The time interval where pulse cal tones can demonstrate curious cross correlation

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Abstract

The pulse cal tones which are injected to the input of the receivers can demonstrate the cross correlation because they created from the high stable frequency standards. If the fringe rate is high enough the cross correlation due to pulse cal tones will be self averaged by fringe rate stopping procedure. But if fringe rate is small this artificial correlation can be large at some time interval. The value of this time interval is figured out as a function of the required reducing of the correlation due to the self averaging, baseline and declination of the source. This time interval can be large for small baseline and the source with high declination.

1 The approach to the problem

The example of the spurious cross correlation spectrum is given at the Fig. 1. The eight pulses separated by 1 MHz are obviously seen at baseline **BR - FD**. The pulses exist at other baselines as well, but they are depressed by fringe stopping procedure because fringe rate is close to zero only for the baseline **BR - FD** and only for the given time.

Let's estimate the time interval where self averaging is not enough to depress the spurious fringes until given value. The expression for fringe rate is given by the following equation:

$$\omega_{fr} = 2\pi\frac{2\pi}{T}B_{xy}\sin(HA-\alpha_b)\cos(\delta) \tag{1}$$

where B_{xy} is projection of the baseline on the equatorial plane in wav elength;
HA is the hour angle of the source at Greenwich;
α_b is longitude of the baseline relatively Greenwich;
δ is the declination of the source;
$T = 24$ hour is duration of the day

The fringe rate is equal zero at the point $HA = \alpha_b$ and can be described by the linear approximation at a small vicinity of this point.

$$\omega_{fr} = 2\pi\left(\frac{2\pi}{T}\right)^2 B_{xy}\cos(\delta)\Delta t = C\cdot t \tag{2}$$

where $C = 2\pi\left(\frac{2\pi}{T}\right)^2 B_{xy}\cos(\delta)$

The phase of the fringe rate used at the fringe rate stopping is determined by the integral of the fringe rate:

$$\Delta\phi = \frac{C}{2}t^2 \tag{3}$$

Under influence of the phase deviation $\Delta\phi$ integrated during time Δt the curious correlation is depressed by the factor F:

$$\frac{1}{F} = \frac{1}{\Delta T}\left|\int_{-\Delta T/2}^{\Delta T/2} \exp(i\Delta\phi)\,dt\right| \tag{4}$$

Substituting (3) into equation (4) we can obtain the following expression for the factor:

$$\frac{1}{F} = \frac{1}{z}\sqrt{C^2(z) + S^2(z)} \tag{5}$$

$$z = \pi\sqrt{2B_{xy}\cos(\delta)\frac{\Delta T}{T}} \tag{6}$$

where $C(z) = \int_0^z \cos\frac{\pi}{2}x^2\,dx$
$S(z) = \int_0^z \sin\frac{\pi}{2}x^2\,dx.$
$C(z), S(z)$ are the Fresnel's integrals [1]

S as function of C in orthogonal coordinates gives so called Cornu's spiral (fig. 2) [1]. Geometric sense of $\sqrt{C^2(z) + S^2(z)}$ is the distance of the given point of Cornu's spiral from the origin of coordinates. This distance, as it is seen from the Fig. 2., is equal $\frac{\sqrt{2}}{2}$ when $z \to \infty$. So the suppressing factor of the pulse cal tones F can be approximated by the straight line:

$$F = \sqrt{2}\cdot z = 2\pi\sqrt{B_{xy}\cos(\delta)\frac{\Delta T}{T}} \tag{7}$$

Fig. 3 shows the suppressing factor of the pulse cal tones as the function of parameter z (equation (5)). The straight line approximation (equation (7)) is shown as well. It is clear that the straight line describes the behavior of the suppressing factor of the pulse cal tones good enough.
Using the linear approximation for the suppressing factor of the pulse cal tones (equation (7)), we can find the expression for the time interval ΔT, where the suppression of the pulse cal tones by fringe rate stopping is less than given factor F.

$$\Delta T = \frac{FT}{2\pi\sqrt{B_{xy}\cos(\delta)}} \tag{8}$$

2 Conclusion

Let's return back to the example of "bad" interval when suppressing factor of the pulse cal tones is not enough to prevent their detection (fig. 1). Let's calculate the length of the "bad" interval using equation (8). Baseline **BR-FD** has the equatorial projection $\sim 1800km$. It is equal $\sim 10^7\lambda$ ($\lambda = 18cm$). Declination of the source is -15 degree. Substituting these values into equation (8), we estimate $\Delta T \sim 4\cdot F$ sec. For $F = 15$ $\Delta T \sim 1$ min. So we were so "lucky" to appear inside of 1 minute bad interval with the baseline **BR-FD**. Now, how big should factor F be selected? The power of pulse cal tones is fixed at the level of 3% of the noise temperature (private message of C. Walker). So if we do not want to see pulse cal tones we have to satisfy the following condition:

$$\frac{0.03}{F} < \frac{1}{\sqrt{\Delta f\tau}} \tag{9}$$

In our example (Fig. 1) $\Delta f = 0.5MHz, \tau = 23s$. So the factor F should selected as large as 100 and the length of the "bad" interval can be as large as 6 min. The length of the "bad" interval can be much bigger for shorter baseline and higher declination of the observed source. For highest declination 90 degrees fringe rate equal zero ever and as a result the length of the "bad" interval is infinite. Fortunately $\cos(\delta)$ is presented under sqrt and this effect is not so strong. Even for declination 80 degrees the "bad" interval is more only in 2.5 times in comparison with declination equal zero.

References

[1] Jahnke, E. and Emde F. Tables of functions with formulae and curves. Dover Publication, Inc. New York, 1945

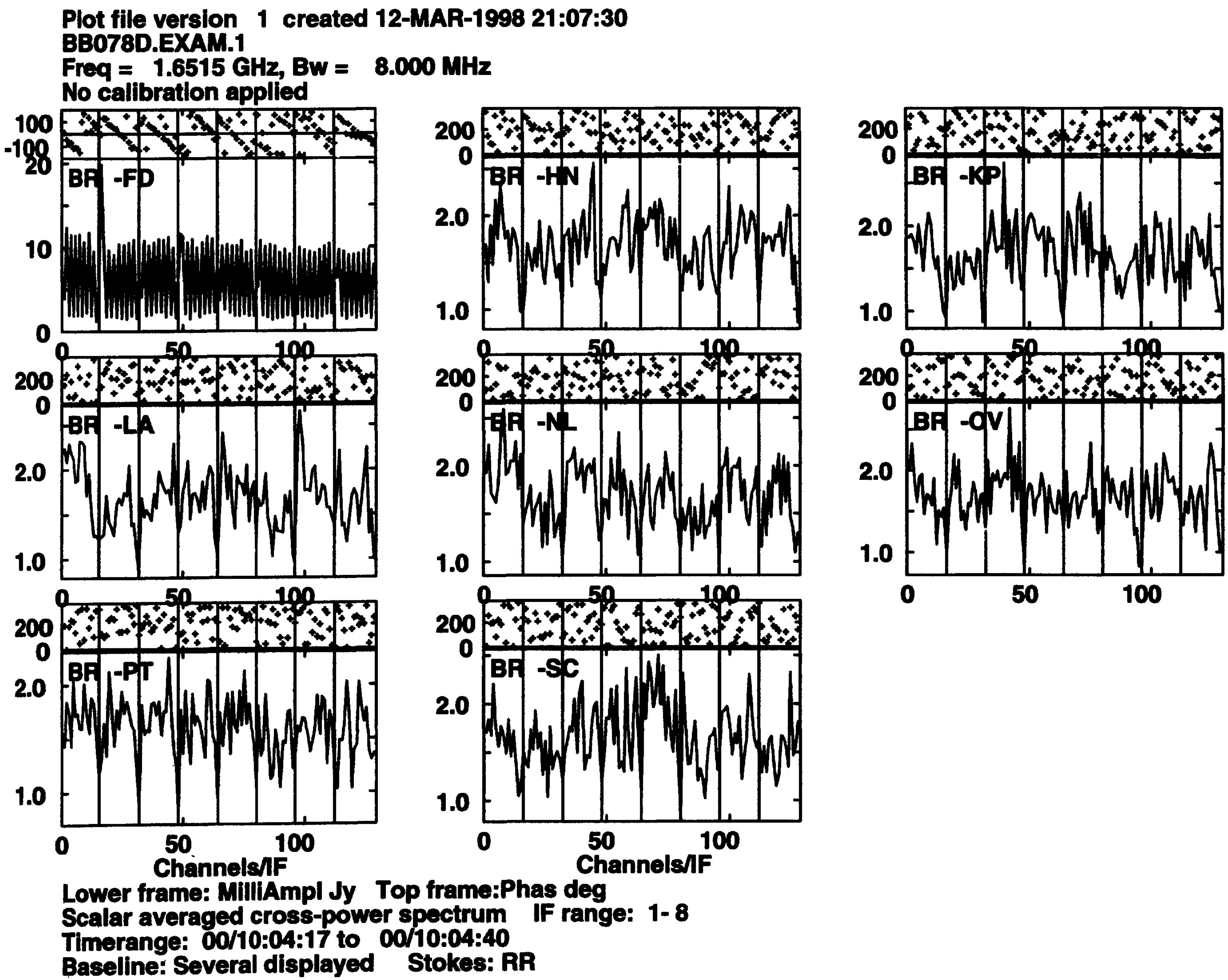
Plot file version 1 created 12-MAR-1998 21:07:30
BB078D.EXAM.1
Freq = 1.6515 GHz, Bw = 8.000 MHz
No calibration applied
BR -FD
BR -HN
BR -KP
BR -LA
BR -NL
BR -OV
BR -PT
BR -SC
Channels/IF
Channels/IF
Lower frame: MilliAmpl Jy Top frame:Phas deg
Scalar averaged cross-power spectrum IF range: 1- 8
Timerange: 00/10:04:17 to 00/10:04:40
Baseline: Several displayed Stokes: RR

Fig 1

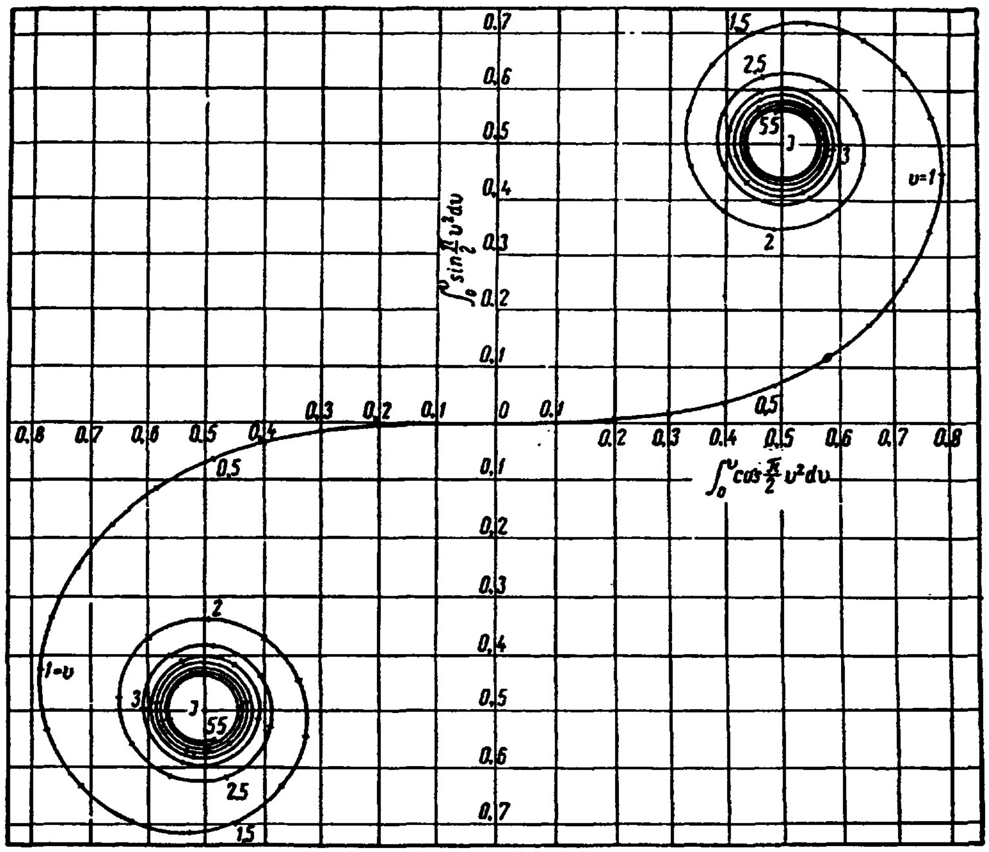
$\int_0^v \sin\frac{\pi}{2}v^2 dv$
$\int_0^v \cos\frac{\pi}{2}v^2 dv$
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
v=1
1=v
1,5
2
2,5
3
55

Fig 2

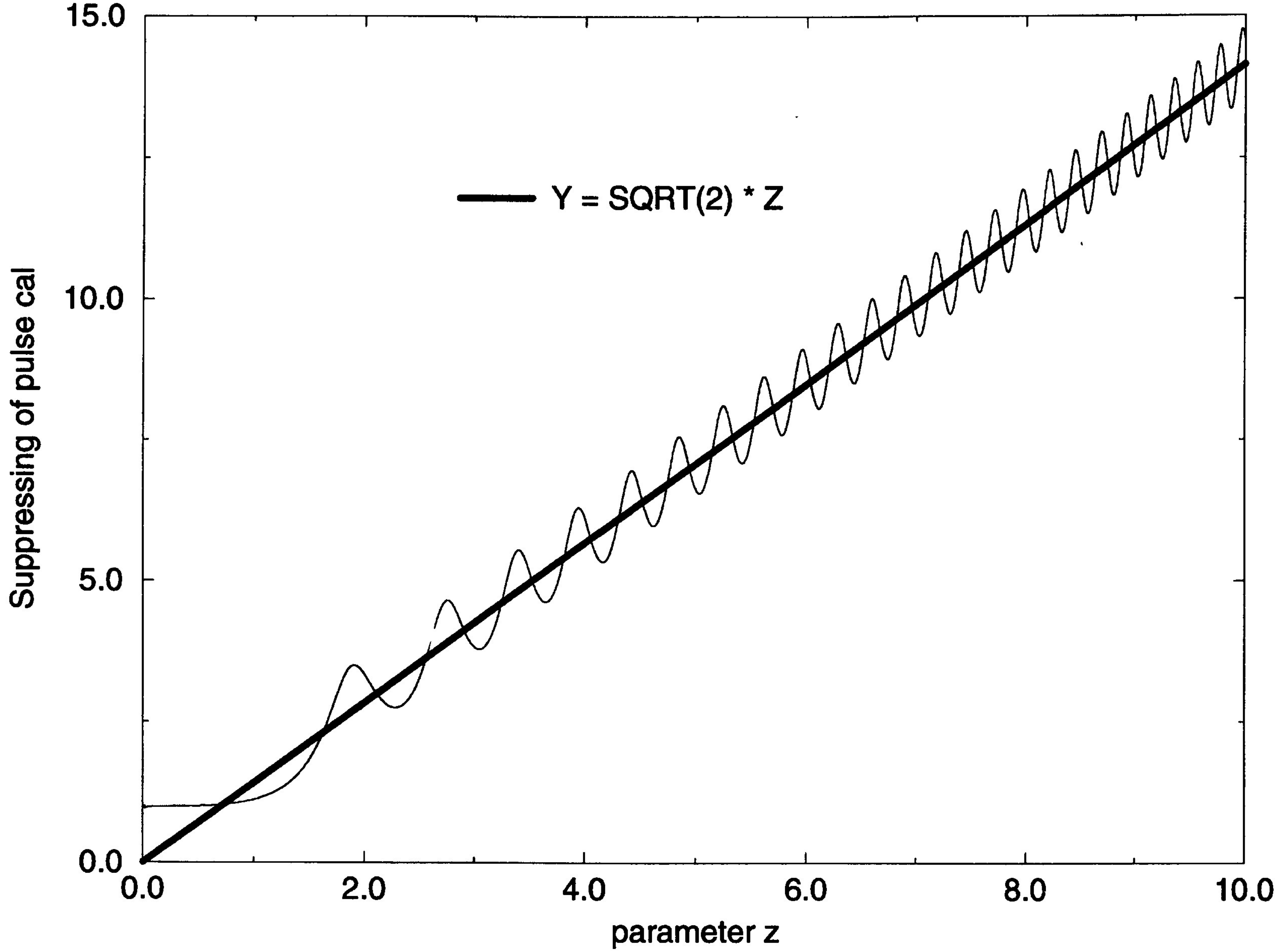
15.0
10.0
5.0
0.0
Suppressing of pulse cal
Y = SQRT(2) * Z
0.0
2.0
4.0
6.0
8.0
10.0
parameter z

Fig 3