VLB ARRAY MEMO No. 324

Broadband mapping of sources with spatially varying spectral index

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Introduction

We describe two algorithms for mapping a source with spatially varying spectral index from observations taken over a large fractional bandwidth (~ 20-30%). We expect that these procedures or something similar will be useful in the context of VLBA imaging of large sources. We will only present algorithms in this memo, results will be published in a subsequent memo.

Using observations at many close frequencies to image a radio source is not a new idea. The idea of filling in the u,v plane by changing λ rather than the baseline $\underline{b}$ is fairly obvious and, for sources which do not change shape with λ, can be easily performed simply by adding the different u,v data sets together with appropriate scaling. If the source does indeed change with λ then some correction must be made prior to producing a final image. In this memo I will describe one simple method for correcting for the spectral index variations prior to CLEANing of the dirty map formed from the total data set, and one variant of CLEAN to acheive the same purpose.

The method

Suppose that we observe a source at N_ν different frequencies so that the i'th observing frequency is ν_i. Let the dirty image, dirty beam, and true image, corresponding to frequency ν_i be $d_\nu(x,y)$, $b_\nu(x,y)$, and $t_\nu(x,y)$ respectively. Furthermore, assume that the spectral index $\alpha(x,y)$ is straight over the wavelength range of interest. Then we have that :

$$d_\nu = b_\nu * t_\nu + \text{noise}$$

or :

$$d_\nu = b_\nu * ((\nu/\nu_0)^\alpha . t_{\nu_0}) + \text{noise}$$

where * denotes convolution.

The effect of the spectral index variations is mild over a small range in λ and so we may use a linear expansion in the offset $\Delta\nu$ of ν from ν_0 :

$$d_\nu \sim b_\nu * (1+\alpha.(\Delta\nu/\nu_0).t_{\nu_0}) + \text{noise}$$

For convenience let η be the fractional offset in frequency :

$$\eta = \Delta\nu/\nu_0$$

Then we have :

$$d_\nu \sim b_\nu * t_{\nu_0} + \eta.b_\nu * (\alpha.t_{\nu_0}) + \text{noise}$$

and there is one such equation for each observing frequency. Therefore we have N_ν equations from which we wish to solve for 2 quantities : t_{ν_0} and α. This overdetermined set of equations can be solved in a variety of ways to optimise signal to noise or final u,v coverage. To illustrate the main points we will ignore signal to noise considerations and simply add the equations first with equal weight, then weighted by η. We thus obtain two equations :

$$d^{(0)} = b^{(0)} * t_{\nu_0} + b^{(1)} * s_{\nu_0} + \text{noise}$$

$$d^{(1)} = b^{(1)} * t_{\nu_0} + b^{(2)} * s_{\nu_0} + \text{noise}$$

where :

$$d^{(n)} = \Sigma_\nu \eta^n . d_\nu$$

$$b^{(n)} = \Sigma_\nu \eta^n . b_\nu$$

and :

$$s_{\nu_0} = \alpha . t_{\nu_0}$$

Rearranging to solve for t and s we find the following equations hold :

$$b^{(2)} * d^{(0)} - b^{(1)} * d^{(1)} = (b^{(0)} * b^{(2)} - b^{(1)} * b^{(1)}) * t_{\nu_0} + \text{noise}$$

$$b^{(0)} * d^{(1)} - b^{(1)} * d^{(0)} = (b^{(0)} * b^{(2)} - b^{(1)} * b^{(1)}) * s_{\nu_0} + \text{noise}$$

which are convolution equations and can thus be solved using any deconvolution algorithm such as CLEAN or even MEM.

The effective beam :

$$b_{eff} = (b^{(0)} * b^{(2)} - b^{(1)} * b^{(1)})$$

determines the success of the reconstruction of t and s. By the convolution theorem we may see that b_{eff} only has significant Fourier coefficients where samples from two <u>different</u> frequencies overlap in the u,v plane. Thus to ensure a good reconstruction using this approach one must sample each point at least twice. Since we are estimating two quantities this is entirely reasonable and will reassure those who believe that one cannot get something for nothing !

If a second order expansion is performed then one finds that a point in the u,v plane must be sampled at three or more different frequencies to enter into the effective beam.

It may seem that a higher order expansion may be required in some cases, Fortunately, one can regard the above procedure as merely one step in an iterative approach to finding t and α. The first order expansion allows estimation of t and α to first order. The values thus found can be removed from the original data and further corrections found using the same first order expansion. No more than 3 or 4 iterations of this approach such be required.

Simultaneous CLEAN

The requirement that all points be sampled at least twice seems to be suprisingly demanding. Since the effect of λ dependent structure is fairly mild one would expect that the countermeasures should be mild also. In practice, arranging for the VLBA to sample a given u,v point twice will be almost impossible whereas it is relatively simple for the VLA to do so. Thus, although the first order expansion works it probably will not be useful for the VLBA or for arrays such as MERLIN. However, it does suggest a further line of investigation : using a deconvolution algorithm such as CLEAN to find t and/or s directly. For CLEAN, the extension to multiple channels is simple. One should CLEAN all N_ν dirty images simultaneously, each with the appropriate beam. If one finds a clean component at the same place in two different images then a straight line can be fitted to predict the clean components which should be removed at the other observing frequencies. In fact since the fit is linear one need not wait until two components have been found at the same location. Instead, any clean component should simply be removed from all data sets with the appropriate weight $w_{\eta,\eta'}$. The form of the weights can be chosen to optimise any of a number of attributes : e.g. signal to noise or accuracy of reconstruction. Using an analogous argument to that used in the previous section we find that:

$$w_{\eta,\eta'} = (1+\eta.\eta'/<\eta^2>)/N_\nu$$

where we have assumed that $<\eta>$ is zero. The work load is considerably increased since one must clean N_ν images and cross subtract the resulting clean components.

For completeness we should mention that the extension of MEM to handle broadband imaging is conceptually trivial. One simply chooses to maximise the entropy of the true image t at the reference frequency. Practically, the horrors of handling the N_ν dirty images and their associated beams are almost too much to comtemplate.

Results from both the first order method and the simultaneous CLEAN approach will be presented in a later memo. We will also consider the practicability for broadband mapping for the VLBA.