VLB ARRAY MEMO No. 629

Frequency Sequences for Reduced Number of Channels, With Applications to the VLBA

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It appears likely that there will be only 8 video converters for our initial geodetic observations with the Pie Town antenna of the VLBA. Even if we borrow extra video converters for Pie Town, there will be other VLBA antenna that we will want to use with some regularity. We can not count on having more than 8 video converters at those stations. Therefore, I have investigated what are the best frequencies for 8 video converters when combined with our usual 6- and 8-frequency sequences for S- and X-band. I have also thought a bit about how the VLBA should use 8 frequencies for its own geodetic or astrometric observations.

I will refer here and there to Alan Rogers's note of 2 March 1988, which spurred me into writing this note.

I have made several assumptions here: 1) large rms spanned bandwidth is desirable; 2) low delay resolution function (DRF) sidelobes are desirable; and 3) wide ambiguity spacings are desirable. (Sort of sounds like motherhood and apple pie, doesn't it!)

Spanned bandwidth and sidelobe levels are rather qualitative, but I can put real limits on ambiguity spacing requirements. Based on our experience from the 1980 MERIT observations, we can expect ionospheric effects on delays to approach 40 nsec at S-band. This is for intercontinental baselines, at times near the peak of the sunspot cycle. In a recent trans-Atlantic experiment (X.Atl-6, November 3, 1987), there is S-band delay scatter of about 25 nsec caused by the ionosphere. The expected effects at X-band are reduced by a factor of about 13, so the largest ionospheric effects at X-band will be only 3 nsec or so.

For ease of ambiguity resolution, ambiguity spacings of greater than 40 nsec are highly desirable at S-band. Almost any spacing will be OK for X-band, although small spacings will make it more difficult to determine source and station parameters when there are large *a priori* uncertainties. On shorter baselines, such as those we expect to use initially with Pie Town, ionospheric effects will have a lot of "common mode," and smaller ambiguity spacings could be tolerated if necessary.

The signal-to-noise ratio and the sidelobe levels in the delay resolution function determine whether or not FRNGE finds the correct group delay. By considering the probability that noise causes the apparent signal level at a sidelobe peak to exceed the amplitude at the true peak, we can determine the likelihood that a false delay value is chosen. The result is

$$P(\text{falsepeak}) = \frac{1}{2} - \Phi\left(\frac{A_m - A_s}{\sqrt{2}\sigma}\right)$$

where A_m is the expected signal value at the true delay, A_s is the expected amplitude of a sidelobe peak, σ is the noise level, and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{t^2/2} dt \quad . \qquad (1)$$

(I thank Clara Kuehn for this derivation.) This value will be an underestimate, by at least a factor of two for high SNR, since there are at least two sidelobes of equal height. (The theoretical DRF is symmetric about the central peak.) For low SNR, a more complicated joint probability should be considered. Table 1 shows the probability of an incorrect delay determination, as a function of σ, the difference between the height of the correct peak and the highest sidelobe, expressed in terms of the noise level. The values in Table 1 are two times the value given by (1).

Table 1
Probability of Choosing Incorrect Peak in Delay Resolution Function

Relative Height of Sidelobe ($A_m - A_s$)	Probability of False Detection
0.5σ	72 %
1σ	48 %
2σ	16 %
3σ	3.4 %
4σ	0.46 %
5σ	0.04 %

For reasonable assurances that the correct delay will be found for nearly all scans, we see that sidelobes should be more than 4σ below the peak of the delay resolution function.

S-Band

Our standard sequence is 0-1-4-10-15-17 for relative frequencies. A multiplier of 5 MHz is applied. Since the unit cell is 5 MHz, the ambiguity spacing is 200 nsec. The rms bandwidth is 33.1 MHz. The largest sidelobes are about 50% of the peak. What are the best 3 (or 4) frequencies to chose from this sequence?

Alan Rogers suggested a 0-4-10 sequence. (I will ignore multipliers for the rest of this note when giving sequences, but I will include them when giving rms bandwidths.) This sequence has ν_{rms} = 20.6 MHz, 100 nsec ambiguities, and 83% sidelobes. The rms bandwidth is significantly less than usual, and the 83% sidelobes require an SNR of about 25 to avoid finding the wrong delay.

I suggest two other sequences. The first is 1-4-10. This is actually the minimum redundancy array (MRA) for three elements, with a unit cell of 15 MHz (when multiplied by 5). It has ν_{rms} = 18.7 MHz, 66.7 nsec ambiguities, and sidelobes of only 60%. The bandwidth is slightly less than Alan's sequence, and the ambiguities are smaller, but still adequate for S-band. The low sidelobe level allows good observations with SNRs of 10.

My preferred 3-frequency sequence for S-band is 0-10-15. This is another version of the three element MRA, with a unit cell of 25 MHz when multiplied by 5. ν_{rms} = 31.2 MHz, just about the same as our regular sequence (33.1 MHz). The sidelobes are still 60%, so modest SNR's are quite acceptable. The ambiguity spacing is only 40 nsec. On long baselines, this would be marginal, but for the shorter western US baselines for which we will use Pie Town, it should be acceptable. (I checked the ionosphere corrections at S-band for HRAS-Mojave for the ATD-4 experiment on April 14, 1987. The total range is about 11 nsec.)

An alternative sequence for S-band is the 4-frequency sequence 1-4-15-17. This set of frequencies has ν_{rms} = 34.4 MHz, 82% sidelobes, and 200 nsec ambiguities. Unfortunately, this leaves only 4 frequencies for X-band. As I discuss below, there are not any very good 4-frequency subsets of our usual X-band sequence. However, if one is not constrained to choose from the pre-existing sequence, there are reasonable 4-frequency sequences that could be used at X-band. In that case, however, one would also be freed from having to choose from our existing sequence at S-band, and would probably make other choices. I comment on such matters at the end of this note.

Figures 1, 2, and 3 show the 3-frequency delay resolution functions discussed in this section.

X-Band

Our usual X-band sequence is 0-1-4-10-21-29-34-36, multiplied by 10 MHz. For this sequence, ν_{rms} = 140.2 MHz, the sidelobes are less than 60%, and ambiguities are 100 nsec. The largest ionospheric effects we have ever seen at X-band are only a few nanoseconds. Hence, except for geometric effects that may be pertinent for new sources and stations, fairly small ambiguity spacings can be tolerated at X-band.

The best 5-frequency subset that I have been able to find is 0-4-10-34-36. This has ν_{rms} = 152.1 MHz (larger by almost 10% than our usual sequence), 80% sidelobes, and 50 nsec ambiguities. Table 2 shows several other 5-frequency sequences. I think the sequences in Table 2 are the only other subsets of our standard X-band sequence that have $\nu_{rms} \gtrsim 140$ MHz and sidelobes $\lesssim 85\%$. These sequences all have 100 nsec ambiguities.

Table 2
5-Element Frequency Sequences for X-Band

Sequence	rms Bandwidth (MHz)	Maximum Sidelobe
0-1-4-21-36 *	140.4	84 %
1-4-10-34-36	149.9	82
0-4-10-29-36	141.8	83
0-1-21-29-34	141.0	82
1-4-21-29-36	137.0	82
0-4-21-29-36	139.6	85

* this sequence suggested by Alan Rogers

The 50 nsec ambiguity of my preferred sequence should not be a problem. (If the *a priori* position of Pie Town has large errors, we may have to home in on it with single band delays and fringe rates.) SNR's should be greater than about 20 to avoid noise problems in determining the correct delay. Figure 4 shows the delay resolution function for my preferred sequence.

Several possible 4-frequency sequences for X-band are shown in Table 3. I could not find any other sequences with both ν_{rms} greater than 130 MHz or so and sidelobes less than about 90%. Even the sequences in Table 3 all have fairly high sidelobes, and would require high SNR ($>$ 25) for correct determination of delay. They all have ambiguity spacings of 50 nsec. Their only advantage is that they would allow four frequencies at S-band, with a corresponding improvement in ambiguity spacing. As I have argued earlier, I think this is not a real problem, at least for western US experiments.

Table 3
4-Element Frequency Sequences for X-Band

Sequence	rms Bandwidth (MHz)	Maximum Sidelobe
0-4-10-36	140.3	87 %
0-4-10-34	131.9	86
0-10-34-36	154.3	87

Recommendations

None of what follows should be a surprise at this point!

At S-band, I recommend that we operate the VLBA video converters at relative frequencies of 0-50-75 MHz as a subset of our usual sequence of 0-5-20-50-75-85 MHz. This results in a nice delay resolution function with low sidelobes and a wide rms bandwidth. the ambiguity spacing is a bit small, at 40 nsec, but I believe this will not cause any problems for our initial use of Pie Town. Any scans which have adequate SNR for other baselines in western US mobile experiments will have adequate SNR for Pie Town baselines so that noise effects will not give incorrect delays.

At X-band, I recommend the sequence 0-40-100-340-360 MHz as a subset of our regular sequence of 0-10-40-100-210-290-340-360 MHz. This sequence appears to be the best compromise between large bandwidth and moderate sidelobes.

The desired SNR for X-band scans involving Pie Town is >20. The least sensitive baseline in our initial experiments involving Pie Town are between Mojave and the mobiles, where I assume an SNR >10 will be achieved (as is routinely the case). The Pie Town antenna has a diameter D of 25 meters *versus* 12.2 m for Mojave. The expected system temperatures and efficiencies at Pie Town are 35 K, 71% at S-band; and 49 K, 69% at X-band (NRAO VLBA Specifications, as of May 1987). We know the ratio of efficiency to system temperature (η/T) at Mojave from measurements of system equivalent flux density (which I take to be 3000 Jy at both S- and X-band for Mojave.)

The sensitivity of an interferometer element goes as $D\sqrt{\eta/T}$. At S-band, Pie Town should be a factor of 3.3 more sensitive than Mojave. At X-band, Pie Town should be a factor of 2.7 more sensitive. The SNR for a given scan also depends on the number of video converter channels. At S-band, Pie Town observations will have half as many channels as other baselines, so the SNR will be reduced by a factor of $\sqrt{2}$, relative to CDP stations, for a given scan length. Thus, the total sensitivity advantage is 2.3 at Pie Town. At X-band, the number of channels is 5 compared to 8, for an additional sensitivity loss of a factor of $\sqrt{5/8}$, and a total sensitivity advantage for Pie Town of 2.1. Thus all SNRs on baselines to Pie Town can be expected to be greater than 20. This may be a little lower than desired at X-band, but only a few scans on baselines to the mobiles will be affected. The fixed antennas will all give very high SNRs on baselines to Pie Town, so we should not have any trouble getting a good position for the new antenna.

VLBA Geodesy and Astrometry

The sequences proposed here are definitely not those that one would choose if starting from scratch, unconstrained. The imposition of picking a subset of the CDP sequences at S- and X-band leads to compromise. Given that 8 frequency channels are available at VLBA antennas, I would propose a pair of 4-frequency sequences.

At S-band, a four frequency minimum redundancy array sequence (pattern 0-1-4-6) is good for modest total bandwidths (less than 100 MHz or so). The unit cell is then less than 16 MHz, and ambiguity spacings are greater than 62.5 nsec. This should be adequate for easy ambiguity resolution on baselines of any length. For larger bandwidths, other sequences would be more desirable. The sidelobes would be somewhat higher, but the unit cell would remain small so that ambiguity resolution would not be a problem. Sequences such as those discussed in the next paragraph should be used.

At X-band, four frequencies are adequate to span several hundred megahertz. The four element zero/minimum redundancy array (0-1-4-6) leads to unit cells of 50 to 100 MHz, and ambiguity spacings of only 20 to 10 nsec. These would be OK if only ionospheric effects were important. Geodetic and astrometric considerations suggest that larger spacings would be more convenient. For instance, 10 nsec corresponds to a position offset of only 3 meters. If a new antenna is used whose position is not known this well *a priori*, it will be a nuisance to find its position. (Bootstrapping from rates and single band delays would probably work, but why bother if there are other ways to avoid this problem.) I suggest that sequences with larger "holes" be considered for X-band. For instance, a 0-1-6-9 sequence, multiplied by 40 MHz has ν_{rms} = 147.0 MHz, 71% sidelobes, and 25 nsec ambiguities. Sequences such as 0-1-3-9 and 0-1-4-10 also look nice, in terms of sidelobes and ambiguity spacings. Figures 5, 6, and 7 show the delay resolution functions for these three sequences.

When (or if) we do experiments with several VLBA antennas and long baselines (more than about 2000 km), I think we will want to use new sequences, rather than using less desirable subsets of our regular sequences.

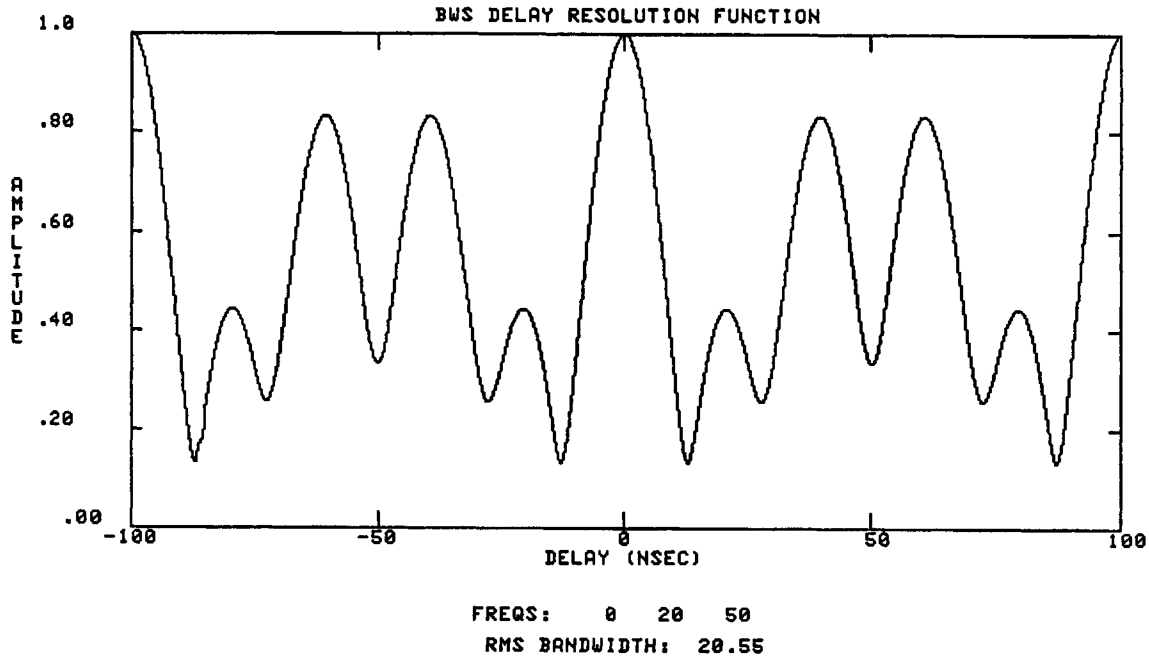


Figure 1. Delay Resolution Function for S-band Sequence 0-4-10, Multiplied by 5 MHz

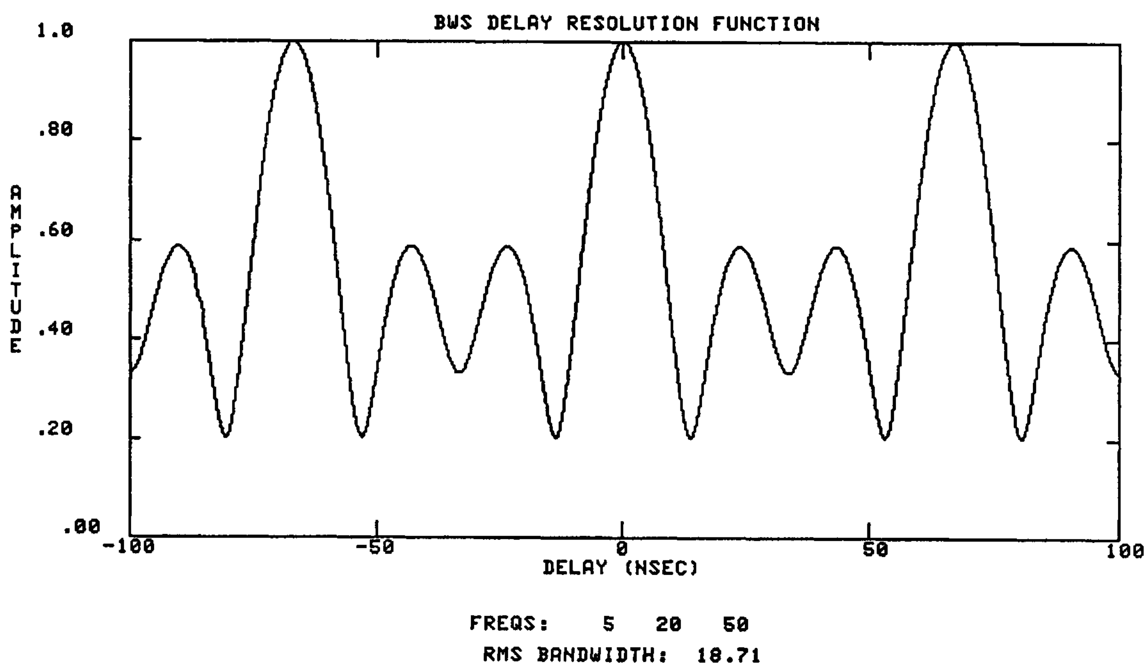


Figure 2. Delay Resolution Function for S-band Sequence 1-4-10, Multiplied by 5 MHz

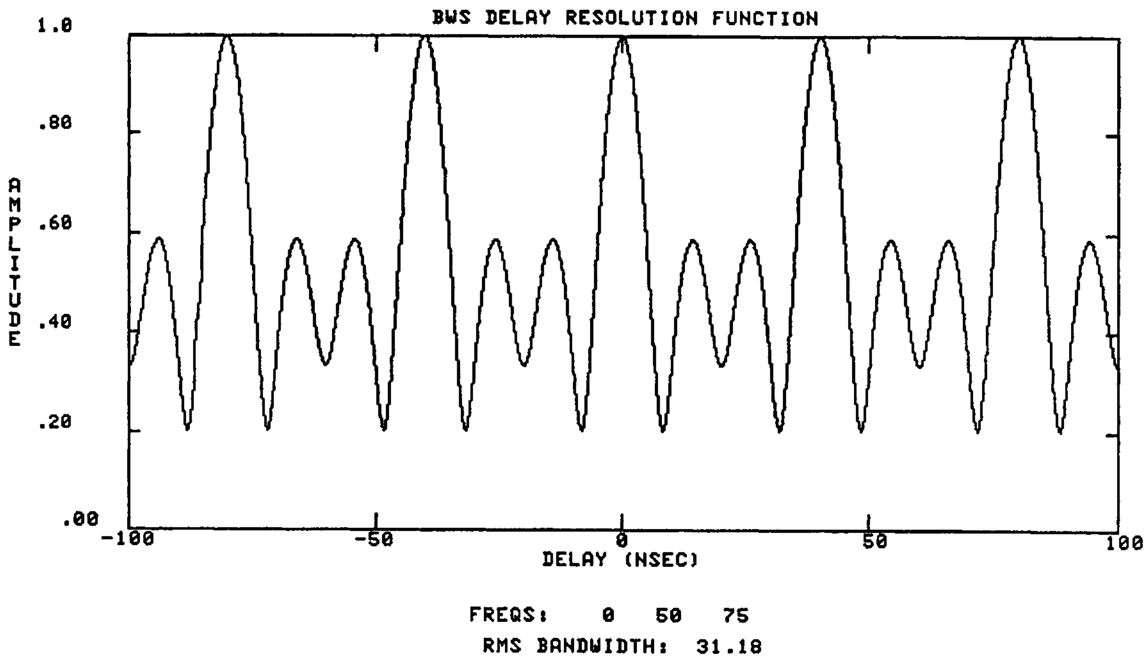


Figure 3. Delay Resolution Function for
S-band Sequence 0-10-15, Multiplied by 5 MHz

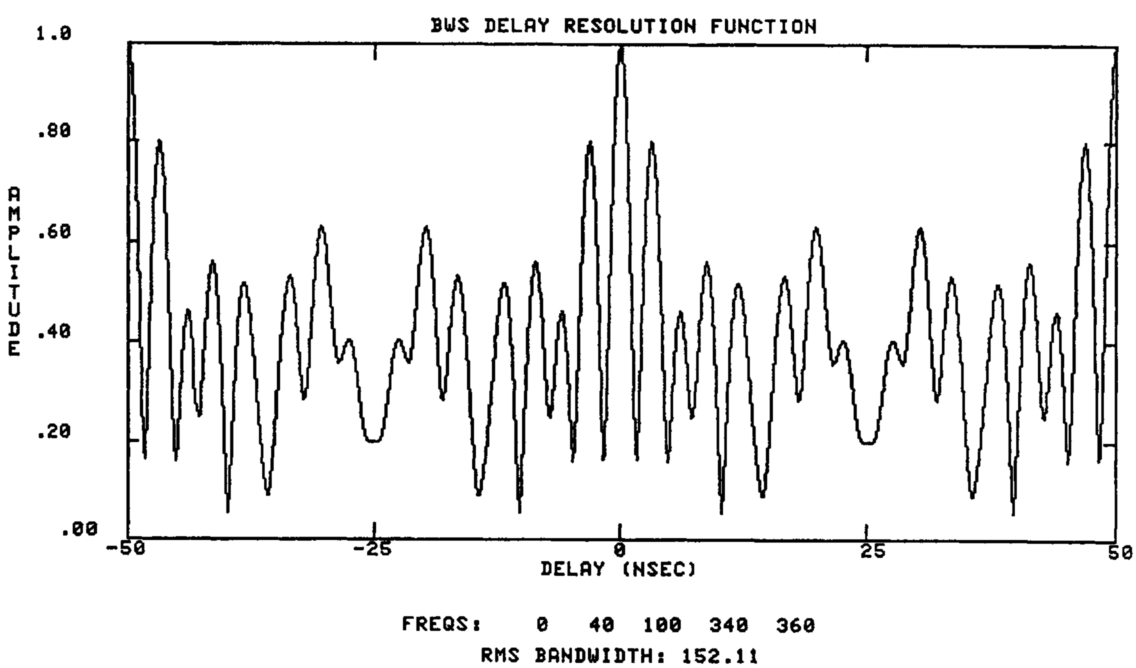
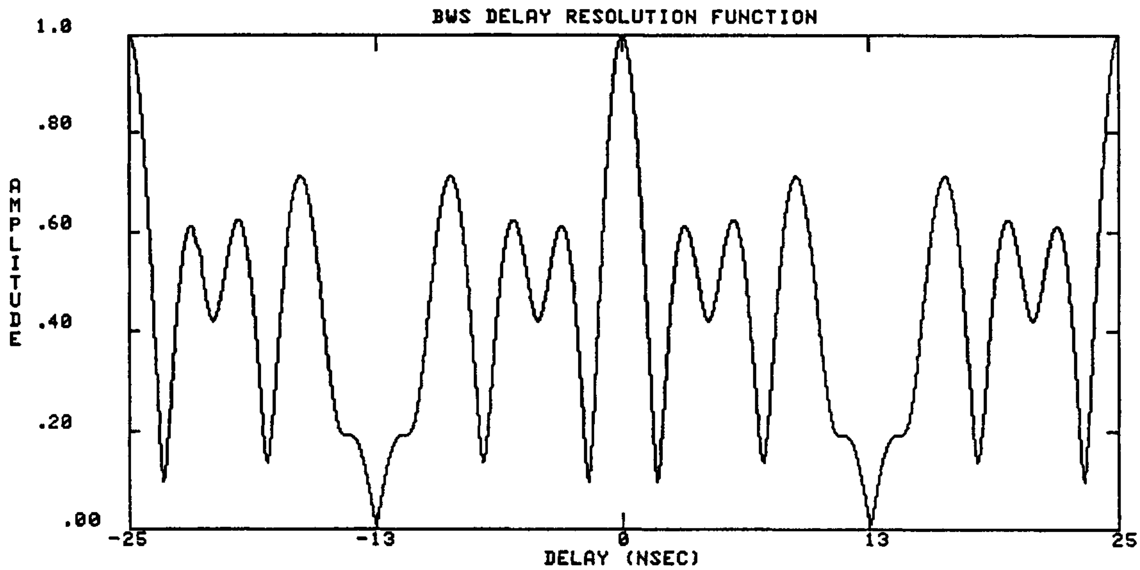
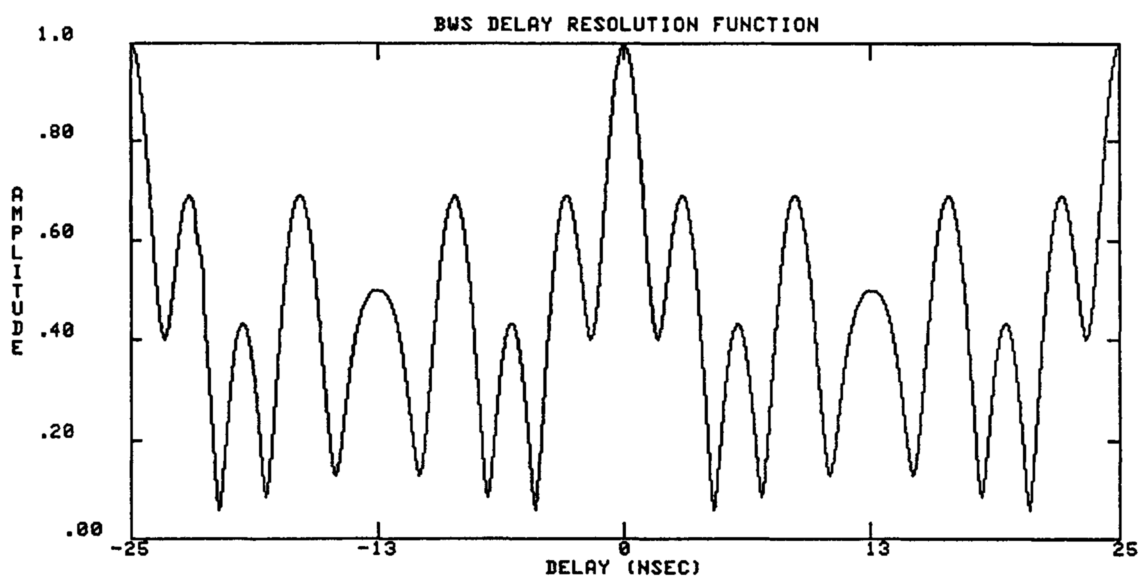


Figure 4. Delay Resolution Function for
X-band Sequence 0-4-10-34-36, Multiplied by 10 MHz



FREQS: 0 40 240 360
RMS BANDWIDTH: 146.97

Figure 5. Delay Resolution Function for Sequence 0-1-6-9, Multiplied by 40 MHz



FREQS: 0 40 120 360
RMS BANDWIDTH: 139.64

Figure 6. Delay Resolution Function for Sequence 0-1-3-9, Multiplied by 40 MHz

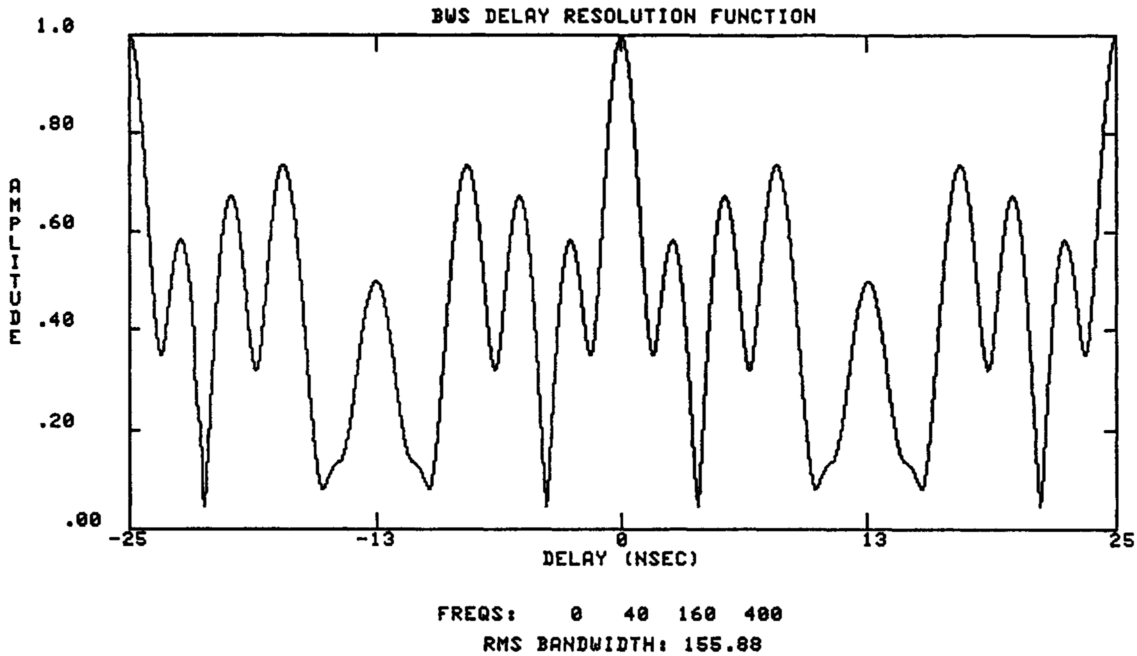


Figure 7. Delay Resolution Function for Sequence 0-1-4-10, Multiplied by 40 MHz