NATIONAL RADIO ASTRONOMY OBSERVATORY
Socorro, New Mexico

VLBA Test Memo Series No. 71

VLBA Wheel and Axle Design

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1.0 Introduction

A VLBA Drive axle failed at Los Alamos and then shortly thereafter the drive axle bearings at Fort Davis and North Liberty failed. A more robust drive axle and bearing assembly was then designed and installed as quickly as possible to minimize antenna down time. Once everything was back up and running a thorough analysis of the failures was completed. This memo presents this failure analysis and describes how the new axle design is optimized to reduce these types of failures in the future.

The VLBA antenna azimuth wheels were designed to be the frustum of a cone with a vertex that intersects the plane of the rail at the center of the antenna. The original antenna design calculations assumed that the wheels would roll in a circular path along the rail in such a way that the bearings on each side of the wheel would share the load equally. In order for this to be true one must assume a perfectly aligned wheel/axle. In practice, slight misalignments exist and since the wheel is constrained to roll in the ideal path instead of its misaligned path, forces develop that are reacted against by the azimuth bearings and axle. These forces are responsible for the premature failures of the VLBA drive axles and bearings.

The new axle is designed to handle the extreme loads but the largest bearings that would fit in the available space will still eventually fail unless the loads can be reduced. These loads can be reduced by aligning the assemblies better and decreasing the coefficient of friction between the rail and the wheels.

2.0 History of drive assembly failures.

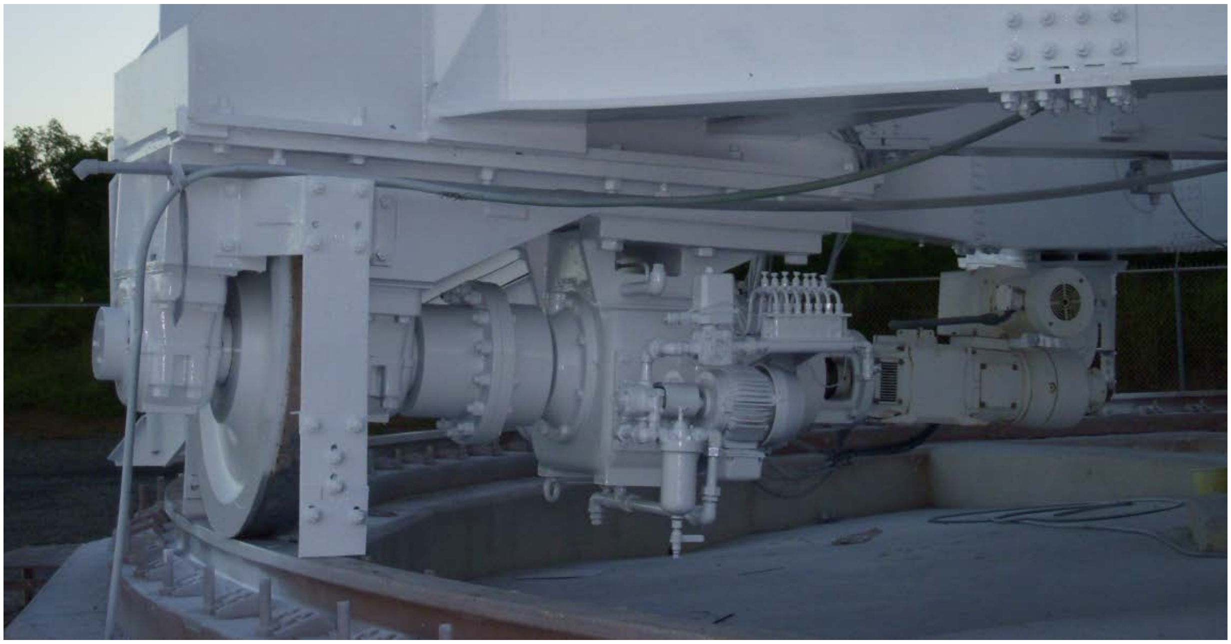

Figure 1, Original design azimuth drive assembly.

The VLBA went into full operation in 1992. Between 1992 and 1997 seven of the azimuth drive wheel bearings failed. It was determined that these bearings were failing in fatigue due to excessive thrust loads [4]. A new bearing and axle arrangement was designed using wheel bearings with a substantially higher load rating. The outside bearing in this design was so large that it required a reduction of the diameter on the end of the axle to accommodate it. The first of these wheel assemblies was installed on the Azimuth Drive #2 of the Brewster VLBA antenna. The axle on the new assembly failed due to fatigue at the reduced axle diameter approximately 13 months later [5]. The design was then modified to utilize a smaller outside bearing allowing for a larger axle diameter. This bearing still had an approximately 25% higher load rating than the original bearing. For clarity I will refer to this improved design as Mark II.

Between 1997 and 2002 seven more of the original style bearings failed.

In October, 2000 the first original style axle broke (Los Alamos drive #2) [6][7]. Two axles that had been removed after several years of service on VLBA antennas were then inspected by Atomic Inspection Laboratory in Albuquerque, NM and were found to be sound with no cracks. In June, 2002 a second axle (Las Alamos drive # 1) broke [8]. This wheel had been previously reported to be misaligned in both the vertical and horizontal direction [9]. We then procured an ultrasonic flaw detector and located several cracked axles. All of the remaining wheel/axle assemblies were then replaced with the Mark II design. It should be noted that the Mark II wheel/axle design was optimized to utilize larger bearings but all of the new axles except one were made from higher quality heat treated 4340 steel that greatly increased the axle fatigue strength. The first axle that was installed at Brewster was made from 4140 steel.

3.0 Mark II Design Wheel/axle Assembly Performance

In 2011, After 12 years of service, the outside Az#2 drive bearing of the Brewster antenna showed metal flakes in the grease and was replaced. This bearing had an approximately ½ inch long spalled area on the outer race. It was later determined that the axle had a very small crack near the wheel axle interface. This axle was the only new style axle that utilized the softer 4140 steel.

Figure 2, Mark II design wheel/axle assembly.

In September 2013, the Los Alamos #1 drive axle failed. This axle was installed in June 2002. This wheel started making popping noises shortly after being installed. The wheel was carefully realigned one month later. After realigning, the wheel was still popping 14-15 times per wheel revolution [10].

In the next six months, both outside drive bearings failed on the Fort Davis antenna and another outside drive bearing failed on the North Liberty antenna.

3.1 Mark II Axle Failure Analyses

At this time we know of two Mark II axles that failed. The first one is the axle removed from the Brewster antenna after 12 years of service. This axle did not break but an approximately 1/8 inch deep crack was found around the axle circumference near the outside edge of the wheel. As shown in Figure 3, there is a stress riser where the wheel attaches to the axle. This is the point of maximum stress and is where all of the other axle failures to date have occurred. A failure of this particular axle is not surprising because it was made from the same material and had the same cross section as the original VLBA axles. This axle was made from softer 4140 steel (tensile strength 89 ksi) than the rest of the Mark II design axles that were made from heat treated 4340 steel (tensile strength 143 ksi).

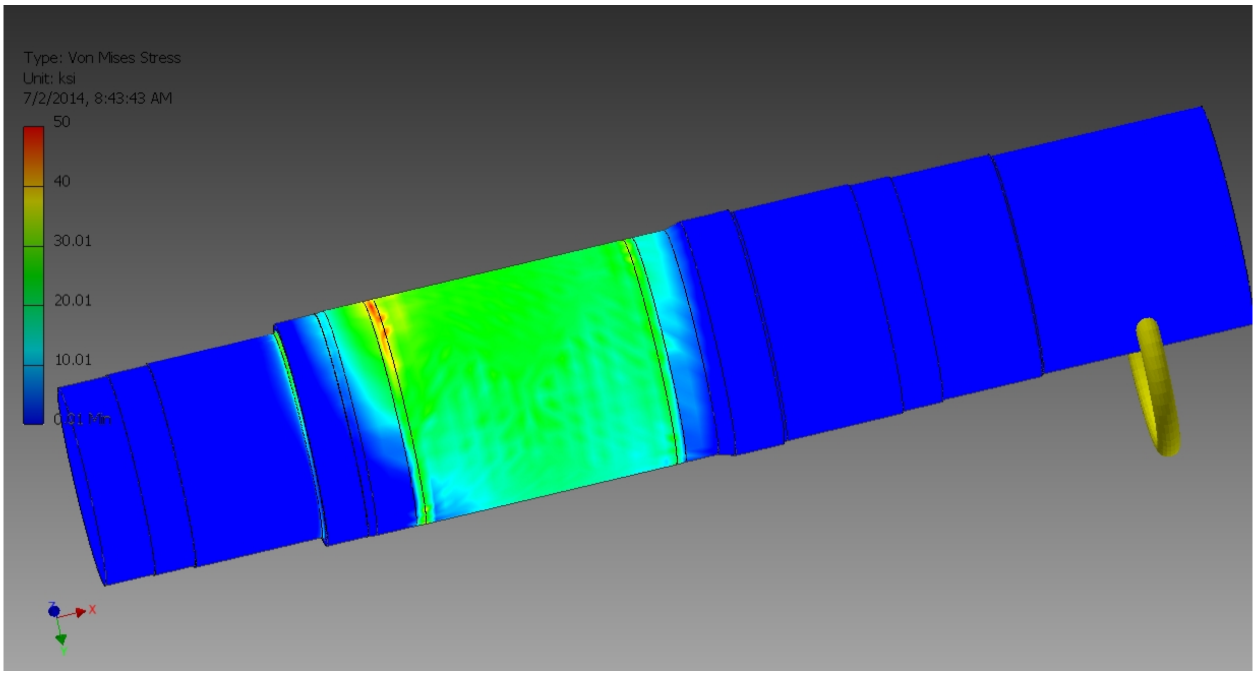

Figure 3, Mark II Axle Von Misses Stress.

The second axle failure occurred at Los Alamos. This axle broke completely after 11 years of service. This axle was made from steel that was significantly stronger than the original axles and therefore should have lasted at least three times as long. This particular axle had a history of making popping noises.   These noises were an indication that there was a slight relative movement or slipping occurring that probably resulted in fretting corrosion. “Fretting is a special wear process that occurs at the contact area between two materials under load and subject to minute relative motion by vibration or some other force” [1].

Fretting occurs by contacting asperities on the mating surfaces continually welding together then breaking. That leads to surface pitting and the transfer of metal particles from one surface to another. In addition, the small fragments of metal which are broken off oxidize, forming oxide particles which, for most engineering metals, are harder than the mated parts. These particles become trapped between the mating surfaces and cause abrasive damage and scoring. A layer of this oxide was found when the Los Alamos axle was disassembled. The above clues lead us to believe that this particular axle prematurely failed in fatigue because of the presence of fretting corrosion.

"Under fretting conditions, fracture cracks can initiate at very low stresses, well below the fatigue limit of non-fretted specimens. Fretting corrosion can reduce the endurance limit of steels to as little as 18% of their original values. The greatest reduction in fatigue strength occurs when the fretting process AND cyclic stressing are applied simultaneously" [2]. Laub [3] does an excellent job of explaining crack growth in the presence of fretting.

The axles that were removed from the Fort Davis and North Liberty antennas were inspected and no cracks were found. These axles also did not have the powdery oxide layer that was evident on the broken Los Alamos axle. These axles were in service between 5 and 10 years.

3.2 Mark II Bearing Failure Analyses

The outside drive wheel bearings on both the Fort Davis and North Liberty antennas failed completely with little or no warning. The original wheel bearings usually shed metal particles for several months before the races cracked. The bearings used in the Mark II design axle seem to shatter with little warning. One of the bearings from the North Liberty antenna was sent back to the manufacturer for an in depth failure analysis. The results of this failure analysis will not be available until later this year.

It is surprising that the larger bearings are not lasting significantly longer that the original bearings. It is entirely possible that the operating conditions have changed. The VLBA antennas are being used more often with reference pointing and the duty cycle of the azimuth drives may have increased. Craig walker claims that the duty cycle probably has increased but it is unlikely that the duty cycle has more than doubled.

Another possibility is that the bearing loads have changed significantly. The Mark II design utilizes a flexible gear coupling to attach the axle to the azimuth drive gearboxes instead of the rigid coupling that was used in the original design. This was done to protect the very expensive gearboxes from non-torsional loading. In the original design, the gearboxes could have been taking a share of the load and thus protecting the bearings.

A third possibility is that the Mark II design bearing housings were not fabricated to specification. We measured several of these bearing housings and found that on some of them the through holes are out of round by as much as 0.007 inch. This is because the bearing housings were made from flame cut steel that was then mounted on a lathe in a four jaw chuck and turned to dimension. As the surface stress on the steel is relieved, the lathe chuck prevents the steel from changing shape. The bearing housing is able to take its true shape only after it is removed from the lathe.

A deflecting bearing housing is also another possibility. The bearing housing could be deflecting in such a manner that it places excessive load on the bearings. In order to use the larger bearings, custom bearing housings had to be designed. The wall thickness of the bearing housing is reduced on the side where it attaches to the antenna. This means that the bearing housing relies on the antenna structure to keep its shape. The Finite Element Analysis results shown in Figure 4 verify that the bearing housing and antenna structure do not deform significantly.

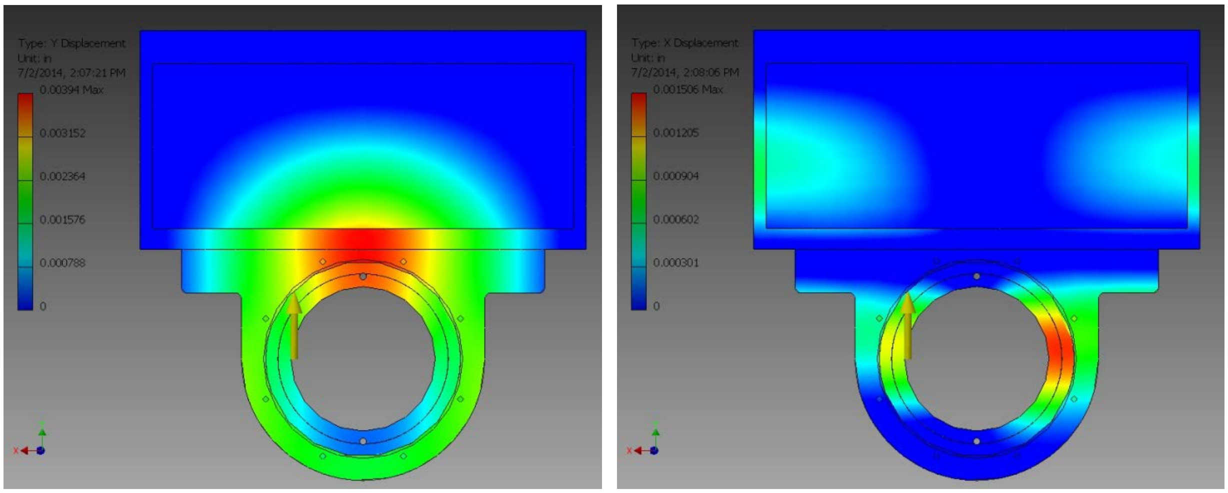

Figure 4, bearing housing displacement under load

It should be noted that impressions were found on the outside bearing housing that was removed from the North Liberty azimuth drive #1. These impressions show that this bearing housing was installed over the jacking bolts that were used to position the original assembly. These jacking bolts probably kept the bearing housing from laying flush against the structure and thereby decreasing the bearing life.

Alignment of the wheel axle assembly has a significant effect on the bearing loads. The VLBA antenna azimuth wheels were designed to be the frustum of a cone with a vertex that intersects the plane of the rail at the center of the antenna. When perfectly aligned the wheel would roll in a circular path around the center of the antenna. However, if the wheel is misaligned in the horizontal direction, the wheel will roll in a tightening radius in one direction and an expanding radius in the other. Since the wheel is constrained to roll in the ideal path instead of its misaligned path, forces develop that are reacted against by the azimuth bearings.

If the wheel is misaligned in the vertical direction, it will roll in a tightening or expanding radius independent of the rotation direction. This is also true if the wheel is set inside or outside the prescribed conic radius. The 7th column in table 1 “distance before or behind actual antenna center” is the distance between the ideal antenna center and where the wheel axis meets the plane of the track. This parameter was not specified in the original antenna alignment procedures but is one I invented to combine the vertical error and the conic radius error. This error is an indicator of the wheels propensity to roll inward or outward regardless of antenna direction. Table 1 demonstrates that the degree of misalignment correlates quite well to failed bearings as all of the bearings with the worst alignment have failed. This demonstrates that the current vertical alignment tolerance needs to be reduced.

Table 1, VLBA Antenna Drive Wheel Installation Parameters.

Specification			300 +/- 0.25	< 0.005	93.44 +/- 0.023	Distance before (positive) or behind actual antenna center.	< .023	Notes
		Date	Conic Radius	Coupling Runout	Vertical Angle		Horizontal Error	
Saint Croix	Az 1	Oct-08	300.186	0.0035	93.456	1.59	0.023	
	Az 2	Oct-08	300.155	0.0025	93.437	-0.09	0.023	
North Liberty	Az1	May-08	299.893	0.001	93.463	1.90	0.004	Failed Bearing
	Az2	Oct-03	299.894	0.001	93.426	-1.31	0.012	Bearings did not fail but had odd wear marks.
Fort Davis	Az1	Jan-07	299.968	0.002	93.464	2.06	0.018	Failed bearing
	Az2	May-07	300.08	0.001	93.462	2.00	0.005	Failed bearing
Los Alamos	Az1	Jun-02	300.039	0.0035	93.441	0.14	0.018	Failed axle Wheel was popping after installation. Tried realignment and still popped. Bearings looked ok
	Az2	Dec-00	299.9	0.005	93.44	-0.08	0.03	Ring Fedder
Hancock	Az1	Sep-07	DATA NOT AVAILABLE					
	Az2	Feb-10	300.26	0.001	93.458	1.84	0.02	
Pie Town	Az1	May-04	300.123	0.002	93.453	1.27	0.02	
	Az2	Aug-08	DATA NOT AVAILABLE					
Kitt Peak	Az1	May-09	300.218	0.004	93.433	-0.38	0.003	
	Az2	Feb-12	300.086	0.003	93.442	0.28	0.013	
Owens Valley	Az1	Aug-08	300.201	0.003	93.447	0.83	0.006	
	Az2	Feb-05	300.133	0.003	93.444	0.50	0.001	
Brewster	Az1	Jul-07	298.888	0.005	93.458	0.46	0.021	
	Az2	Oct-99	299.93	0.004	93.483	3.65	0.011	Data is for original assembly. Replaced 9-2011 due to outside bearing failure. The axle was cracked at outside wheel /axle interface. This axle was fabricated from 4140 steel not the harder 4340 steel used on later model axles.
Mauna Kea	Az1	Aug-06	300.266	0.002	93.426	-0.94	0.023	
	Az2	Oct-09	300.185	0.004	93.445	0.64	0.02	

4.0 New Design Wheel/Axle Assembly

The newly designed wheel/axle assembly (Mark III) shown in Figure 5, incorporates a stronger axle, a wheel to axle locking device designed to eliminate fretting corrosion, enhanced lubrication and a larger outside bearing. This design reuses the wheel, gear coupling and pillow blocks used in the Mark II design.

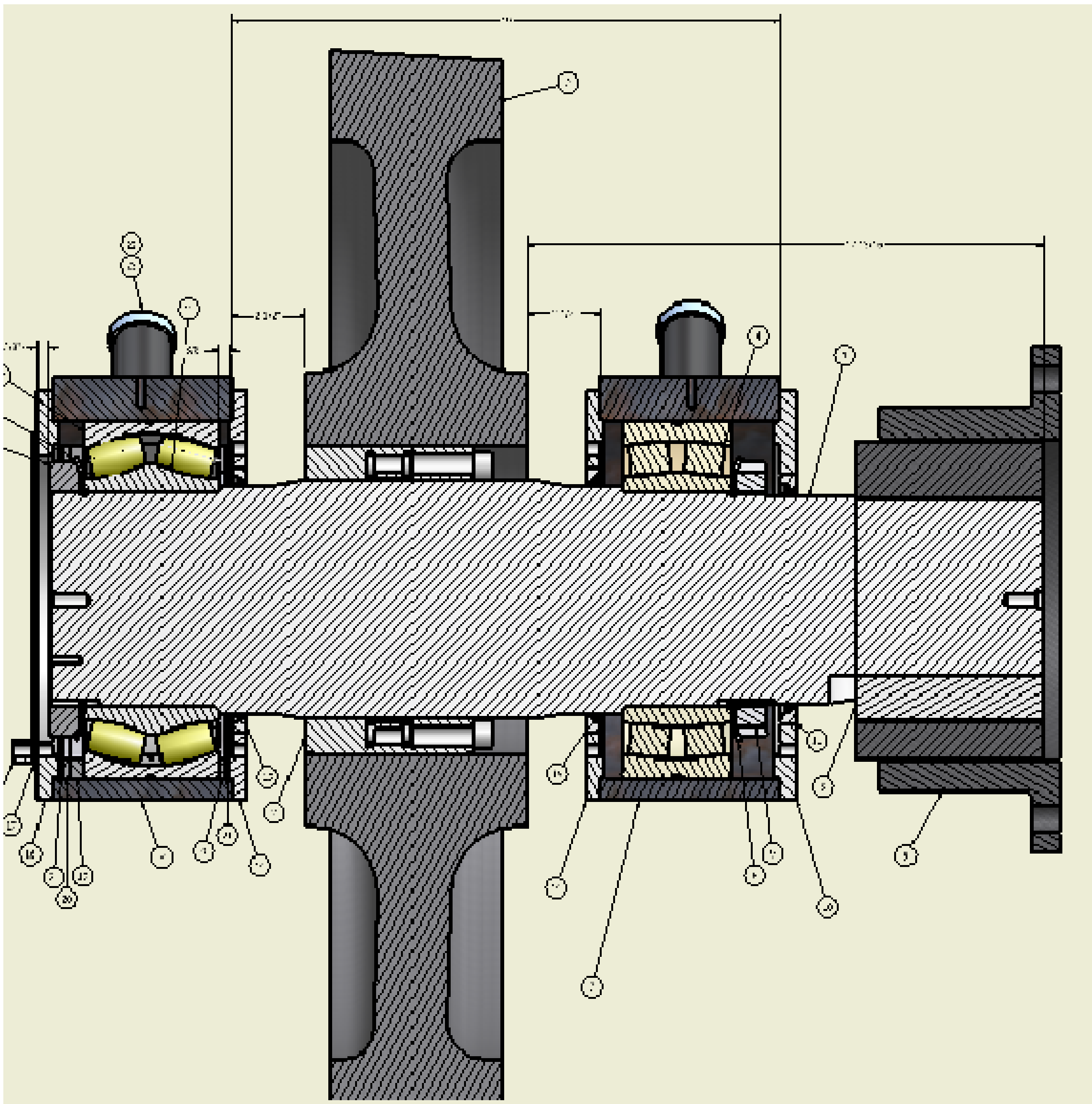

Figure 5, Mark III wheel/axle assembly section view.

4.1 Wheel/axle design

The RBC taper couplings that were used to connect the wheels to the axles in the original wheel/axle design were re-used in the Mark II design. It is evident from the Los Alamos axle failure that these couplings may not be griping the axles tight enough to prevent fretting corrosion from occurring. All failed drive wheels will be replaced with a newly designed wheel/axle assembly that use double taper, B-Loc keyless bushings well suited to transmit the rotating/ reversed bending moment loads that develop on the VLBA axles. These bushings should eliminate the possibility of wheel axle fretting.

The axle diameter at the wheel-axle interface was also increased from 6.9375 inch diameter to 7.875 inch diameter. The larger diameter results in an increase in the section modulus of about 45%. This is the largest diameter axle that can be used with the existing VLBA wheels and the B-Loc type bushings. B-Loc bushings exert so much radial force on the wheel that a larger bushing could possibly break the wheel.

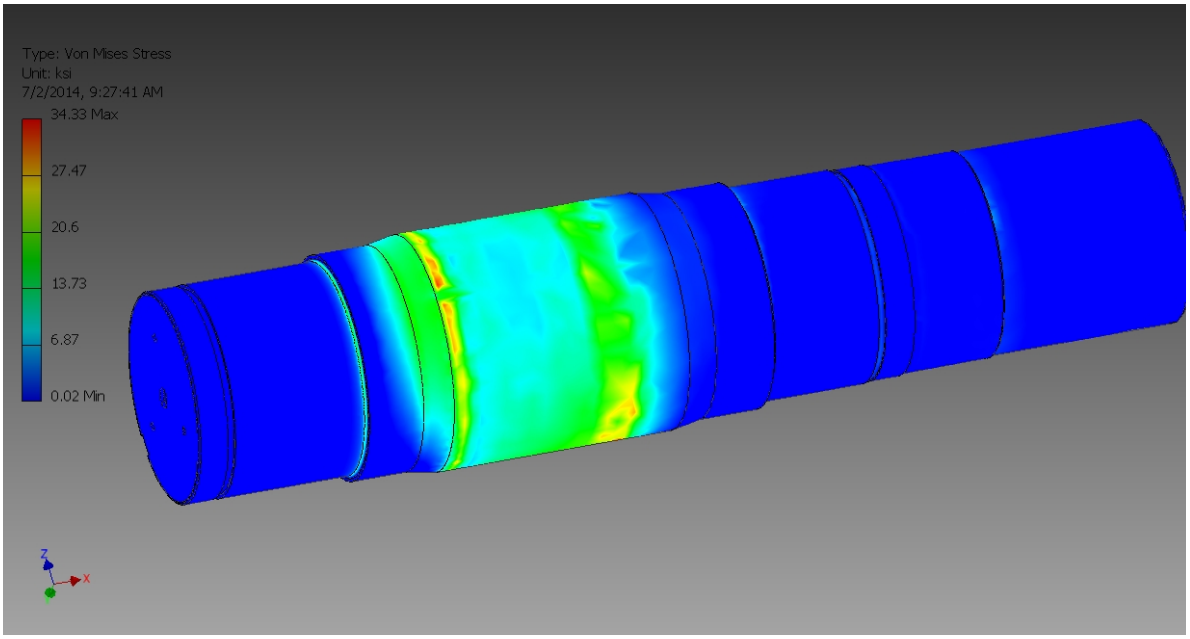

Figure 6, Mark III Axle Von Misses Stress.

The part of the axle with the second highest level of stress is the stress concentration at the shoulder used to position the outside bearing. The shaft diameter at the outside bearing was increased from 6.3 inches to 7.1 inches. This diameter increase and corresponding decrease in the stress concentration factor reduces the stress at this point by approximately 27%.

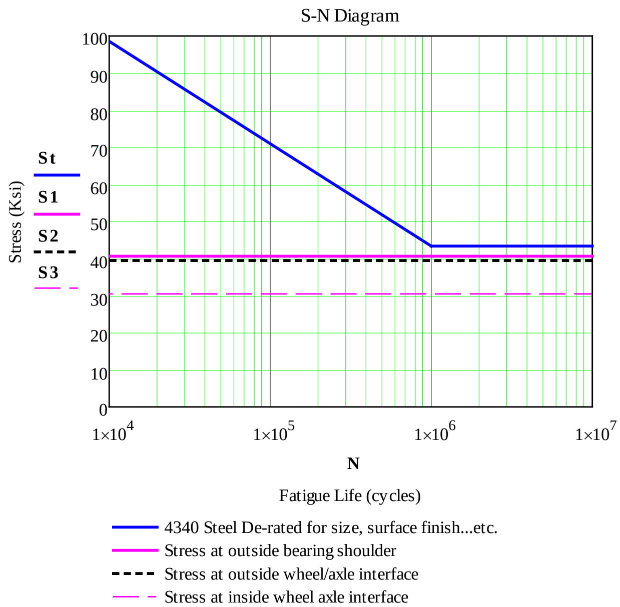

Figure 7, S-N diagram for Mark III axle

The material that we are making the VLBA axles from has the fatigue properties shown in the S-N diagram on the right. This diagram illustrates that a 27% reduction in the stress level would significantly increase the fatigue life of the axle. The knee on the curve is the point where failure will not occur no matter how great the number of cycles. For a VLBA axle, with a 90% reliability factor, this knee (endurance limit) occurs at 43.4 ksi. The stress values on the chart are higher than shown on the FEA results in Figure 6 because a stress concentration factor of 2.0 was used for the wheel axle interface. This is higher than the stress concentration factor of 1.5 that was calculated using the FEA model. The 2.0 stress concentration number is conservative and was recommended by Warren Palmer a product engineer employed by the manufacturer of the B-Loc couplings.

Unless a VLBA drive axle is aligned perfectly, the stress in the axle is dependent on the friction coefficient between the wheel and the track. The friction coefficient is discussed in depth in Section 5.0 of this report. The stress levels on the S-N diagram shown assume an average coefficient of friction (μ) of 0.25.

4.2 Larger Outside Bearings.

As stated earlier, most of the VLBA drive failures have been due to bearings failing. The original VLBA axle design used #23038 spherical bearings. The Mark II design replaced the outside bearings with a # 23232 spherical bearing. The new wheel assembly design uses a #24136 outside bearing. These bearings have a greater dynamic load capacity and also have a lower thrust load factor which makes them more tolerant of axial loads. The calculated bearing life of the new design bearings is about 25% higher than the Mark II design bearings. We would like to see a greater bearing life but the 24136 bearing was the best bearing we could find that did not require significant changes to the antenna structure. The 24136 bearings also have smaller rollers and we are hoping that they will fail slowly like the original 23038 bearings instead catastrophically like the 23232 bearings.

Table 2, Bearing life comparison.

Bearing #	23038	23232	24136
Dynamic Load Rating	164100 lbs.	245900 lbs.	258400 lbs.
Thrust Load Factor	4	2.91	2.74
Bearing Width	2.95	4.09	4.64
Internal Diameter	7.48	6.30	7.09
Outside Diameter	11.42	11.42	11.81
Calculated Bearing Life (L^{10}) μ=.25, 0.2 RPM	1.5 years	11.9 years	15.9 years

The bearing life calculations shown in Table 2 are used for comparison purposes only and are not valid for bearings that turn as slow as ours. The 23232 bearings that failed did not last eight times longer than the original bearings like the bearing life calculations predict. Reasons for this discrepancy were discussed in Section 3.2.

4.3 Enhanced lubrication

One of the factors in play is our bearings turn so slow that the lubricant does not work as well as it should. In a high speed bearing the lubricant is dragged into the contact zone and the surfaces become fully separated. When a bearing is at rest the lubricant is squeezed out from beneath the rollers resulting in metal to metal contact. Bearings running at very low speed operate somewhere between these two extremes. Bearings running this slow require continuous lubrication. The VLBA wheel bearings are currently greased every six months. The new wheel/axle design uses SKF LAGD WA2 automatic lubricators that continuously supply a minute quantity of grease. The site techs will continue to flush the grease at six month intervals to ensure cleanliness and to inspect for metal particles.   A small pressure relief valve is installed in the new wheel/axle assembly to ensure that the grease in the pillow blocks is slightly pressurized to minimize contamination. The enhanced lubrication should extend the life of our bearings.

4.4 Pillow block improvements

The outside pillow blocks need to be bored out to accommodate the larger diameter 24136 bearings. The pillow blocks will be inspected before assembling on the bearings to ensure that the inside diameter is not out of round. Additional bolt holes are also being drilled into the seal plates and pillow blocks. The additional bolts will allow the seal plates to carry some of the load and help maintain the proper pillow block shape under load.

5.0 Load Control

The new wheel/ axle design has been improved and is as strong as possible without requiring major structural modifications to the antenna. I am very confident that the new axle design is robust enough that it will not break. Even though the new outside bearing is larger and has improved lubrication, it is probably not large enough that it will not eventually fail. The next step is to try and decrease the load on the bearings. The load from the antenna weight is fixed and cannot be easily changed. However, it is possible to decrease the load that is introduced due to wheel/axle misalignment.

Since the bearing failures are occurring on the wheels that are farthest out of alignment, it makes sense that we should try and align the wheels better. This may require significantly more time during installation but if it increases the life of the assembly, it is well worth the effort. The parameter that makes the most difference is the vertical alignment error. The original VLBA wheel installation procedure required that the axles be installed at a 93.44 +/- 0.023 degree angle. In the future, we will try and install the axles at 93.44 +/- 0.01 degrees.

Since the wheel is constrained to roll in the ideal path instead of its misaligned path, forces develop that are reacted against by the azimuth bearings. These forces will build up until the wheel slides laterally on the rail and relieves the stress. The point where the wheel slides is dependent on the coefficient of friction between the wheel and the rail. When the wheel is trying to move in an outward direction, the radial load is increased on the outside bearing and decreased on the inside bearing. The coefficient of friction on a dry wheel and track can vary between 0.05 and 0.57. The table below shows the bearing loads and axle stress that correspond to different coefficients of friction. A similar table could be developed for the case where the wheel is trying to roll inward. Rolling inward is preferred because the inside bearing is not constrained in the axial direction and therefore is subjected to only radial loading.

Table 3, Bearing life and axle stress vs coefficient of friction.

Coefficient of Friction	Inside Bearing Radial Load (kips)	Outside Bearing Radial Load (kips)	Outside bearing Axial Load (kips)	Calculated Bearing Life (L^{10}) years	Maximum Stress on Axle (ksi)
.05	62.5	97.2	17.6	94.8	40.5
.25	32.0	127.7	49.6	15.9	40.5
.42	6.1	153.6	76.8	5.0	43.1
.57	-16.7	176.4	100.8	2.3	46.7

The maximum axle stress is above the endurance limit when the coefficient of friction exceeds 0.44.

The obvious solution to reducing the coefficient of friction is to grease the rail. I am sure that this would not be a popular option with the VLBA site techs. However, there is another option that is often used on flanged crane wheels. These are solid lubrication sticks (Figure 7) that apply a dry lubricant to the surface of the wheel. These sticks run on the surface of the wheel in a spring loaded applicator like a brush on an electric motor. These sticks can be purchased to provide coefficients of frictions as low as 0.05 to as high as 0.3 when controlled traction is required. One manufacturer claims that the lube sticks last approximately six months when installed on shipyard cranes that operate 24 hours a day. The cost of these particular sticks was about $7.00 each.

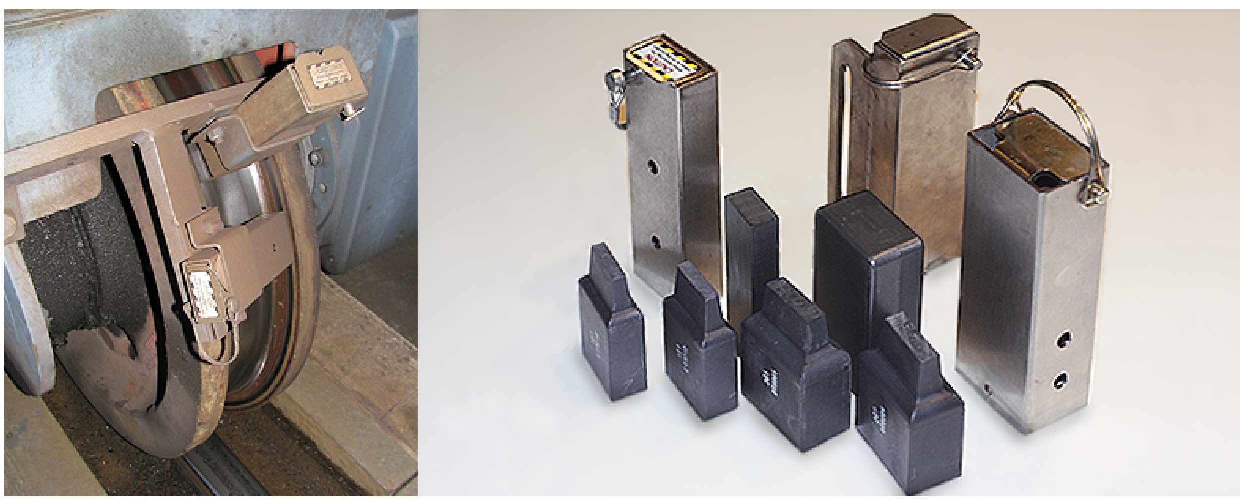

Figure 7. Crane rail lubrication sticks.

Some experimentation will be required to see how much we can decrease the coefficient of friction and still drive the antenna without wheel slipping. If this experiment yields satisfactory results, these lubrication sticks will be incorporated into the VLBA array.

References:

1) Fatigue and Fracture Handbook, ASM International, V-19, 1996.

2) www.epi-eng.com/mechanical_engineering_basics/fretting_corrosion.htm

3) Laub, S., “Fretting Induced Fracture of Coupling Driven Shafts” (1980). International Compressor Engineering Conference. Paper 310. Http://Docs.lib.purdue.edu/icec/310

4) J.E. Thunborg, “VLBA Test Memo No.54, Investigation of VLBA Azimuth Wheel Bearing Failures”, July, 1997.

5) Jim Ruff, “VLBA Antenna Memo # 24, Azimuth Drive Wheels”, November, 1999.

6) Jim Ruff , “VLBA Antenna Memo #28, Los Alamos Drive #2 Wheel replacement”, December, 2000.

7) J. E. Thunborg , “VLBA Antenna Memo #30, VLBA Azimuth Axle Failure – Follow Up”, March, 2001.

8) J. E. Thunborg, “VLBA Antenna Memo Series # 39, Los Alamos – Drive #1 Axle Repair”, June, 2002.

9) Jim Ruff, “VLBA Antenna Memo Series #32, Los Alamos Maintenance Visit”, April, 2001.

10) Jim Ruff, “VLBA Antenna Memo Series #42, Los Alamos Az drive #1”, July 2002.

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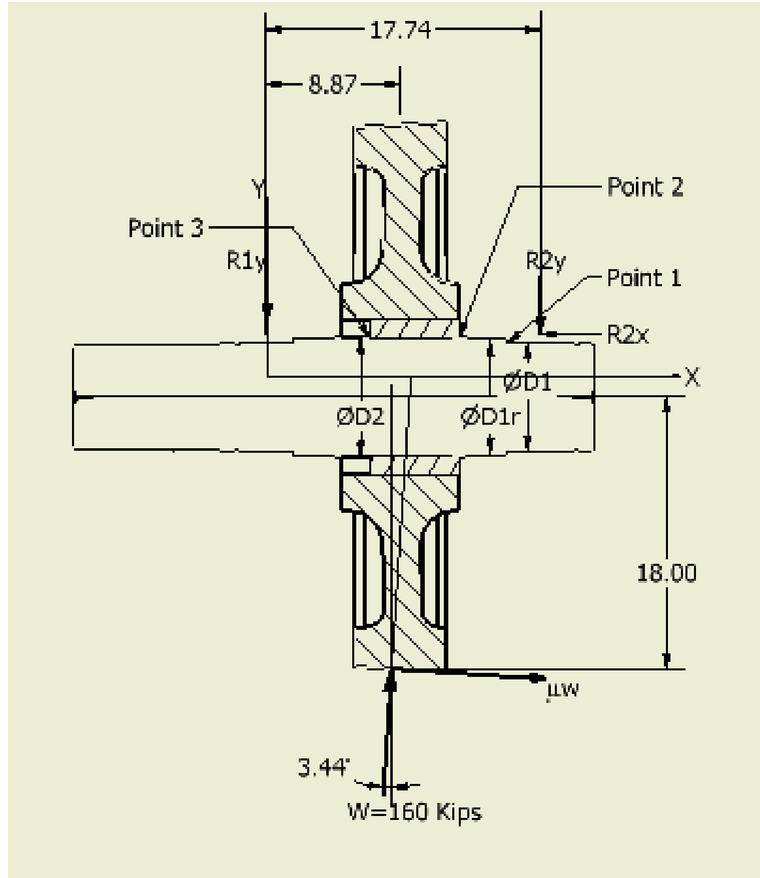
VLBA AXLE CALCULATIONS

The track exerts a 160 kip load on the wheel at a 3.44 Deg. angle relative to the axle. Spherical roller bearings at R1 and R2 react against this force. The roller bearing at R2 also resists axial loads.



$W := 160000 \cdot lbf$	Load on wheel
$R1X := 0 \cdot in$	Distance along X axis to R1
$R2X := 17.7425 \cdot in$	Distance along X axis to R1
$Wx := 8.875 \cdot in$	
$\mu := .25$	Coefficient of Friction Steel on Steel
$B := 134 \cdot mm$	Coupling Width
$Dia1 := 7.09 \cdot in$	Diameter at Outside Bearing
$Dia2 := 7.87 \cdot in$	Diameter at Outside wheel
$Dia3 := 7.87 \cdot in$	Diameter at Inside wheel
$Dia1r := 7.5 \cdot in$	Diameter at Bearing Shoulder
$Rad2 := .1 \cdot in$	Radius at Bearing Shoulder

Material Properties
4340 Steel:

$Fy := 120 \cdot ksi$	Yield Strength
$Fut := 140 \cdot ksi$	Ultimate Strength
$q := .92$	q = Notch Sensitivity For 4340 steel From Figure 5-19, Shigley

Reactions

$\Sigma m_{R1} := 0$ $\qquad \theta := 3.44 \cdot \frac{\pi}{180}$

$$R2y := \frac{[W \cdot \cos(\theta) \cdot Wx + (W \cdot \sin(\theta) - W \cdot \mu \cdot \sin(\theta) + W \cdot \mu \cdot \cos(\theta)) \cdot 18 \cdot in]}{R2X}$$

$\Sigma fy := 0$ $\qquad R2y = 127701.9108 \cdot lbf$

$R1y := W \cdot \cos\left(3.44 \cdot \frac{\pi}{180}\right) - R2y$ $\qquad R1y = 32009.7981 \cdot lbf$

$\Sigma fx := 0$

$R2x := W \cdot \sin\left(3.44 \cdot \frac{\pi}{180}\right) + W \cdot \mu$ $\qquad R2x = 49600.5219 \cdot lbf$

Bending Moment

$L1 := 2.275 \cdot in$ Distance from Center of outside bearing to Point 1 $\quad M1 := L1\, R2y \quad M1 = 290521.8472 \cdot lbf \cdot in$

$L2 := 5.15 \cdot in$ Distance from Center of outside bearing to Point 2 $\quad M2 := L2 \cdot R2y \quad M2 = 657664.8408 \cdot lbf \cdot in$

$L3 := 6.87 \cdot in$ Distance from Center of inside bearing to Point 3 $\quad M3 := L3 \cdot R1y \quad M3 = 219907.3131 \cdot lbf \cdot in$

Torque at Point 3 If input torque is transmitted to the track at the wheel then we can assume: $\tau 1 := 0$ $\quad \tau 2 := 0$

Operating torque $\quad HP := 15 \quad RPM := 3500$

Motor torqe $\quad Mt := 5252 \cdot \frac{HP \cdot ft \cdot lbf}{RPM} \quad Mt = 22.5086 \cdot ft \cdot lbf$

Gearbox torque ratio = 848 : 1 $\quad Mt3 := 848 \cdot Mt \quad Mt3 = 19087.2686 \cdot ft \cdot lbf$

Torsional Shear Stress

$\tau 3 := 16 \cdot \frac{Mt3}{\pi \cdot Dia3^3}$ $\qquad \tau 3 = 2393.156\, psi$

Maximum Load Condition

Shear Stress at Points 1, 2 and 3 :

$$\mathbf{Ss1} := \frac{(16\mathbf{W})}{3\pi\cdot\mathbf{Dia1}^2} \qquad \mathbf{Ss1} = 5403.5151\cdot\mathbf{psi}$$

$$\mathbf{Ss2} := \frac{(16\mathbf{W})}{3\pi\cdot\mathbf{Dia2}^2} \qquad \mathbf{Ss2} = 4385.5026\cdot\mathbf{psi}$$

$$\mathbf{Ss3} := \frac{(16\mathbf{W})}{3\pi\cdot\mathbf{Dia3}^2} \qquad \mathbf{Ss3} = 4385.5026\cdot\mathbf{psi}$$

Total Shear Stress

$$\tau\mathbf{xy1} := \mathbf{Ss1} + \tau\mathbf{1} \qquad \tau\mathbf{xy1} = 5403.5151\cdot\mathbf{psi}$$

$$\tau\mathbf{xy2} := \mathbf{Ss2} + \tau\mathbf{2} \qquad \tau\mathbf{xy2} = 4385.5026\cdot\mathbf{psi}$$

$$\tau\mathbf{xy3} := \mathbf{Ss3} + \tau\mathbf{3} \qquad \tau\mathbf{xy3} = 6778.6582\cdot\mathbf{psi}$$

Bending Stress:

$$\sigma\mathbf{b1} := 32\cdot\frac{\mathbf{M1}}{\pi\cdot\mathbf{Dia1}^3} \qquad \sigma\mathbf{b1} = 8303.0986\cdot\mathbf{psi}$$

$$\sigma\mathbf{b2} := 32\cdot\frac{\mathbf{M2}}{\pi\cdot\mathbf{Dia2}^3} \qquad \sigma\mathbf{b2} = 13742.9679\cdot\mathbf{psi}$$

$$\sigma\mathbf{b3} := 32\cdot\frac{\mathbf{M3}}{\pi\cdot\mathbf{Dia3}^3} \qquad \sigma\mathbf{b3} = 4595.3181\cdot\mathbf{psi}$$

Axial Stress:

$$\sigma\mathbf{a1} := \frac{\mathbf{4R2x}}{\pi\cdot\mathbf{Dia1}^2} \qquad \sigma\mathbf{a1} = 1256.3305\cdot\frac{\mathbf{lbf}}{\mathbf{in}^2}$$

$$\sigma\mathbf{a2} := \frac{\mathbf{4R2x}}{\pi\cdot\mathbf{Dia2}^2} \qquad \sigma\mathbf{a2} = 1019.6401\cdot\frac{\mathbf{lbf}}{\mathbf{in}^2}$$

$$\sigma\mathbf{a3} := 0$$

Principal Stress

$$\sigma\mathbf{x1} := \sigma\mathbf{b1} + \sigma\mathbf{a1} \qquad \sigma\mathbf{x1} = 9559.4291\cdot\frac{\mathbf{lbf}}{\mathbf{in}^2}$$

$$\sigma\mathbf{x2} := \sigma\mathbf{b2} + \sigma\mathbf{a2} \qquad \sigma\mathbf{x2} = 14762.608\cdot\frac{\mathbf{lbf}}{\mathbf{in}^2}$$

$$\sigma\mathbf{x3} := \sigma\mathbf{b3} + \sigma\mathbf{a3} \qquad \sigma\mathbf{x3} = 4595.3181\cdot\frac{\mathbf{lbf}}{\mathbf{in}^2}$$

Von Mises Failure Criteria:

$\sigma y1 := 0 \quad \sigma y2 := 0 \quad \sigma y3 := 0$

$\tau max1 := \left[\left(\frac{\sigma x1 - \sigma y1}{2}\right)^2 + \tau xy1^2\right]^{.5} \quad \tau max1 = 7214.1282 \cdot psi$

$\tau max2 := \left[\left(\frac{\sigma x2 - \sigma y2}{2}\right)^2 + \tau xy2^2\right]^{.5} \quad \tau max2 = 8585.8187 \cdot psi$

$\tau max3 := \left[\left(\frac{\sigma x3 - \sigma y3}{2}\right)^2 + \tau xy3^2\right]^{.5} \quad \tau max3 = 7157.4747 \cdot psi$

Factor of Safety:

$\sigma 11 := \frac{\sigma x1 + \sigma y1}{2} + \tau max1 \quad \sigma 21 := \frac{\sigma x1}{2} - \tau max1 \quad \sigma 11 = 11993.8428 \cdot psi \quad \sigma vm1 := \left(\sigma 11^2 - \sigma 11 \cdot \sigma 21 + \sigma 21^2\right)^{.5} \quad Fvm1 := \frac{Fy}{\sigma vm1}$

$\sigma 21 = -2434.4137 \cdot psi \quad \sigma vm1 = 13378.214 \cdot psi$

$Fvm1 = 8.9698$

$\sigma 12 := \frac{\sigma x2 + \sigma y2}{2} + \tau max2 \quad \sigma 22 := \frac{\sigma x2}{2} - \tau max2 \quad \sigma 12 = 15967.1227 \cdot psi \quad \sigma vm2 := \left(\sigma 12^2 - \sigma 12 \cdot \sigma 22 + \sigma 22^2\right)^{.5} \quad Fvm2 := \frac{Fy}{\sigma vm2}$

$\sigma 22 = -1204.5146 \cdot psi \quad \sigma vm2 = 16602.1834 \cdot psi$

$Fvm2 = 7.228$

$\sigma 13 := \frac{\sigma x3 + \sigma y3}{2} + \tau max3 \quad \sigma 23 := \frac{\sigma x3}{2} - \tau max3 \quad \sigma 13 = 9455.1338 \cdot psi \quad \sigma vm3 := \left(\sigma 13^2 - \sigma 13 \cdot \sigma 23 + \sigma 23^2\right)^{.5} \quad Fvm3 := \frac{Fy}{\sigma vm3}$

$\sigma 23 = -4859.8157 \cdot psi \quad \sigma vm3 = 12608.2343 \cdot psi$

$Fvm3 = 9.5176$

Fatigue Calculations .

Endurance Limit Modifying Factors

For most steels, the mean endurance limit of rotating beam specimens is approximately 1/2 of the ultimate strength of the material. This endurance limit corresponds to approximately 1 million cycles. The endurance limit is further reduced by the following Endurance Limit Modifying Factors.

Surface Factor	Figure 5-17 Polished Specimen = 1 Ground = .89 Machined = .65 - .8	$\mathbf{Ka} := .8$
Size Factor	Kb =1 d<0.3 in Kb=.85 0.3<d<2 in Kb=.75 d>2 in	$\mathbf{Kb} := .75$
Reliability factor	Reliability 0.5 Zr=1 Reliability 0.9 Zr=1.288 Reliability 0.95 Zr=1.645 Reliability 0.99 Zr=2.326 Reliability 0.999 Zr=3.091 Reliability 0.9999 Zr=3.719	$\mathbf{Zr} := 1.288$
	$\mathbf{Kc} := 1 - 0.08 \cdot \mathbf{Zr}$	$\mathbf{Kc} = 0.897$
Temperature factor	Kd = 1 T<160 F T > 160 F	$\mathbf{Kd} := 1$

Sum of all effects

$\mathbf{K} := \mathbf{Ka} \cdot \mathbf{Kb} \cdot \mathbf{Kc} \cdot \mathbf{Kd}$ $\quad \mathbf{K} = 0.538$

Endurance Limit

$\mathbf{Se} := \mathbf{0.577} \cdot \mathbf{Fut} \cdot \mathbf{K}$ $\quad \mathbf{Se} = 43.4739 \cdot \mathbf{ksi}$

Fatigue at Point 1

Maximum Shear Stress (Torque + Shear)

$$\tau xy1min := \tau 1 - Ss1 \qquad \tau xy1max := \tau 1 + Ss1$$

$$\tau xy1mean := \frac{\tau xy1max + \tau xy1min}{2} \qquad \tau xy1a := \tau xy1max - \tau xy1min \qquad \tau xy1mean = 0 \cdot ksi \qquad \tau xy1a = 10.807 \cdot ksi$$

Bending and Axial Stress

$$\sigma x1min := -\sigma b1 - \sigma a1 \qquad \sigma x1max := -\sigma b1 + \sigma a1$$

$$\sigma x1mean := \frac{\sigma x1max + \sigma x1min}{2} \qquad \sigma x1a := \frac{\sigma x1max - \sigma x1min}{2} \qquad \sigma x1mean = -8.3031 \cdot \qquad \sigma x1a = 1.2563 \cdot ksi$$

$$\sigma y1mean := 0 \qquad \sigma y1a := 0$$

Von Mises Failure Criteria:

$$\tau xy1mean := \left[\left(\frac{\sigma x1mean - \sigma y1mean}{2}\right)^2 + \tau xy1mean^2\right]^{.5} \qquad \tau xy1mean = 4151.5493 \cdot psi$$

$$\tau xy1a := \left[\left(\frac{\sigma x1a - \sigma y1a}{2}\right)^2 + \tau xy1a^2\right]^{.5} \qquad \tau xy1a = 10825.271 \cdot psi$$

$$\sigma 11mean := \frac{\sigma x1mean + \sigma y1mean}{2} + \tau xy1mean \qquad \sigma 21mean := \frac{\sigma x1mean - \sigma y1mean}{2} - \tau xy1mean \qquad \sigma 11mean = 0 \cdot psi$$

$$\sigma 21mean = -8303.0986 \cdot psi$$

$$\sigma vm1mean := \left(\sigma 11mean^2 - \sigma 11mean \cdot \sigma 21mean + \sigma 21mean^2\right)^{.5} \qquad \sigma vm1mean = 8.3031 \cdot ksi$$

$$\sigma 11a := \frac{\sigma x1a + \sigma y1a}{2} + \tau xy1a \qquad \sigma 21a := \frac{\sigma x1a}{2} - \tau xy1a \qquad \sigma 11a = 11453.4362 \cdot psi$$

$$\sigma 21a = -10197.1058 \cdot psi$$

$$\sigma vm1a := \left(\sigma 11a^2 - \sigma 11a \cdot \sigma 21a + \sigma 21a^2\right)^{.5} \qquad \sigma vm1a = 18.7604 \cdot ksi$$

Because the radius at point 1 is a stress riser we must further reduce the endurance limit of the steel. The Stress concentration Factor from Roark's Formulas for Stress and Strain (Table 37 Case 17). Valid for hr3 between 2 and 20.

$$h1 := \frac{Dia1r - Dia1}{2} \qquad hr1 := \frac{h1}{Rad2} \qquad hr1 = 2.05$$

$$k11 := 1.225 + 0.831 \cdot hr1^{.5} - .01hr1 \qquad k11 = 2.3943$$

$$k21 := -1.831 - .318hr1^{.5} - .049hr1 \qquad k21 = -2.3868$$

$$k31 := 2.236 - .522 \cdot hr1^{.5} + .176hr1 \qquad k31 = 1.8494$$

$$k41 := -.63 + .009hr1^{.5} - .117 \cdot hr1 \qquad k41 = -0.857$$

$$Kt1 := k11 + k21 \cdot \left(2 \cdot \frac{h1}{Dia1}\right) + k31 \cdot \left(2 \cdot \frac{h1}{Dia1}\right)^2 + k41 \cdot \left(2 \cdot \frac{h1}{Dia1}\right)^3 \qquad Kt1 = 2.262$$

$$Ke1 := \frac{1}{1 + q \cdot (Kt1 - 1)} \qquad Ke1 = 0.463$$

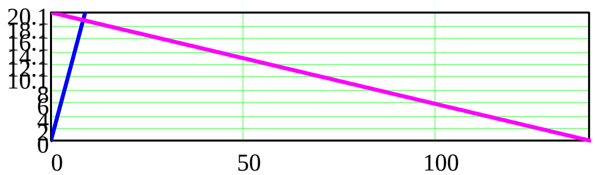
$$Se1 := Se \cdot Ke1 \qquad Se1 = 20.1145 \cdot ksi$$

$$Ke1 := \frac{1}{[1 + q \cdot (Kt1 - 1)]} \qquad Ke1 = 0.463 \qquad Se1 := Se \cdot Ke1 \qquad Se1 = 20.1145 \cdot ksi$$

In the presents of fluctuating stress A modified Goodman Diagram can be used to relate the stress to the strength.

Modified Goodman Line $\mathbf{i} := 0 .. 150$ $\quad x_i := i$ $\quad y1_i := \frac{-Se1 \cdot x_i}{Fut} + \frac{Se1}{ksi}$ $\quad Sm1 := \frac{Se1}{\left(\frac{Se1}{Fut} + \frac{\sigma vm1a}{\sigma vm1mean}\right)}$ $\quad Sa1 := \frac{\sigma vm1a \cdot Sm1}{\sigma vm1mean}$

$$ymg1_i := \frac{\sigma vm1a \cdot x_i}{\sigma vm1mean}$$



Stress Components
Modified Goodman line
Intersection

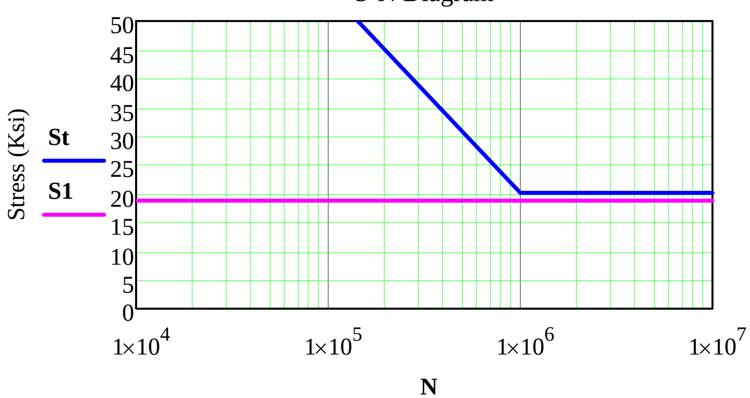
$St_0 := \frac{Fut}{ksi}$ $\quad St_1 := .9 \cdot \frac{Fut}{ksi}$ $\quad St_2 := \frac{Se1}{ksi}$ $\quad St_3 := \frac{Se1}{ksi}$

$N_0 := 100$ $\quad N_1 := 1000$ $\quad N_2 := 1000000$ $\quad N_3 := 10000000$

$\mathbf{i} := 1 .. 4$

$$S1_i := \frac{\sigma vm1a}{ksi}$$

S-N Diagram



Fatigue Life (cycles)

4340 Steel De-rated for size, surface finish...etc.
Stress Level at point 1

Fatigue at Point 2

Maximum Shear Stress (Torque + Shear)

$\tau xy2 := \tau 2 + Ss2 \quad \tau xy2 = 4385.5026 \cdot psi \quad \tau xy2min := \tau 2 - Ss2 \quad \tau xy2max := \tau 2 + Ss2$

$$\tau xy2mean := \frac{\tau xy2max + \tau xy2min}{2} \quad \tau xy2a := \tau xy2max - \tau xy2min \quad \tau xy2mean = 0 \cdot ksi \quad \tau xy2a = 8.771 \cdot ksi$$

$\sigma x2min := -\sigma b2 - \sigma a2 \quad \sigma x2max := \sigma b2 - \sigma a2$

$$\sigma x2mean := \frac{\sigma x2max + \sigma x2min}{2} \quad \sigma x2a := \frac{\sigma x2max - \sigma x2min}{2} \quad \sigma x2mean = -1.0196 \cdot ksi \quad \sigma x2a = 13.743 \cdot ksi$$

$\sigma y2mean := 0 \quad \sigma y2a := 0$

Von Mises Failure Criteria:

$$\tau xy2mean := \left[\left(\frac{\sigma x2mean - \sigma y2mean}{2}\right)^2 + \tau xy2mean^2\right]^{.5} \quad \tau xy2mean = 509.82 \cdot psi$$

$$\tau xy2a := \left[\left(\frac{\sigma x2a - \sigma y2a}{2}\right)^2 + \tau xy2a^2\right]^{.5} \quad \tau xy2a = 8771.0052 \cdot psi$$

$$\sigma 12mean := \frac{\sigma x2mean + \sigma y2mean}{2} + \tau xy2mean \quad \sigma 22mean := \frac{\sigma x2mean - \sigma y2mean}{2} - \tau xy2mean \quad \sigma 12mean = 0 \cdot psi$$

$\sigma 22mean = -1019.6401 \cdot psi$

$$\sigma vm2mean := \left(\sigma 12mean^2 - \sigma 12mean \cdot \sigma 22mean + \sigma 22mean^2\right)^{.5} \quad \sigma vm2mean = 1.0196 \cdot ksi$$

$$\sigma 12a := \frac{\sigma x2a + \sigma y2a}{2} + \tau xy2a \qquad \sigma 22a := \frac{\sigma x2a}{2} - \tau xy2a$$

$$\sigma 12a = 18013.6482 \cdot psi$$

$$\sigma 22a = -4270.6803 \cdot psi$$

$$\sigma vm2a := \left(\sigma 12a^2 - \sigma 12a \cdot \sigma 22a + \sigma 22a^2\right)^{.5}$$

$$\sigma vm2a = 20.4856 \cdot ksi$$

Stress concentration at the wheel interface.

$$Kt2 := 2$$

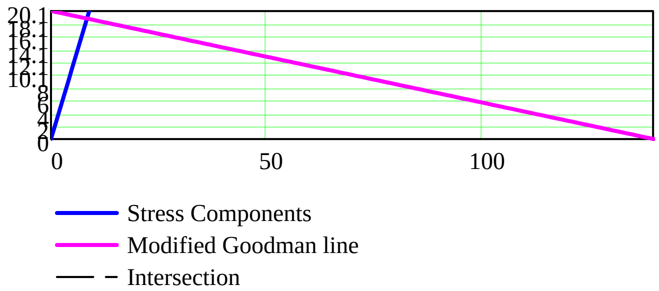
$$Ke2 := \frac{1}{1 + q \cdot (Kt2 - 1)} \qquad Ke2 = 0.521$$

$$Se2 := Se \cdot Ke2 \qquad Se2 = 22.6426 \cdot ksi$$

Modified Goodman Line $\mathbf{i} := 0 .. 150$ $\mathbf{x_i} := \mathbf{i}$ $\mathbf{y2_i} := \frac{-\mathbf{Se2} \cdot \mathbf{x_i}}{\mathbf{Fut}} + \frac{\mathbf{Se2}}{\mathbf{ksi}}$ $\mathbf{Sm2} := \frac{\mathbf{Se2}}{\left(\frac{\mathbf{Se2}}{\mathbf{Fut}} + \frac{\sigma\mathbf{vm2a}}{\sigma\mathbf{vm2mean}}\right)}$

$$\mathbf{ymg2_i} := \frac{\sigma\mathbf{vm2a} \cdot \mathbf{x_i}}{\sigma\mathbf{vm2mean}}$$

$$\mathbf{Sa2} := \frac{\sigma\mathbf{vm2a} \cdot \mathbf{Sm2}}{\sigma\mathbf{vm2mean}}$$

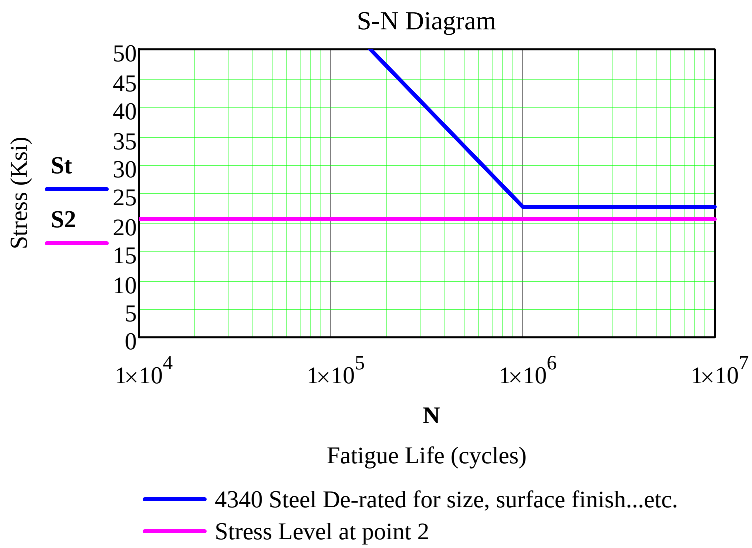


$\mathbf{St_0} := \frac{\mathbf{Fut}}{\mathbf{ksi}}$ $\mathbf{St_1} := .9 \cdot \frac{\mathbf{Fut}}{\mathbf{ksi}}$ $\mathbf{St_2} := \frac{\mathbf{Se2}}{\mathbf{ksi}}$ $\mathbf{St_3} := \frac{\mathbf{Se2}}{\mathbf{ksi}}$

$\mathbf{N_0} := 100$ $\mathbf{N_1} := 1000$ $\mathbf{N_2} := 1000000$ $\mathbf{N_3} := 10000000$

$\mathbf{i} := 1 .. 4$

$$\mathbf{S2_i} := \frac{\sigma\mathbf{vm2a}}{\mathbf{ksi}}$$



Fatigue at Point 3

Maximum Shear Stress (Torque + Shear)

$\tau xy3min := \tau 3 - Ss3 \qquad \tau xy3max := \tau 3 + Ss3$

$\tau xy3mean := \dfrac{\tau xy3max + \tau xy3min}{2} \qquad \tau xy3a := \tau xy3max - \tau xy3min \qquad \tau xy3mean = 2.3932 \cdot ksi \quad \tau xy3a = 8.771 \cdot ksi$

$\sigma x3 = 4595.3181 \cdot psi \qquad \sigma x3min := -\sigma x3 \qquad \sigma x3max := \sigma x3$

$\sigma x3mean := \dfrac{\sigma x3max + \sigma x3min}{2} \qquad \sigma x3a := \dfrac{\sigma x3max - \sigma x3min}{2} \qquad \sigma x3mean = 0 \cdot ksi \qquad \sigma x3a = 4.5953 \cdot ksi$

$\sigma y3mean := 0 \qquad \sigma y3a := 0$

Von Mises Failure Criteria:

$\tau xy3mean := \left[\left(\dfrac{\sigma x3mean - \sigma y3mean}{2}\right)^2 + \tau xy3mean^2\right]^{.5} \qquad \tau xy3mean = 2393.1556 \cdot psi$

$\tau xy3a := \left[\left(\dfrac{\sigma x3a - \sigma y3a}{2}\right)^2 + \tau xy3a^2\right]^{.5} \qquad \tau xy3a = 8771.0052 \cdot psi$

$\sigma 13mean := \dfrac{\sigma x3mean + \sigma y3mean}{2} + \tau xy3mean \qquad \sigma 23mean := \dfrac{\sigma x3mean - \sigma y3mean}{2} - \tau xy3mean \qquad \sigma 13mean = 2393.1556 \cdot psi$

$\sigma 23mean = -2393.1556 \cdot psi$

$\sigma vm3mean := \left(\sigma 13mean^2 - \sigma 13mean \cdot \sigma 23mean + \sigma 23mean^2\right)^{.5} \qquad \sigma vm3mean = 4.1451 \cdot ksi$

$$\sigma 13a := \frac{\sigma x3a + \sigma y3a}{2} + \tau xy3a \qquad \sigma 23a := \frac{\sigma x3a}{2} - \tau xy3a$$

$$\sigma 13a = 11364.6194 \cdot psi$$

$$\sigma 23a = -6769.3013 \cdot psi$$

$$\sigma vm3a := \left(\sigma 13a^2 - \sigma 13a \cdot \sigma 23a + \sigma 23a^2\right)^{.5}$$

$$\sigma vm3a = 15.8716 \cdot ksi$$

$$Kt3 := 2$$

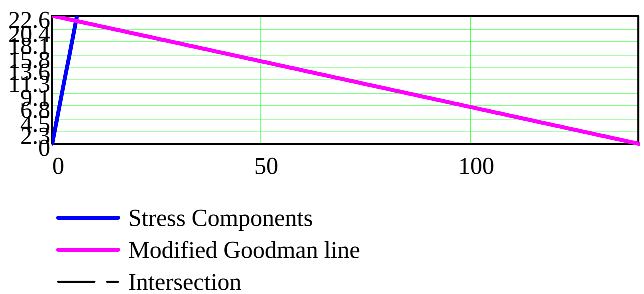
$$Ke3 := \frac{1}{1 + q \cdot (Kt3 - 1)} \qquad Ke3 = 0.521$$

$$Se3 := Se \cdot Ke3 \qquad Se3 = 22.6426 \cdot ksi$$

Modified Goodman Diagram

Modified Goodman Line $\quad i := 0 .. 150 \quad x_i := i \quad y3_i := \frac{-Se3 \cdot x_i}{Fut} + \frac{Se3}{ksi} \quad Sm3 := \frac{Se3}{\left(\frac{Se3}{Fut} + \frac{\sigma vm3a}{\sigma vm3mean}\right)} \quad Sa3 := \frac{\sigma vm3a \cdot Sm3}{\sigma vm3mean}$

$$ymg3_i := \frac{\sigma vm3a \cdot x_i}{\sigma vm3mean}$$



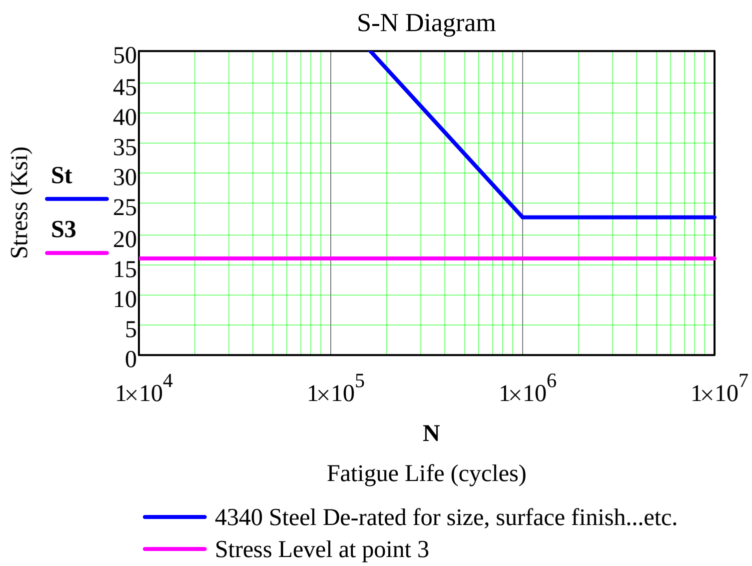
$Sm3 = 5.6737 \cdot ksi \qquad Sa3 = 21.725 \cdot ksi$

$St_0 := \frac{Fut}{ksi} \quad St_1 := .9 \cdot \frac{Fut}{ksi} \quad St_2 := \frac{Se3}{ksi} \quad St_3 := \frac{Se3}{ksi}$

$N_0 := 100 \quad N_1 := 1000 \quad N_2 := 1000000 \quad N_3 := 10000000$

$i := 1 .. 4$

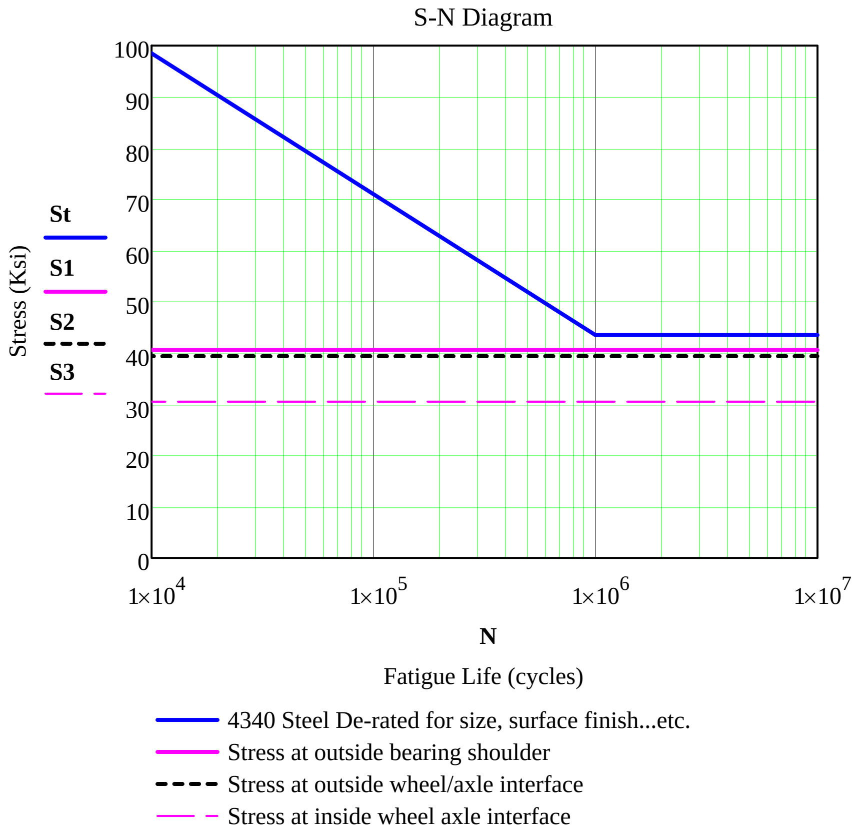
$S3_i := \frac{\sigma vm3a}{ksi}$



$\mathbf{St}_0 := \frac{\mathbf{Fut}}{\mathbf{ksi}}$ $\mathbf{St}_1 := .9 \cdot \frac{\mathbf{Fut}}{\mathbf{ksi}}$ $\mathbf{St}_2 := \frac{\mathbf{Se}}{\mathbf{ksi}}$ $\mathbf{St}_3 := \frac{\mathbf{Se}}{\mathbf{ksi}}$

$\mathbf{N}_0 := 100$ $\mathbf{N}_1 := 1000$ $\mathbf{N}_2 := 1000000$ $\mathbf{N}_3 := 10000000$

$\mathbf{i} := 1 .. 4$ $\mathbf{S1}_\mathbf{i} := \frac{\mathbf{\sigma vm1a}}{\mathbf{Ke1 \cdot ksi}}$ $\mathbf{S2}_\mathbf{i} := \frac{\mathbf{\sigma vm2a}}{\mathbf{Ke2 \cdot ksi}}$ $\mathbf{S3}_\mathbf{i} := \frac{\mathbf{\sigma vm3a}}{\mathbf{Ke3 \cdot ksi}}$



$\mathbf{St}_3 = 43.4739$

$\mathbf{S1}_1 = 40.5474$

$\mathbf{S2}_1 = 39.3324$

$\mathbf{S3}_1 = 30.4735$

B - Loc Calculations

Series B112 pn B121220

$\mathbf{Lc} := 134 \cdot \mathbf{mm}$ Contact Length

$\mathbf{Dhb} := 260 \cdot \mathbf{mm}$ Outside diameter

$\mathbf{Dshaft} := 200 \cdot \mathbf{mm}$ Shaft diameter

Minimum Hub diameter

$\mathbf{HW} := 7.25 \cdot \mathbf{in}$ Hub width

Hub contact Ratio From Catalog

$\mathbf{Hrat} := \frac{\mathbf{HW}}{\mathbf{Lc}}$ $\mathbf{Hrat} = 1.3743$ $\mathbf{Csr} := .8$

$\mathbf{Ph} := 19870 \cdot \frac{\mathbf{lbf}}{\mathbf{in}^2}$

$\mathbf{Syhub} := 65000 \cdot \frac{\mathbf{lbf}}{\mathbf{in}^2}$ Yield strength of hub

$\mathbf{Dn} := \mathbf{Dhb} \cdot \left[\frac{\mathbf{Syhub} + (\mathbf{Ph} \cdot \mathbf{Csr})}{\mathbf{Syhub} - (\mathbf{Ph} \cdot \mathbf{Csr})} \right]^{.5}$ $\mathbf{Dn} = 13.1385 \cdot \mathbf{in}$

Radial load capacity

$\mathbf{Pshaft} := \mathbf{Ph} \cdot \frac{\mathbf{Dhb}}{\mathbf{Dshaft}}$ $\mathbf{Pshaft} = 25831 \cdot \frac{\mathbf{lbf}}{\mathbf{in}^2}$

$\mathbf{Frad} := \mathbf{Dshaft} \cdot \mathbf{Lc} \cdot \left(\mathbf{Ph} \cdot \frac{\mathbf{Dhb}}{\mathbf{Dshaft}} \right)$ $\mathbf{Frad} = 1.073 \times 10^6 \cdot \mathbf{lbf}$

Outside Bearing Life Calculations:

$\mathbf{hours} := 3600 \cdot \mathbf{sec}$

$\mathbf{years} := 8760 \cdot \mathbf{hours}$

$\mathbf{R} := \mathbf{R2y}$ Radial Load

$\mathbf{T} := \mathbf{R2x}$ Thrust Load

$\mathbf{Nr} := .2$ Operating Speed (rpm)

The operational speed is unknown. The value above were used for comparative purposes.

Original VLBA Outside Bearing

PN 23038 **Bearing Number**

$\mathbf{e} := .23$ Load Test

$\mathbf{X}_0 := 1 \quad \mathbf{X}_1 := .67$ Radial Load Factor

$\mathbf{Y}_0 := 2.69 \quad \mathbf{Y}_1 := 4$ Thrust Load Factor

$\mathbf{C} := 164100 \cdot \mathbf{lbf}$ Dynamic Load Rating

$$\mathbf{i} := \left(\frac{\mathbf{T}}{\mathbf{R}} > \mathbf{e}\right) \qquad \mathbf{i} = 1$$

$$\mathbf{P} := \mathbf{X}_{\mathbf{i}} \cdot \mathbf{R} + \mathbf{Y}_{\mathbf{i}} \cdot \mathbf{T}$$ Equivalent Load

$$\mathbf{P} = 283962 \cdot \mathbf{lbf}$$

$$\mathbf{L}_{10} := \frac{16667 \cdot \mathbf{hours}}{\mathbf{Nr}} \cdot \left(\frac{\mathbf{C}}{\mathbf{P}}\right)^{3.33333}$$

Expected minimum life for 90% of the bearings in a given population.

$$\mathbf{L}_{10} = 1.5 \cdot \mathbf{years}$$

Jim Ruff Design Outside Bearing

PN 23232 **Bearing Number**

$e := .34$ Load Test

$X_0 := 1 \quad X_1 := .67$ Radial Load Factor

$Y_0 := 1.96 \quad Y_1 := 2.91$ Thrust Load Factor

$C := 245900 \cdot lbf$ Dynamic Load Rating

$$i := \left(\frac{T}{R} > e\right) \qquad i = 1$$

$P := X_i \cdot R + Y_i \cdot T$ Equivalent Load

$P = 7396743\, ft \cdot s^{-2} \cdot lb$

$$L_{10} := \frac{16667 \cdot hours}{Nr} \cdot \left(\frac{C}{P}\right)^{3.33333}$$

Expected minimum life for 90% of the bearings in a given population.

$L_{10} = 11.9 \cdot years$

New Design Outside Bearing

PN 24136 **Bearing Number**

$e := .37$ Load Test

$X_0 := 1 \quad X_1 := .67$ Radial Load Factor

$Y_0 := 1.84 \quad Y_1 := 2.74$ Thrust Load Factor

$C := 258400 \cdot lbf$ Dynamic Load Rating

$$i := \left(\frac{T}{R} > e\right) \qquad i = 1$$

$P := X_i \cdot R + Y_i \cdot T$ Equivalent Load

$P = 7125449\, ft \cdot s^{-2} \cdot lb$

$$L_{10} := \frac{16667 \cdot hours}{Nr} \cdot \left(\frac{C}{P}\right)^{3.33333}$$

Expected minimum life for 90% of the bearings in a given population.

$L_{10} = 15.9 \cdot years$