VLB ARRAY MEMO No. 674

FREQUENCY SEQUENCES FOR BANDWIDTH SYNTHESIS ON THE VLBA

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Bandwidth synthesis observations for geodesy and astrometry use several narrow bandwidths at separate frequencies to obtain very wide effective bandwidths for accurate delay measurements. The delay is determined by finding the peak in the Fourier transform of the phases from each of the narrow bands or, equivalently, by fitting for a phase slope across the full spanned bandwidth. The number of separate narrow bands is set by the number of available baseband converters (video converters in Mark III terminology). It can be extended by frequency switching, but this has not been very satisfactory in the past.

The Fourier transform of the phases is characterized by a central peak at the true delay surrounded by sidelobes at other delays. If the frequencies are integer multiples of some unit frequency spacing, there will be a 100% sidelobe at the delay corresponding to the inverse of that unit spacing. The optimum choice of the frequencies of the individual bands is one that provides the lowest maximum sidelobe between the central peak and a delay just outside the range allowed by the a-priori knowledge of the geometry or by the delay determined using phase slopes within the individual narrow bands (the "single band delay"). In general, the optimum choice has a unit spacing such that there is a 100% sidelobe ("ambiguity") just outside the desired delay range.

In addition to low sidelobes, it is desirable to have the maximum "rms spanned bandwidth" for the most accurate delay measurements. Simply put, of various sequences that provide good sidelobes, one that has the individual bands concentrated near the ends of the spanned bandwidth should be chosen.

Current state-of-the-art geodetic observations, such as the Crustal Dynamics Project R&D observations, span bandwidths of 725 MHz at 4 cm (X band) and 125 MHz at 13 cm (S Band) using the Mark III system with a total of 14 channels. The ambiguity spacing used is 50 ns at 4 cm and 200 ns at 13 cm. The maximum sidelobes at delays less than the ambiguity are in the neighborhood of 50 to 65%. Six frequencies are used at 13 cm while 8 are used at 4 cm. These parameters are comfortable in the sense that the chances of the fringe fitting ending up on a sidelobe are small and the ambiguities are well outside the range to which the delays can be constrained by the experiment a-priori information. The single band delays are not used.

The VLBA system has only 8 baseband converters and so can only use 8 individual frequencies without frequency switching. The CDP observations at Pie Town to date have used frequency switching to increase the number of frequencies. However the experience with this has been poor in terms of data reliability and sensitivity. Dave Shaffer has written a memo to the CDP that suggests some frequency sequences that avoid the switching at the cost of having to tolerate high sidelobes and lower ambiguity spacing. It should be possible to use such sequences on the VLBA antennas because of the high sensitivity of the VLBA. It is likely that the CDP observations in 1991 will use one of these sequences.

This memo shows, in rather general terms, the constraints placed on sidelobe levels and ambituity spacings by the limited number of channels available on the VLBA. It also suggests a viable scheme for obtaining bandwidth synthesis data on the final VLBA without the need to tolerate high sidelobes and without frequency switching.

Figure 1 is a plot of the lowest peak sidelobe levels obtainable with 3, 4, and 5 separate frequencies as a function of the number of unit frequency spacings (as discussed above) in the spanned bandwidth (the number of "unit cells"). Note that for a given spanned bandwidth, a larger number of "unit cells" implies a smaller unit frequency spacing and, hence, a larger ambiguity spacing in delay. The plotted points were determined by calculating the transforms for all possible frequency sequences of each length. For each transform, the peak sidelobe at delays between 0 and the ambiguity spacing (where there is a 100% sidelobe) was found and recorded. Then the lowest such peak sidelobe level for all transforms of a given length was determined and plotted. This then represents the best sidelobe performance that can be obtained with a frequency sequence with that number of "unit cells". These calculations were carried out in a few hours on a PC.

There are usually several different frequency sequences with similar performance, so the rms bandwidth can also be taken into account in making a final choice. The rms bandwidth determines the width of the central delay peak of the transform and, hence, the accuracy with which the delay can be determined. The final sequence chosen is likely to have somewhat worse sidelobe performance than shown in the figure because of this compromise. No more will be said about rms bandwidths in this memo, although they should be considered when actual sequences are picked.

Figures 2 and 3 are the same as Figure 1, except that the scale is shown in terms of the ambiguity spacing for spanned bandwidths of 125 MHz and 725 MHz respectively. If one can determine the minimum ambiguity spacing that can be tolerated, these plots show approximately the magnitude of the sidelobes that must be tolerated.

I have asked a number of people from the geodetic community about the minimum tolerable ambiguity spacing if the single band delays are not used. The answers are typically about 100 ns at 13 cm where the ionosphere can cause fluctuations at nearly this level and somewhat less than 50 ns at 4 cm. With these constraints, it is clear that 3 frequencies are inadequate at either wavelength so the only option is to use 4 at both. This in turn implies that the sidelobes will be a bit less than 80% at 13 cm and just under 90% at 4cm. This is probably ok for geodetic observations using strong sources thanks to the high sensitivity of the VLBA systems (by the standards of the geodetic community which uses many small antennas). However, it is not very good for astrometry of weak sources.

Figure 4 shows the constraints, as a function of SNR, that can be placed on the synthesized delay using the single band delays for individual channel bandwidths of 2, 4, 8, 16, and 32 MHz. A one sigma error of $1/(1.8 * BW * SNR)$ is assumed for the determination of the single band delay and the constraint plotted is that given by 5 sigma. Using the figure, one can determine the minimum ambiguity spacing that can be tolerated for data of a given channel bandwidth and minimum SNR.

Current practice is to use 2 or 4 MHz channels with the Mark III system. For 2 MHz channels, it is clear that the single band delays cannot provide enough of a delay constraint to allow the ambiguity spacings to be reduced over current practice unless the SNR is very high. Channels of 4 MHz are better, but still do not really allow enough reduction of the ambiguity spacing to permit the use of only 3 channels at 13 cm. Therefore, with the Mark III system, the single band delays are not of much help.

With the VLBA system, however, the single band delays can be a large help. SNR's of about 12 with 8 MHz channels, or as low as 7 for 16 MHz channels should provide sufficient delay constraints from the single band delays to allow the use of ambiguity spacings as low as 25 ns in the bandwidth synthesis. With this ambiguity spacing, it should be possible to achieve sidelobe levels below 70% with 3 channels when spanning 125 MHz at 13 cm. The 5 remaining channels can be used at 4 cm allowing sidelobe levels below 70% to be used at that wavelength also.

For either system, the "single band" delay constraint can be improved by recording the upper and lower sideband from each video converter or baseband converter. This allows effective bandwidths of 8 MHz for Mark III and 32 MHz for VLBA to be obtained if the required sidebands are recorded. This may be useful on the VLBA for weak sources. It may also be useful in order to obtain 8 MHz effective bandwidth on experiments involving both Mark III and VLBA systems. For such experiments, the number of different frequencies is limited by the VLBA and the channel bandwidth is limited by Mark III. However, limitations of the Mark III phase cal system may make it difficult to relate the single band delays to the multiband delay.

In current practice, there is only one phase cal tone per channel. As a result, the bandwidth synthesis delays, which are derived after correcting for the phase cal phases, refer to the point of the injection of the phase cal tones which, in the geodetic systems, is always at the reciever. The single band delays, however, are not corrected using phase cals and so refer to the sample clock. The two differ in a time dependent manner by any electronic delays between the phase cal injection point and the samplers. If there are fluctuations in the delays through the cables and electronics, this can make it hard to relate the single band delays to the synthesized delay and limits the ability to use the single band delays to resolve ambiguities. The VLBA phase cal system is being designed to allow more than one tone to be measured within each channel. This will allow the single band delays to be refered to the same point as the synthesized delays. Therefore, on the VLBA, the single band delays can be used to constrain the synthesized delays.

The maximum sustained bandwidth of the VLBA is 128 MBits per second which would allow the use of 8 channels of 8 MHz each. Any higher recording rate requires either more frequent tape changes or a duty cycle of less than one. However, the achieved SNR is purely a function of the number of bits recorded (if coherence considerations can be ignored as they can be here). Therefore, a mode that uses 8 channels of 16 MHz each, with a duty cycle of 0.5, uses the same amount of tape and achieves the same SNR, but allows much improved single band delays to be determined. There is also no effect on the time taken to correlate the two cases. With 8 MHz channels, there will be a factor of 2 speedup with the VLBA correlator but the full elapsed time must be correlated. With 16

MHz channels, there is no speedup, but the tapes are only moving half of real time on record but can be moving all the time on playback so the correlator still does the job in half of observe time.

A duty cycle of 0.5 is well matched to the typical geodetic observing style. The CDP R&D observations use 90 second scans on each source and keep the tape stopped while the antennas move to the next source. For observations all over the sky, the total time spent slewing is probably similar to that spent on source giving a natural duty cycle near 0.5.

Therefore, it appears that a VLBA observing mode that spans bandwidths similar to those used by the CDP R&D projects and uses 8 channels of 16 MHz each with a 50% duty cycle for tape motion will provide high quality geodetic and astrometric data. The single band delays will constrain the synthesized delays sufficiently tightly that frequency sequences can be found that give sidelobe levels well below 70%, even with data of limited SNR.

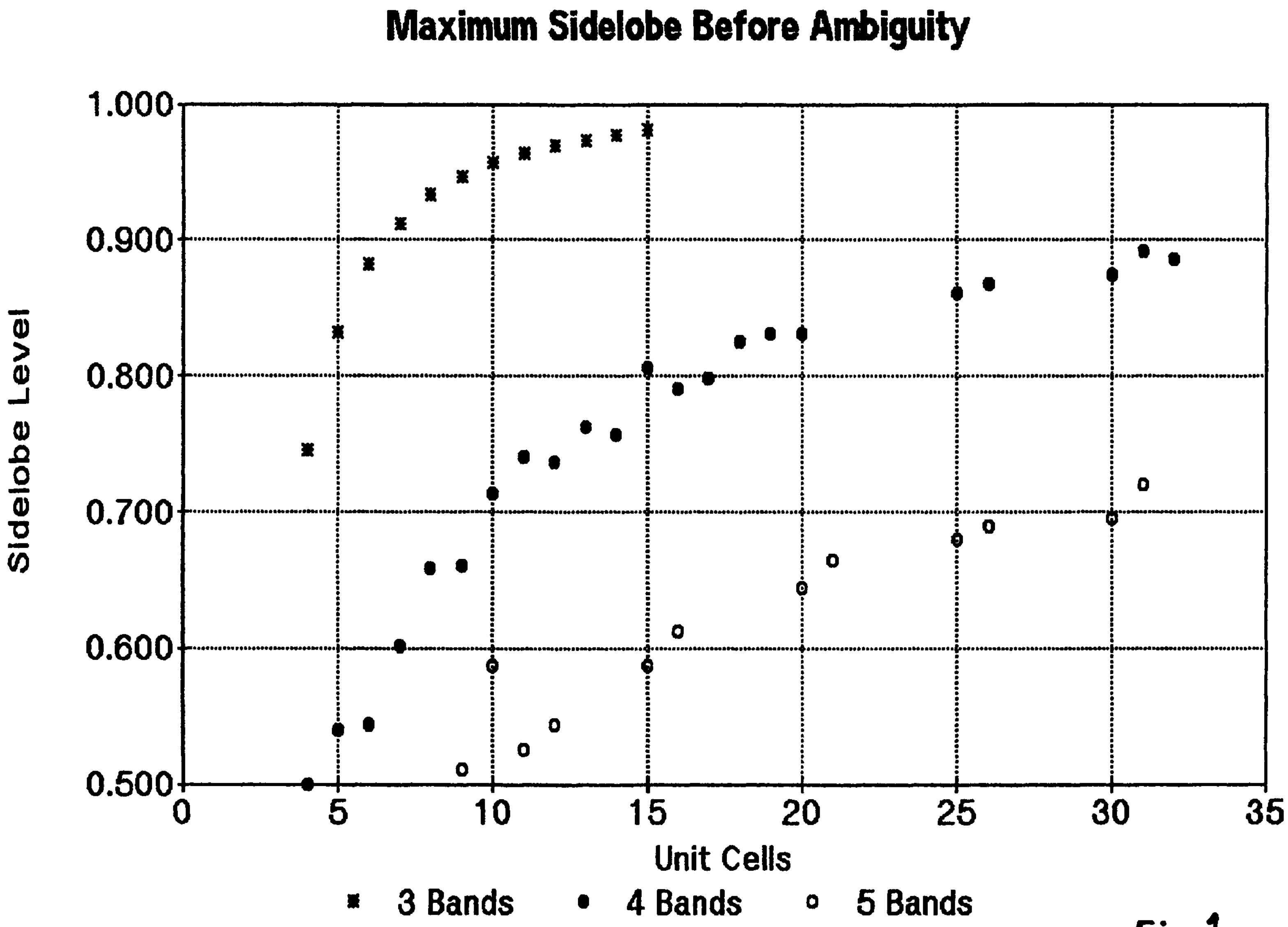
Maximum Sidelobe Before Ambiguity
Sidelobe Level
1.000
0.900
0.800
0.700
0.600
0.500
0
5
10
15
20
25
30
35
Unit Cells
3 Bands
4 Bands
5 Bands

Fig 1

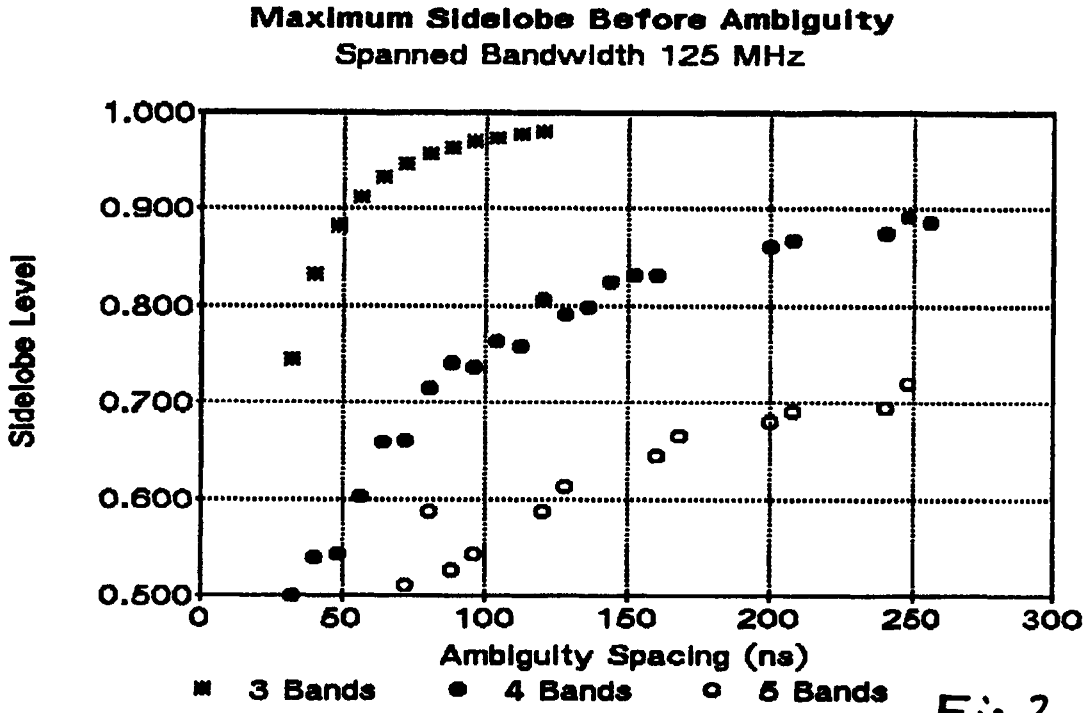
Maximum Sidelobe Before Ambiguity
Spanned Bandwidth 125 MHz

Fig 2

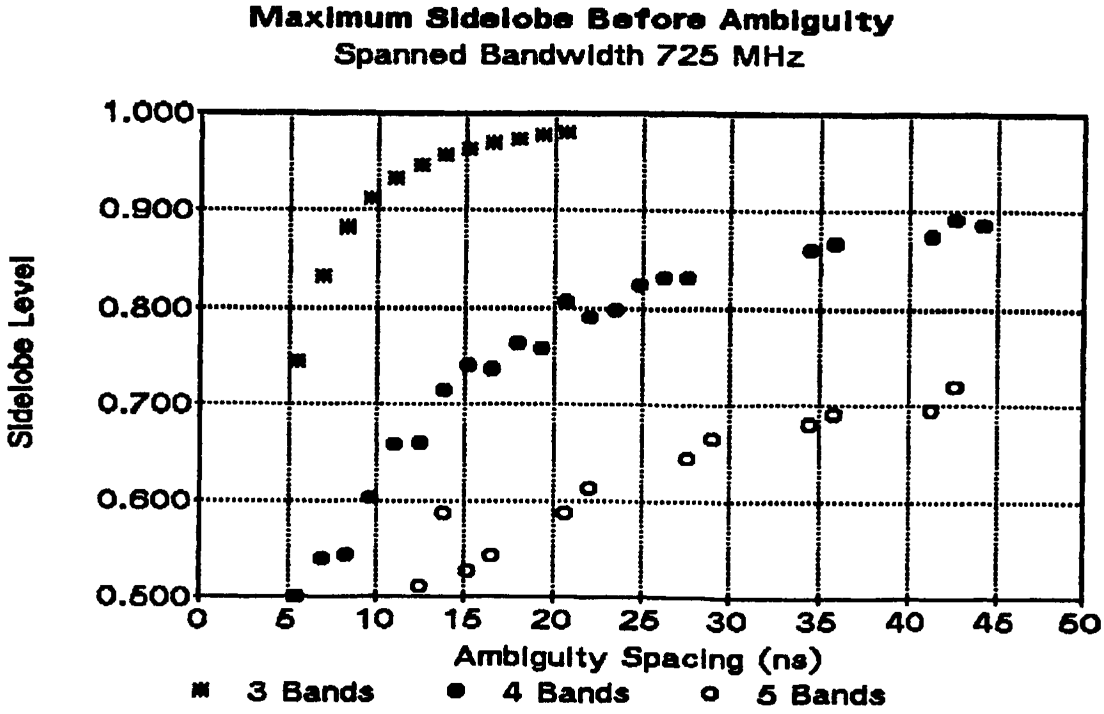
Maximum Sidelobe Before Ambiguity
Spanned Bandwidth 725 MHz

Fig 3

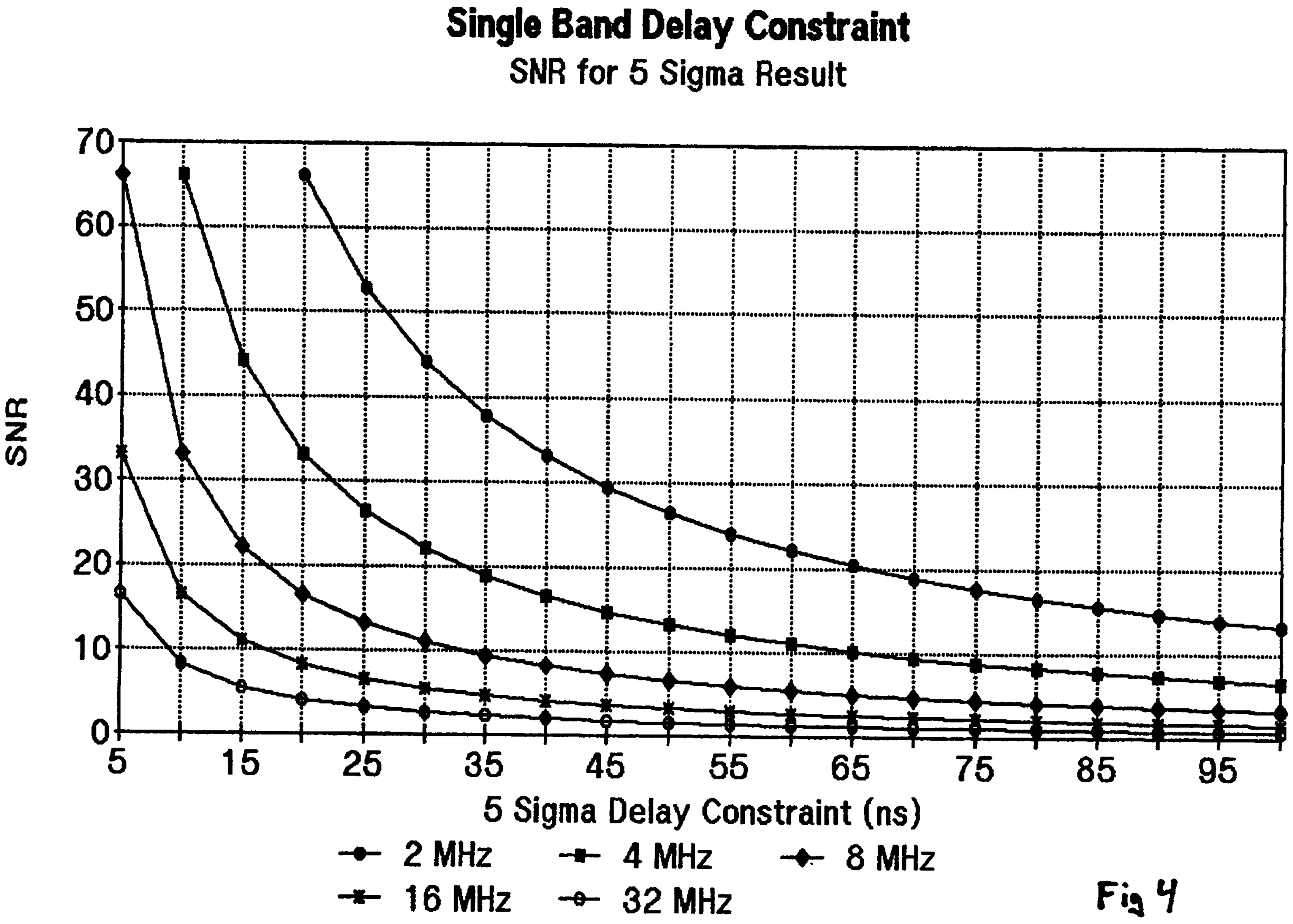
Single Band Delay Constraint
SNR for 5 Sigma Result
SNR
70
60
50
40
30
20
10
0
5
15
25
35
45
55
65
75
85
95
5 Sigma Delay Constraint (ns)
2 MHz
4 MHz
8 MHz
16 MHz
32 MHz

Fig 4