Weak Source Detection: Comparison of Two Methods

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Introduction

Previous memos in this series have discussed the sensitivity of global fringe fitting. For the VLBA the improvement has been estimated to be 2.3 or 2.7 over a single baseline. Either value is significant and clearly emphasizes the potential of global methods. While there is little doubt that the existing global methods improve the images in the high SNR case we need a clear demonstration that the detection threshold can be improved by a global fringe search using current software.

True multidimensional search

A maximum likelihood estimate (MLE) of the station based delays and rates and phases can be made by maximizing

$$Re \sum_{baselines} \sum_{frequency} \sum_{time} a(w,t) e^{-iw(\tau_i - \tau_j)} e^{-it(R_i - R_j)} e^{-i(\theta_i - \theta_j)} \tag{1}$$

For three stations this becomes a search in four dimensions (delay and rate for stations 2 and 3). In Memo #3, I showed that the detection threshold for this method at least $(N/2)^{1/2}$ better than on a single baseline.

Method using one- and two-baseline combinations of phasors

The algorithm in AIPS [2,3] searches for a maxima of

$$F_{ij} = \sum_{frequency} \sum_{time} \{ w_{ij} e^{iD_{ij}(w,t)} + \sum_k w_{ikj}\, e^{iD_{ik}(w,t)} e^{-iD_{kj}(w,t)} \}\ e^{-iw(\tau_i - \tau_j)} e^{-it(R_i - R_j)} e^{-i(\theta_i - \theta_j)} \tag{2}$$

where I have omitted three baseline combinations. Where $D_{ij}(w,t)$ is the phase of the cross-spectral function on baseline ij. The advantage of this algorithm is that the search is only 2-dimensional, F_{ij} can be separately maximized on each baseline. The disadvantage is that the two baseline phasor is the product of two phasors and suffers a loss when the SNR of each data sample has a low SNR. This loss factor [4] which becomes $(\pi/8)^{1/2}(SNR)$ when $SNR << 1$ and is shown in figure for a wide range of SNR) reduces the signal in the 2 baseline phasor. For small search ranges the data samples can be averaged to maximize the SNR of each segment [4] to ameliorate this problem.

For a search over n rates (assuming we already know the delay) the SNR of each segment will be approximately $n^{-1/2}$ relative to that of a single baseline. If the 2-baseline combinations are optimally weighted for a given apriori single baseline signal-to-noise the weight in the low SNR case should be

$$w_{ikj} \approx (\pi/8)^{1/2}\ SNRB\ n^{-1/2} \tag{3}$$

where SNRB is the SNR of a single baseline using the MLE algorithm, and the overall of the global search *SNRG* becomes

$$SNRG \approx SNRB \left\{ 1 + (N-2) \left(\frac{\pi}{8}\right) SNRB^2 \, n^{-1} \right\}^{1/2} (\pi/4)^{1/2} \tag{4}$$

$$\approx 5 \;\; when \;\; N=10, \;\; n=10, \;\; SNRB = 3$$

Because of the loss factor 3 baseline combinations will add little if any SNR in the low SNR regime. $(\pi/4)^{1/2}$ is the SNR loss factor in the low SNR limit associated with using phasors instead of complex amplitudes.

The more general form of the phasor loss factor *L(s)* which is plotted in the figure is given by

$$L(s) = s \left(\pi/8\right)^{1/2} e^{-s^2/4} \left[I_o \left(s^2/4\right) + I_1 \left(s^2/4\right) \right] \tag{5}$$

where s = SNR of each segment = SNRB $n^{-1/2}$

and I_o, I_1 are hyperbolic Bessel functions. The total loss of a product of phasors is multiplicative or additive when measured in dB and thus severely limits the sensitivity of the "triple product" or "bispectrum" used in optical interferometry using closure information [4,5].

Conclusions

For a wide global search it would be desirable to use an optimal multi-dimensional search were it not for the mind boggling computing task. Unless the windows are very narrow ($\leq$ 10 delay and/or rate channels) the current algorithm in the AIPS global search may suffer losses making it no better for fringe detection than the Alef and Porcas [6] method of constrained windows.

References

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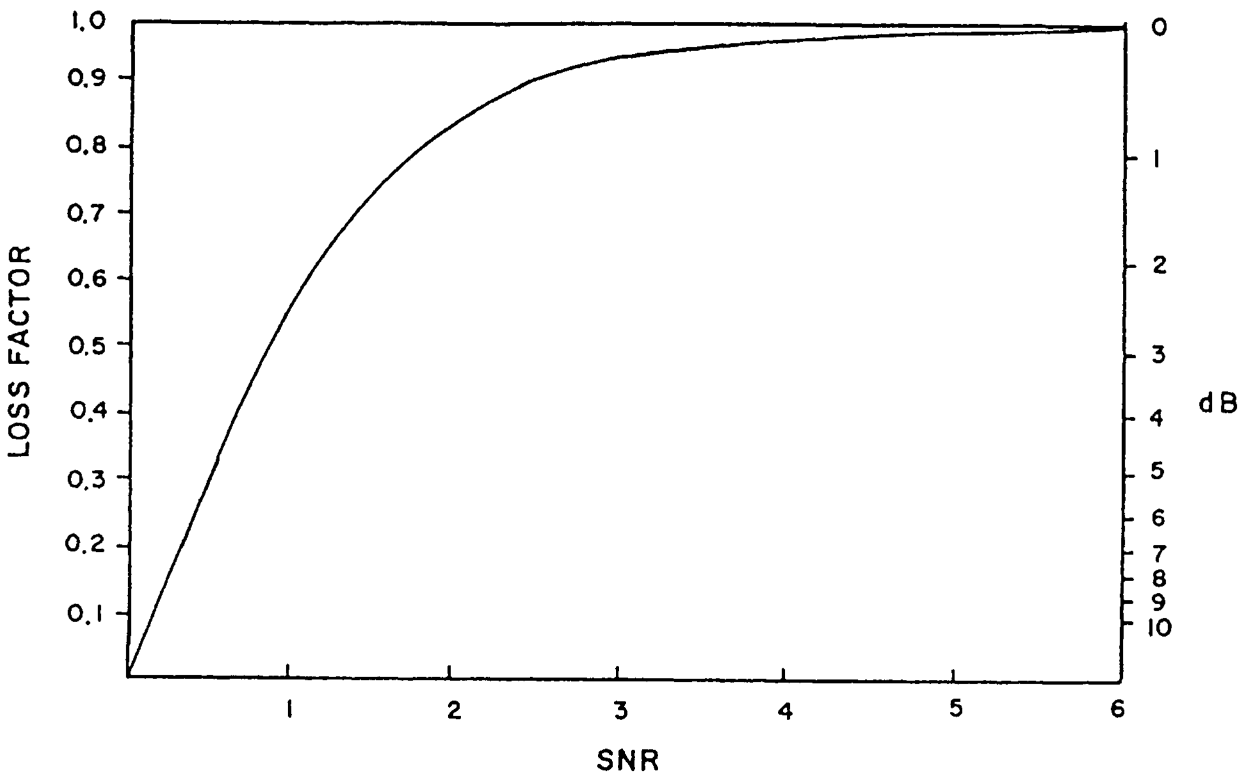

Loss factor as a function of segment SNR from Rogers, et al. [5]