrm{INT}|S|$$

$\left(\frac{M-N}{2}\right) = 0$ when $M + N$ is even - since $M = N$

$= +\frac{1}{2}$ when $M > N$ in which case $FB < 0$

$= -\frac{1}{2}$ when $M < N$ " $FB > 0$

The third term is zero at band center, plus $\frac{\pi}{2}$ at the top of the band and $-\frac{\pi}{2}$ at D.C. For a large number of shifts the 2nd term tends to zero.



SLOPE = S

$$\frac{FB}{\Delta} = S$$

FIGURE NORMALISED PHASE SAW TOOTH WHICH RESULTS FROM BIT SHIFTING