TOLERANCES ON POLARIZATION MISMATCH

A. R. Thompson
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In VLBA Memo No. 253, Alan Rogers has pointed out that variation of the polarization characteristics of the antennas leads to closure errors. In this memorandum the problem will be treated in a manner similar to that used by Thompson and D'Addario (1982).

We start with the general formula for the response of a pair of arbitrarily polarized antennas to a signal of arbitrary polarization, as first derived by Morris, Radhakrishnan and Seielstad (1964). Since we are concerned with the effects of randomly polarized radiation, we take only the term in Stokes' parameter I from equation 8 of the above paper, which can be written as

$$R = \cos(\phi_m-\phi_n)\cos(\theta_m-\theta_n) + j\sin(\phi_m-\phi_n)\sin(\theta_m+\theta_n). \qquad (1)$$

Here ϕ and θ are the parameters of the polarization ellipse of an antenna, and subscripts m and n refer to two antennas of an array. As shown in Fig. 1, ϕ is the position angle of the major axis of the polarization ellipse, and θ is the arctangent of the ratio of the axes. Note that R is unity if $\phi_m = \phi_n$ and $\theta_m = \theta_n$. Thus the effect that we are concerned with depends upon the mismatch of the polarization for different antennas, rather than upon the deviation from any particular standard.

For circular polarization θ_m and θ_n are ideally $(\pm)\pi/4$, and if we write $\theta_m = \pi/4 + \delta\theta_m$, $\theta_n = \pi/4 + \delta\theta_n$, (1) becomes

$$R_c = \cos(\phi_m-\phi_n)\cos(\delta\theta_m-\delta\theta_n) + j\sin(\phi_m-\phi_n)\cos(\delta\theta_m+\delta\theta_n). \qquad (2)$$

The term $(\phi_m-\phi_n)$ and the δ terms all result from constructional tolerances and are all small. Thus we may expand the sine and cosine functions in (2) to obtain

$$R_c = 1-\tfrac{1}{2}[(\phi_m-\phi_n)^2 + (\delta\theta_m-\delta\theta_n)^2] + j(\phi_m-\phi_n). \qquad (3)$$

Since the imaginary term in (2) is small, we have approximated the cosine part of it by unity.

For linear polarization θ_m and θ_n are ideally zero, and following the above procedure we obtain:

$$R_\ell = 1-\tfrac{1}{2}[(\phi_m-\phi_n)^2 + (\theta_m-\theta_n)^2] + j\,(\phi_m-\phi_n)(\theta_m+\theta_n). \qquad (4)$$

To determine the tolerance mismatch of the polarization parameters, equations (3) and (4) are used to compute the antenna-pair responses for a model group of antennas with various parameter deviations. Best-fit antenna gains are then derived using the algorithm by L. R. D'Addario that was used in the Thompson and D'Addario (1982) paper. The differences between the derived gains and the gains assumed in (3) and (4) represent the closure errors that would be introduced in the visibility data. The maximum tolerable residual is taken to be 1%. In selecting the model antenna parameters, the aim is to attempt (by intuition plus some trial-and-error calculations) to obtain a simple system which roughly maximizes the closure errors. Here we consider three different antenna polarizations, with two antennas of each type. The inclusion of two antennas for each polarization clearly increases the residuals since pairs with identical polarization have the maximum response and unmatched pairs have reduced responses. As considered here the polarization is characterized by two parameters, ϕ and θ. Since we have no a' priori information on the relative magnitudes of the deviations in ϕ and θ, it is simplest to consider the case where they are of equal magnitude. The values of ϕ and θ are therefore represented by a single parameter Δ, as shown in Table 1, which also gives the corresponding expression for the antenna-pair responses. Note that the pair gains include both maximum and reduced values, and that the imaginary part appears with both signs. The effects of given polarization variations should therefore be large.

The solutions for circular and linear polarization that correspond to 1% maximum closure errors are shown in Figs. 2 and 3, and correspond to $\Delta = 3.6°$ for circular polarization and $\Delta = 4.9°$ for linear polarization. The rms closure errors are about half the maximum in each case, so use of the maximum value for the tolerance criterion is fairly conservative. For circular polarization, in which we are chiefly interested, the results indicate that ϕ and θ should not deviate by more than 3.6° from their average values over the antennas. These angles are determined chiefly by the feeds, and the mechanical tolerances in the feed placement on the antenna structure should be well within ±3.6° in rotation. The tolerance on θ indicates that the axial ratio of the polarization ellipse should not be greater than 1.12. The ellipticity of the feed is commonly specified in terms of the variation of the intensity of the linear component of the field with the position angle ϕ. The axial ratio of 1.12 corresponds to 1.0 dB maximum variation, which is well within the value of 0.4 dB that is achievable with the polarizers used on the VLA and specified for the VLBA.

In the case of the VLBA, the use of altazimuth antenna mounts and widely separated antennas causes the parallactic angles to vary at different rates as the antennas track, and this effect can introduce a large additional component in the term $(\phi_m-\phi_n)$. How does this affect the closure errors? Consider the effect for circularly polarized antennas, with which we are mostly concerned. Equation (2) can now be written as

$$R_c = \cos(\phi_m-\phi_n+\Phi)\cos(\delta\theta_m-\delta\theta_n) + j\sin(\phi_m-\phi_n+\Phi)\cos(\delta\theta_m-\delta\theta_n) \qquad (5)$$

where Φ is the difference in parallactic angles. With perfect circularly polarized feeds the parallactic rotation introduces a phase shift Φ into the visibility. Φ is accurately calculable and the visibility phase can be corrected, but a small error is introduced. The phase angle of the complex response R_c in (5) is equal to

$$\tan^{-1}\left[\tan(\phi_m-\phi_n+\Phi)\frac{\cos(\delta\theta_m+\delta\theta_n)}{\cos(\delta\theta_m-\delta\theta_n)}\right] \qquad (6)$$

When Φ is subtracted from (6) we obtain $(\phi_m-\phi_n+\delta\phi)$ where $\delta\phi$ is the error in the correction for Φ:

$$\delta\phi = \tan^{-1}\left[\tan(\phi_m-\phi_n+\Phi)\frac{\cos(\delta\theta_m+\delta\theta_n)}{\cos(\delta\theta_m-\delta\theta_n)}\right] - (\phi_m-\phi_n)-\Phi \qquad (7)$$

Now for the cases analyzed above, the maximum deviation of $[\cos(\delta\theta_m+\delta\theta_n)/\cos(\delta\theta_m-\delta\theta_n)]$ from unity is $(\cos 2\Delta)^{-1} = \sec 7.2° = 1.008$. Then the maximum value of (7), which occurs when $(\phi_m-\phi_n+\Phi)$ is about 45°, is equal to 0.23°. This is small compared with the range of $(\phi_m - \phi_n)$ in Table 1, which takes values from zero to 2Δ (=7.2°). Thus for the magnitude of the polarization mismatch that we are considering, the inaccuracy of the correction for the variation of parallactic angle does not significantly affect the closure residuals.

In conclusion it appears that the existing polarizer design should give adequate uniformity of polarization between antennas, at least near the centers of the beams. The variation of the polarization differences over the beams is more difficult to estimate. However, in the VLBA the bandwidth will generally limit the field of view to a small fraction of the antenna beam. In observations where it is necessary to make maps of several source components in different parts of the beam, it seems reasonable to assume that there will usually be one near enough to the beam center to allow the antenna gain factors to be established accurately.

REFERENCES

Morris, D., V. Radhakrishnan and G. A. Seielstad, On the Measurement of Polarization Distributions over Radio Sources, Astrophys. J., 139, 551-559, 1964.

Thompson, A. R. and L. R. D'Addario, Frequency Response of a Synthesis Array: Performance Limitations and Design Tolerances, Radio Science, 17, 357-369, 1982.

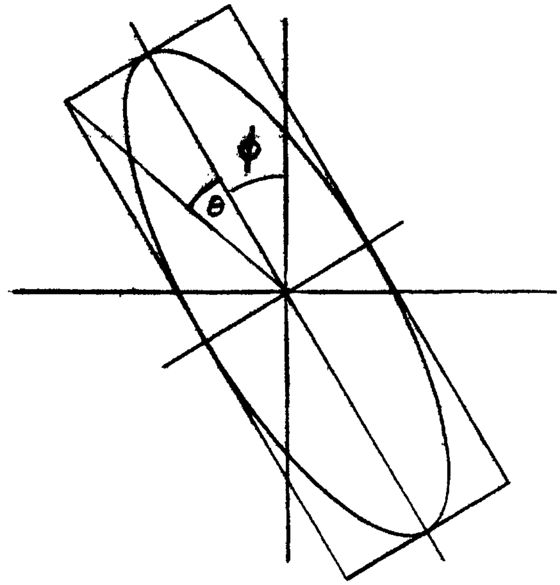

Fig. 1. Polarization ellipse specified in terms of ϕ and θ

Table 1. Polarization Parameters used in the Model Calculations.

ANTENNAS	ϕ	θ	ANTENNA PAIRS	PAIR RESPONSES	
1	0	0	1-2, 3-4, 5-6.	1	CIRCULAR POLARIZATION
2	0	0	1-3, 1-4, 2-3, 2-4.	$1-\Delta^2+j\Delta$	
3	$-\Delta$	$-\Delta$	1-5, 1-6, 2-5, 2-6.	$1-\Delta^2-j\Delta$	
4	$-\Delta$	$-\Delta$	3-5, 3-6, 4-5, 4-6.	$1-4\Delta^2-j2\Delta$	
5	Δ	Δ			
6	Δ	Δ			
1	0	0	1-2, 3-4, 5-6	1	LINEAR POLARIZATION
2	0	0	1-3, 1-4, 2-3, 2-4.	$1-\Delta^2+j\Delta^2$	
3	Δ	$-\Delta$	1-5, 1-6, 2-5, 2-6.	$1-\Delta^2-j\Delta^2$	
4	Δ	$-\Delta$	3-5, 3-6, 4-5, 4-6.	$1-2\Delta^2$	
5	Δ	Δ			
6	Δ	Δ			

Output of Program CLOSUR.SAI

Antenna-pair	Input		Output		Closure Discrepancy	
	Amp	Pha(deg)	Amp	Pha(deg)	Amp(%)	Pha(deg)
1-2	1.000000	.0000000	1.001617	.2163331a-4	.1616836	.2163331a-4
1-3	.9960000	3.600000	.9955933	3.596707	-.4082993a-1	-.3293335a-2
1-4	.9960000	3.600000	.9955933	3.596705	-.4083067a-1	-.3295004a-2
1-5	.9960000	-3.600000	.9955933	-3.596670	-.4082993a-1	.3329665a-2
1-6	.9960000	-3.600000	.9955933	-3.596672	-.4082993a-1	.3328234a-2
2-3	.9960000	3.600000	.9955933	3.596685	-.4082993a-1	-.3314942a-2
2-4	.9960000	3.600000	.9955933	3.596683	-.4083067a-1	-.3316701a-2
2-5	.9960000	-3.600000	.9955933	-3.596692	-.4082993a-1	.3308058a-2
2-6	.9960000	-3.600000	.9955933	-3.596693	-.4082993a-1	.3306627a-2
3-4	1.000000	.0000000	.9896061	-.1725483a-5	-1.039394	-.1725483a-5
3-5	.9840000	-7.190000	.9896061	-7.193377	.5697221	-.3377020a-2
3-6	.9840000	-7.190000	.9896061	-7.193379	.5697221	-.3378511a-2
4-5	.9840000	-7.190000	.9896061	-7.193375	.5697221	-.3375232a-2
4-6	.9840000	-7.190000	.9896061	-7.193377	.5697206	-.3376722a-2
5-6	1.000000	.0000000	.9896061	-.1401955a-5	-1.039393	-.1401955a-5

RMS Values of Discrepancy: amplitude= .4829415 %, phase= .2981575a-2 deg.

Complex-Gain Solutions for Antennas.

Antenna	Real	Imaginary
1	1.000808	.0000000
2	1.000808	-.3778773a-6
3	.9928301	-.6240623a-1
4	.9928301	-.6240620a-1
5	.9928301	.6240560a-1
6	.9928301	.6240562a-1

No. of Iterations = 26

Fig. 2. Solution for circular polarization, $\Delta = 3.6^{\circ}$.

Output of Program CLOSUR.SAI

Antenna-pair	Input		Output		Closure Discrepancy	
	Amp	Pha(deg)	Amp	Pha(deg)	Amp(%)	Pha(deg)
1-2	1.000000	.0000000	.9972032	.1874049a-4	-.2796836	.1874049a-4
1-3	.9930000	.4170000	.9936901	.1400015	.6949455a-1	-.2769985
1-4	.9930000	.4170000	.9936901	.1400001	.6949455a-1	-.2770000
1-5	.9930000	-.4170000	.9936901	-.1399699	.6949455a-1	.2770301
1-6	.9930000	-.4170000	.9936901	-.1399711	.6949455a-1	.2770289
2-3	.9930000	.4170000	.9936901	.1399828	.6949455a-1	-.2770172
2-4	.9930000	.4170000	.9936901	.1399813	.6949455a-1	-.2770187
2-5	.9930000	-.4170000	.9936901	-.1399886	.6949455a-1	.2770114
2-6	.9930000	-.4170000	.9936901	-.1399899	.6949455a-1	.2770101
3-4	1.000000	.0000000	.9901894	-.1456703a-5	-.9810627	-.1456703a-5
3-5	.9860000	.0000000	.9901894	-.2799714	.4248858	-.2799714
3-6	.9860000	.0000000	.9901894	-.2799726	.4248858	-.2799726
4-5	.9860000	.0000000	.9901894	-.2799700	.4248858	-.2799700
4-6	.9860000	.0000000	.9901894	-.2799712	.4248858	-.2799712
5-6	1.000000	.0000000	.9901894	-.1220936a-5	-.9810627	-.1220936a-5

RMS Values of Discrepancy: amplitude= .4292582 %, phase= .2486539 deg.

Complex-Gain Solutions for Antennas.

Antenna	Real	Imaginary
1	.9986006	.0000000
2	.9986006	-.3266253a-6
3	.9950796	-.2431467a-2
4	.9950796	-.2431442a-2
5	.9950796	.2430918a-2
6	.9950796	.2430940a-2

No. of Iterations = 19

Fig. 3. Solution for linear polarization, $\Delta = 4.9^{o}$.